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The measure of friction angles for different types of granular material

DEGANUTTI Andrea Maria[1] https://orcid.org/0000-0002-9882-5566; e-mail: andrea.maria.deganutti@irpi.cnr.it

TECCA Pia Rosella[1] https://orcid.org/0000-0001-5568-3169; e-mail: pia.tecca@irpi.cnr.it

GENEVOIS Rinaldo[2] https://orcid.org/0000-0001-7065-8500; e-mail: rinaldo.genevois@unipd.it

* *Corresponding author*

1 CNR-IRPI, Corso Stati Uniti 4, 35127 Padova, Italy

2 Department of Geosciences, University of Padova, Via Gradenigo 6, 35131 Padova, Italy

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Abstract: The aim of this research is to deepen the knowledge of the role of friction on the dynamics of granular media; in particular the friction angle is taken into consideration as the physical parameter that drives stability, motion and deposition of a set of grains of any nature and size. The idea behind this work is a question: is the friction angle really that fundamental and obvious physical parameter which rules stability and motion of granular media as it seems from most works which deal with particle dynamics? The experimental study tries to answer this question with a series of laboratory tests, in which different natural and artificial granular materials have been investigated in dry condition by means of a tilting flume. The characteristic friction angles, both in deposition (repose) and stability limit (critical) conditions, were measured and checked against size, shape, density and roughness of the considered granular material. The flume tests have been preferred to "classical" geotechnical apparatuses (e.g. shear box) since the flume experimental conditions appear closer to the natural ones of many situations of slope stability interest (e.g. a scree slope). The results reveal that characteristic friction angles depend on size and shape of grains while mixtures of granules of different size show some sorting mechanism with less clear behaviour.

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Introduction

Virtually any field of interest of the earth sciences deals, more or less often, with the concept of friction and closely linked, of friction angle. In particular, with reference, to granular media, the friction angle is among the parameters that rule the initiation, the dissipation of energy during motion and the deposition processes, with or without the presence of an interstitial fluid.

Very often in literature values for friction angles are taken as standard, kind of some general knowledge, by many scholars in geomorphological studies and in numerical models applied to geological and rheological processes. The term and concept of friction angle comes from the empirical description of friction given by Coulomb where the tangent of the maximum angle (generally indicated with the Greek letter Φ) of stability for a body on a inclined plane is postulated as the friction coefficient for the two surfaces in contact (a simple, and useful, relation for a complex phenomenon):

$$\tau = \sigma \tan \Phi \quad (1)$$

where τ, σ respectively shear and normal stresses acting between the sliding surface and the sliding body; tanΦ = tangent of the maximum angle of stability for a body on a inclined plane.

In spite the fact that the ability of granular particles to move sliding and rotating one in contact (with the influence of the packing stage) makes things much more complicated than the simple case of a block slide, this equation has been extended to granular material (e.g., a classical work with reference to soil, Casagrande 1936). But nomenclature problems soon arose: especially among geomorpholigists, the friction angle for natural granular deposits (especially talus slopes) is often referred as "repose angle" but also, especially by civil engineers, as the angle of shearing resistance of granular material in a loose state of packing (Zhou et al. 2002) . Then again, for the maximum gradient of a grain mass in stationary conditions the friction angle is again termed repose (Rousé 2014) and (or) static (in analogy with the block slide) or also angle of initial yield (Allen 1969), in contrast with the gradient assumed by the material when it stops after a phase of motion (e.g. deposition angle, residual angle after shearing, also dynamic friction angle). On the other hand, scholars, depending on their particular field of interest, often make no difference between these two limit conditions of stability for granular material, terming one or the other in various ways with a terminology with some ambiguity (e.g. again: repose, dynamic, static, critical, internal, natural, stopping, residual, neutral, bulk, etc.) (e.g. Carson 1977; Holz and Kovacs 1981; Pouliquen and Renaut 1996; Calvetti et al. 2000; Pudasaini and Hutter 2007; Fischer et al. 2008).

Nevertheless, when dealing with granular material, the friction angle is a useful "tool" given it can be considered as a global measure of the various effects that affect the stability and, as a consequence, the motion of a lump of a large number of bodies in contact as it takes into account the whole set of physical factors involved (inter-particle surface friction, cohesion (if any) shape factor...). Nevertheless, in spite of its usefulness, there is not a standardized procedure to measure the two characteristic friction angles: "static" and "dynamic" with reference to the two limit conditions for granular material as described above. This is due, at least in part, to the fact that authors who worked on granular media have focused their studies on particular phenomena in which matter in grains was involved, so experimental or field conditions and relative apparatuses generally varied greatly from a study to another. As a consequence, the values of measured friction angles for the considered material in field or laboratory situations are generally rather variable, up to more than 30% (Metcalf 1965; Bagnold 1966; Cundall and Strack 1979; Hungr and Morgenstern 1984; Metha and Barker 1994; Rousé 2014; Montanari et al. 2017).

For example, for most geomorphologists the angle of repose of talus scree corresponds to the straight part of talus slope profiles but, on experimental side, different test procedures produced highly variable results being the measured friction angles affected by the procedures besides the material itself. While geologists measure the angle of repose by means of static (fixed height cone - Cornforth method, fixed base cone etc.) and dynamics (rotating drum, titling table), geotechnical engineers use the standard direct shearbox or the ring shearbox (Hubbert apparatus) or by triaxial tests and moreover, the rotating drum is becoming more popular among granular media scholars as an easily standardisable testing method (Powers 1953; Carson 1977; Statham 1977; Lumay et al. 2012; Rousé 2014; Montanari et al. 2017).

For these reasons, in literature a univocal and unambiguous definition of friction angle for granular material in the different fields of application does not exist and the terminology related to friction of granular material and to friction angle abounds without any standardization of field or laboratory measure procedures.

Focusing the attention on geological studies on natural hazards in mountainous environment, many scholars (Statham 1976; Santomaso et al. 2003) have indicated that granular materials typically display two "angles of repose": a steeper angle at which movement of the debris begins (initial yield angle), and a gentler angle at which mass movement ceases (angle of rest). However, traditional usage of the term “angle of repose” commonly refers to the angle of accumulation of a cone built up by mass supply from above after an avalanching has ceased (Carson 1977) so considering this procedure as a sort of physical model of a natural scree slope.

In engineering geology the problem of the friction angles of natural granular material received particular attention in recent years for their importance in the development of numerical models for debris avalanche and debris flow runout prediction (Hungr et al. 2002; Rickenmann 2005; Zhang et al. 2011; Zhang and Yin 2013; De Blasio and Crosta 2014, 2015; Perinotto et al. 2015; Zhang et al. 2016; Zhang and McSaveney 2017) and field researchers proposed some procedures for friction angle measure (e.g. Metcalf 1965; Hutter and Koch 1991; Pudasaini and Hutter 2007).

In the present work we consider a critical (or yielding) friction angle, defined as the gradient at which a granular mass becomes unstable and starts to slide and a deposition (or repose) friction angle, defined as the uniform angle to the horizontal assumed by the free surface of a mass of granular material which arranges itself on a inclined slope when it comes at rest after a slide of the whole mass.

Deganutti and Scotton (1997) proposed a measure of critical friction angles for movement initiation and for movement ceasing for a mass of 2-3 mm PVC grains in dry and wet conditions by means of a rotative cone and plate rheometer in controlled normal stress conditions. The aim of the present study is to present a testing procedure closer to the natural field situation, e.g. a scree talus. Various laboratory techniques have been proposed to study the motion and the friction characteristic of granular material (rotating drums e.g. Santomaso et al. 2003; Yan Liu et al. 2005); chutes (Savage 1979), with the general aim to study the dynamics of a granular mass in motion (mostly constituted by artificial granules). Nevertheless, when referring to loose granular rock materials (such as scree) on a slope, where the material is potentially prone to originate debris flows, the study of yielding condition has to take into consideration the superficial instability, where there is no confinement pressure and the granules are free to move and to dilate (Reynolds 1885).

Recent developments (Daerr and Douady 1999; Pouliquen 1999; GDR MiDi 2004; Jop et al. 2006) focused on mechanics of dense granular flows both from theoretical and experimental point of view, taking into consideration artificial granular material (generally glass beads) in some particular laboratory conditions (steady flows, monodisperse grain size distribution etc.), giving to those studies characteristics which are far from the natural conditions of a scree slope (which, for its strict relation to dangerous instability phenomena, is our centre of interest).

The dynamic effects during dry granular flow are essentially given by the friction among grains and the exchange of momentum due to their collisions; these two effects together, with a larger importance of the first one in case of slow movement, rule the rolling-sliding interactions of moving granular particles.

Friction and interlocking of grains rule the critical yielding angle at the onset of instability, while the energy dissipation due to collisions has a more important role for the movement ceasing-deposition stage.

It must be said that the granular materials used for the tests are not intended to reproduce in laboratory the behaviour of natural granular mixtures in the field, which are widely variable in size and properties (cohesion, shape etc.) since the purpose of the research is to deepen the rheological knowledge on the behaviour of granular masses with particular attention on the phases of onset of instability and the final stage of deposition.

Both phenomena are of special interest for debris flows numerical modellers who need rheological knowledge when dealing with the simulation of initiation and deposition processes of debris flows.

The use of granular material different in size, shape and density gives anyway a clue to the effect of these parameters on critical and deposition angles for natural materials in the field.

In order to test the effect of a vertical load on the used material and to get a comparison between the friction angles measured with the presented procedure and those measured with a well-established geotechnical method, some shear box tests have performed.

1 Experimental Techniques and Procedures

When studying the physic-mechanical characteristics of natural materials by means of laboratory techniques, a problem arises in understanding whether a property exhibited by a material reflects a true feature of the mechanical behaviour of that material in the field or is an experimental artefact coming from the particular interaction between the used apparatus and the material itself. Trying to keep our tests as much as possible close to the field conditions, we used a tilting plane where the material was free to slide under gravity, reducing to a minimum the effects coming from apparatus and testing procedures as the motion of grains depends only on gravity and the interaction with the other grains.

1.1 Experimental setup

Figure 1 shows the setup of the experimental apparatus used for this study. It is constituted by a 2 m × 1.5 m tilting plane with an adjustable slope from 0° to 38°; a 23.2 cm wide flume is mounted on the plane. The bottom surface of the flume is roughened with coarse sand paper in order to prevent slippage of the granular mass on it, in which case the mass movement would be, at least partly, a translation of a semi-rigid body along a surface, while this study is focused on the interactions among particles and their effect on the dynamics of flowing granular material in quasi-static conditions.

The tilting plane is hinged to a fixed horizontal plane (1.5 m long and 1.5 m wide) where the material deposits when it exits from the flume.

The tilting movement of the plane is controlled by a synchronous electric motor which provides a slow constant rate of tilting (around 2.5×10^{-3} rad s^{-1}) through rotating screwed bar; this system has proved to be very good for the smoothness of movement and for the absence of vibrations on the plane itself, both characteristics of great importance when testing the features of the flow of granular material.

The depositional features of the granular material were recorded by means of a laboratory class laser-scanner, installed through a spherical joint to a movable and adjustable frame (Figure 1).

The laboratory laser-scanner scanned at high precision (recording accuracy of 0.38 mm) the surface of the granular mass yielding a 3D restitution of it. From a simple elaboration of those data, the average surface angle to the horizontal was obtained. The irregularities on the grain surfaces were generally within 2 particle diameters.

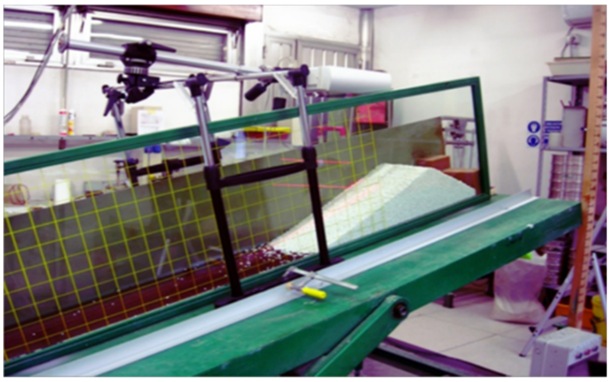

Figure 1 Flume and laser-scanner apparatus while probing the material surface.

1.2 Methods

In literature various methods to measure critical angles for granular material have been proposed, methodologies often subject to uncertainties derived from not well controlled experimental conditions, e.g. pouring material on a surface and measuring the slope of the obtained heap, not taking into account that the resulting angle of deposition/repose is very sensitive to the method used to create the heap (Lumay et al. 2012).

Otherwise, in order to measure the characteristic friction angles of granular material in a more controlled and reproducible way, we adopted, as mentioned above and given the available laboratory facilities, a method based on the idea to reach the material critical conditions as much as possible close to the natural ones. The procedure was to accumulate a heap of dry material in the rising end of the flume, then the plane was slowly and continuously tilted until the material slided; at this time frame the flume tilting motor was stopped and the slope of the flume was measured.

The downward angle of the material surface after the first sliding was not taken into consideration as in that case the starting condition was "unnatural" (the material was initially simply poured in the flume) and the resulting heap was in a loose and casual state.

After the first slide, the flume was increasingly tilted up in order to cause a second material sliding; the flume slope increase necessary to cause the new instability of material was measured and its value added to the previous deposition angle. The resulting value gives the critical angle for stability of the material under test.

The uniform surfaces assumed by the granular mass after each slide (Figure 2), have been measured by a least squares interpolation of the mathematical surface recorded by the laser scanner (Figure 3), along the middle longitudinal section, in order to minimize possible disturbances due to "side-effects" of flume walls (effect seen only in case of very fine material).

A double-check of angle measurements was accomplished taking a digital photo of the material sidewise normal to the glass wall of the flume and then treated by means of a photo processing software.

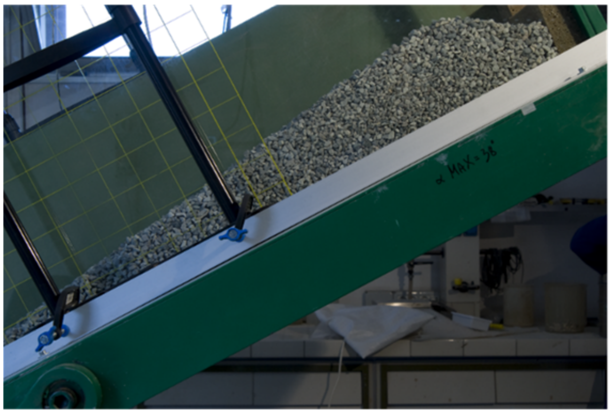

Figure 2 Uniform surface assumed by the granular mass (material *m13*) after the fifth slide.

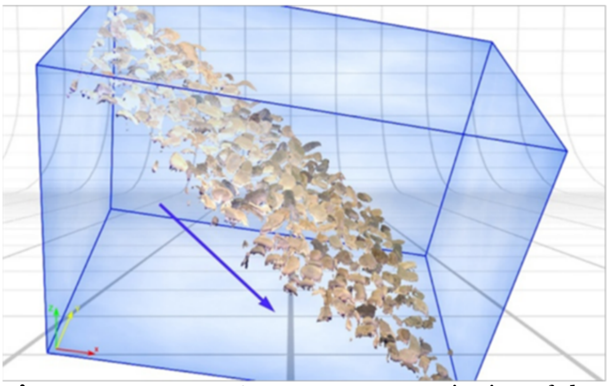

Figure 3 3-D Laser Scanner survey restitution of the surface of material *m3* after a slide in the flume.

1.3 Materials

Fourteen types of granular cohesionless material, synthetic and natural, different in density, size and shape, have been tested in dry condition. In order to check the effect of grain size, mixtures of materials of different size have been tested as well.

Eight size classes, labelled with a letter from *a* to *h*, have been chosen to characterize the tested materials from the size point of view; materials themselves have been named from *m1* to *m17* and their main characteristics are presented in Table 1.

Materials *m1* and *m2* are PVC cylindrical granules whose density is respectively 1200 and 1500 kg m^{-3}.

From *m3* to *m8* and *m15* materials are constituted by limestone, rounded river gravel with equivalent sizes ranging from 4.0 to 31.5 mm and a density of 2700 kg m^{-3}.

Materials from *m9* to *m11* and *m13* are coarse-grained sub-angular gravel, obtained by limestone rock crushing, with equivalent sizes ranging from 4.0 to 16.0 mm and the same density of the rounded gravel above.

Materials *m12*, *m14* and *m16* consist of graded sands with equivalent sizes ranging from 0.075 to 2.0 with fines (less than 5%-6%); *m14* represents the finer fraction of *m12*.

Materials *m8*, *m11* and *m12* result by a mixing of different grain size classes as follows: *m8* by 20% each of (values in mm) *a* (22.4 to 31.5), *b* (16.0 to 22.4), *c* (11.2 to 16.0), *d* (8.0 to 11.2) and *e* (4.0 to 8.0) size classes; *m11* by 50% of *c* (11.2 to 16.0) and *d* (8.0 to 11.2) size classes; *m12* by 50% of *g* (0.5 to 2.0) and *h* (<0.5 mm) size classes.

Material *m17* is a coarse grained material obtained by mixing at 50% rounded and sub-angular gravels with a grain size distribution from 2.0 to 4.0 mm (size class *f*).

2 Results

A total of 143 tests have been done; tests with a material were repeated at least twice in the same experimental conditions in order to check test repeatability.

Table 2 reports, for every tested material, the average values of the critical and repose angles and their differences; these delta values can be considered as a measure of the yield strength of the various granular media.

The standard deviation for the values of measured angles is less than one degree for about 60% of tests performed for every material. Repeated tests in the same experimental conditions gave differences generally smaller than 2 degrees,

Table 1 Parameters for tested material

Material	Type	Size (mm)	Size class
m1	Cylindrical black PVC grains	$2r$ = 3.1, h = 3	
m2	Cylindrical white PVC grains	Same size as m1	
m3	Rounded gravel	22.4 < d < 31.5	*a*
m4	Rounded gravel	16.0 < d < 22.4	*b*
m5	Rounded gravel	11.2 < d < 16.0	*c*
m6	Rounded gravel	8.0 < d < 11.2	*d*
m7	Rounded gravel	4.0 < d < 8.0	*e*
m8	Rounded gravel	4.0 < d < 31.5	*a+b+c+d+e*
m9	Subangular gravel	11.2 < d < 16.0	*c*
m10	Subangular gravel	8.0 < d < 11.2	*d*
m11	Subangular gravel	8.0 < d < 16.0	*c+d*
m12	Sand	d < 2.0	*g+h*
m13	Subangular gravel	4.0 < d < 8.0	*e*
m14	Sand	d < 0.5	*h*
m15	Rounded gravel	4.0 < d < 8.0	*e*
m16	Coarse sand	0.5 < d < 2.0	*g*
m17	Mixed shape gravel	2.0 < d < 4.0	*f*

Note: $2r$, h: cylinder diameter and height; d: particle equivalent diameter.

Table 2 Average values of deposition and critical angles

Material	Material type	Size class	Mean Dep angle (°)	Mean Crit angle (°)	Angle Delta (°)
m1	PVC LD		31.0	37.2	6.2
m2	PVC HD		28.6	35.2	6.6
m3	Rounded river gravel	*a*	35.0	40.8	5.8
m4		*b*	35.1	39.9	4.8
m5		*c*	33.7	38.4	4.7
m6		*d*	33.2	37.5	4.3
m7		*e*	31.5	36.2	4.7
m15		*e*	30.9	34.0	3.1
m8		*a+b+c+d+e*	34.0	36.8	2.8
m9	Angular gravel	*c*	35.3	42.2	6.9
m10		*d*	35.6	39.4	3.8
m13		*e*	35.2	39.1	3.8
m11		*c+d*	36.0	40.3	4.3
m17	Mixed shape	*f*	30.8	34.3	3.5
m12	Sand	*g+h*	Irregular behaviour		
m14		*h*	Irregular behaviour		
m16		*g*	32.1	34.4	2.3

Note: Dep: Deposition; Crit: Critical; Angle Delta: Critical- Deposition.

value which is about our apparatus and measurement system accuracy range.

For cross-experimental systems comparison, a series of direct shear tests were conducted using direct shear box apparatus to determine the consolidated-drained shear strength of some of the materials tested in the flume (Standard Reference: ASTM D 3080 - Standard Test Method for Direct Shear Test of Soils Under Consolidated Drained Conditions): PVC materials (*m1* and *m2*), coarse natural sands (*m16*) and mixed rounded and sub-angular gravels (*m17*) have been tested.

Mohr's failure envelopes have been obtained using the peak stresses recorded by shear box tests. PVC (*m1* and *m2*) materials gave quasi-static friction angles ranging as a mean from 30.6° to 33.9°, differences being due mainly to difficulties in obtaining the same initial bulk density and to the distribution of particles main axis with respect to the shear plane. High values of the quasi-static friction angles have been obtained for *m16* (35.0°) and *m17* (46.8°) materials. In these cases, however, the materials showed a contractive behaviour during shear that increased the initial bulk density up to quite high values and this in spite of the relatively high value of the normal stress (50 kPa). The same contractive behaviour has been found with similar natural material in field conditions in Acquabona site (Genevois et al. 2000).

A series of flume tilting tests for some materials using a different volume of the granular mass have been conducted in the same experimental conditions, in order to check a possible "size effect" due to the volume of the tested mass on the resulting limit angles. However, such effect was found irrelevant.

3 Discussion

The tests showed a regular behaviour of the granular material characterized by very well defined planar configuration (i.e. uniform plane surface) after slides with constant (in cited limits) deposition and critical angles (Figures 1 and 2). Nevertheless materials *m12* and *m14*, formed by a mixture of sand and finer particles, showed an irregular behaviour with a mechanism of separation of grain sizes: when the bigger particles slided, the finer grains remained in their previous position with a slope steeper than the overlying sand grains, forming an inside separation surface of irregular shape (Figure 4). Our results would suggest some

Figure 4 Irregular surface assumed by the granular mass (material *m14*) after the fifth slide; black dashed and dotted line: separation surface between sand and finer grains.

sorting mechanism, kind of "Brazilian nut effect" (Herrmann & Luding 1998).

The lowest deposition angle was showed by high density PVC (28.6°) probably because of the smoothness of grain surface and ~~to~~ the cylindrical shape, giving to the grains themselves a low inter-particle friction resistance and some ability to rotate.

Surprisingly, the lowest critical (yielding) angle is not given by the high density PVC as expected, but by the material *m15* of size class *e* (4.0 < *d* < 8.0) rounded gravel with 34.0°, against 35.2° of high density PVC.

This fact shows that the difference between the two characteristic angles ("Delta" angle) is not constant for different grain size and this states the complexity of the involved dynamic effects, in spite of the simplicity of the performed tests.

This result can be considered another proof that the yielding and deposition are physically different processes (e.g. Costa 1984; Pouliquen 1999; Benda and Cundy 1990; Imaizumi et al. 2017) and that it is generally wrong to consider a single generic "friction angle" to characterize both the yielding and deposition processes of granular material.

Longo and Lamberti (2002), in experiments conducted with a rotating drum to measure the angle of repose, use both terms, static and dynamic friction. For the initiation of a granular flow, the static angle of repose, greater than the dynamic angle, must be exceeded. The mobilization of the material into a flow necessarily involves dilation, which reduces the number of contacts and therefore the contact forces. Once a granular mass is moving, the momentum and reduced friction cause it to run out below the static angle of repose, to reach a dynamic angle. The force needed to overcome the static friction is greater than the one needed to keep on the motion, so the coefficient of kinetic friction is lower than the one of static friction. Our tests confirm Longo & Lamberti findings even if the experimental conditions were very different.

The highest value of critical angle was showed by material m9 of size class c (11.2<*d*<16.0) subangular gravel, showing a high resistance to yielding (given its angular shape). Its deposition angle (35.3°) is just slightly higher than that of rounded gravel of the same size (33.7°), suggesting that the grain shape is relatively less important in the deposition process than in the ceasing one.

The size of grains of similar shape has the clear effect to give a higher stability to bigger grains: with size going from 4 to 31.5 mm, values of critical angle for rounded gravel go progressively from 34.0° to 40.8°. The same gravel displays the same effect for the deposition angle as well, even if with slighter variations. Consequently, the difference between yielding and deposition angles decreases with size. This result is consistent with the experimental studies of Cagnoli and Piersanti (2015), who found that the mobility of a granular mass increases as grain size decreases.

The subangular gravel has a similar behaviour with regard to yielding angles, while its deposition angles seem to be unaffected by size. The influence of grain shape on granular flow has been studied by different authors (e.g. Petit et al. 2001).

Tests with a mixing of different grain size gave angle values which are in the range of the single size classes, but in different order: (the deposition angle was close to the lower ones and the yielding angle was close to the higher) ones. The result is that the angles "Delta" of some materials, e.g. *m8*, resulting from a mixture of 20% in weight from *a*, *b*, *c*, *d*, e class sizes, is the lowest for rounded river gravel.

Again, mixed subangular gravel (*m11*) showed a behaviour similar to river gravel, with smaller variations.

No sorting/separation effects were noted for mixtures of different size grains, given the quasi-static experimental conditions, with the exception of mixed sands.

The tests with PVC granules showed that the density of material has not a clear role in the yielding and deposition processes since the low-density PVC has angles higher than those of the high-density PVC, both for critical and deposition conditions. In Author's opinion, this occurrence may be explained by the higher friction showed by the "rubbery" surface of the low-density PVC granules rather than an effect of density difference.

From the analysis of video shots of the granular slope failures, it was possible to recognize two failure ways: (i) a general yielding of the slope starting from the top and quickly propagating downward. (ii) a failure starting from the toe and climbing back the slope in a negative wave fashion, only for materials *m12* and *m14* (mixture of sand and finer grains).

Finally, the results of our tests confirm that granular materials, with the exclusion, as explained above, of very fine material, present the characteristic behaviour to assume the evident deposition feature of a plane and regular slope, substantially invariant with flume inclination.

4 Conclusion

The Authors presented a series of laboratory tests in order to characterize two values of friction angles showed by different materials in particular limit conditions: a stability critical one and the deposition angle.

Critical angle measurements with an adapted tilting plane resulted repeatable within 1-2 degrees and were not affected by the volume of employed material.

The use of natural cohesionless granular material is of particular interest for its close relation with mountain scree slopes which are prone to debris-flow triggering, especially in the Dolomites area.

These angles, here defined and measured, are of paramount importance for the rheology of granular materials and in particular for debris flow studies, where the consideration of a generic friction angle (e.g. in debris flow numerical models) as characteristic for both yielding and deposition of a flowing mass, could lead to results far from reality and in particular to an underestimation of the maximum runout of debris flows and other granular flows.

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