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Quantum cloning and universal NOT gate by teleportation

F. Sciarrino *, C. Sias, M. Ricci, F. De Martini

Dipartimento di Fisica and Istituto Nazionale per la Fisica della Materia, Università di Roma "La Sapienza", Roma 00185, Italy

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Abstract

The universal optimal quantum cloning machine (UOQCM) and the universal NOT gate can be implemented contextually by modifying the quantum state teleportation network. We report the experimental realization of the probabilistic UOQCM with polarization encoded qubits. This is achieved by combining on a symmetric beam-splitter the input qubit with an ancilla in a fully mixed state.

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Quantum information is encoded in qubits, the quantum analogue of the classical bits. Transformations of these systems follow the laws of quantum mechanics. This fundamental requirement implies theoretical limitations as far as the "cloning" and/or "spin-flipping" processes of qubits are concerned. For instance, it has been shown that an arbitrary unknown qubit cannot be perfectly cloned: $|\Psi\rangle \rightarrow |\Psi\rangle|\Psi\rangle$, a consequence of the so-called "no cloning theorem" [1, 2]. Another "impossible" device is the quantum NOT gate, the transformation that maps any qubit into the orthogonal one $|\Psi\rangle \rightarrow |\Psi^{\perp}\rangle$ [3]. In the last years a great deal of theoretical investigation has been devoted to finding the best approximations allowed by quantum mechanics to these two processes and to establish for them the corresponding "optimal" values of the "fidelity" $F < 1$. This problem has been solved in the general case [4–6]. In particular, it was found that a one-to-two universal optimal quantum cloning machine (UOQCM), i.e., able to clone one qubit into two qubits ($1 \rightarrow 2$), can be realized with a fidelity $F_{\mathrm{CLON}} = 5/6$. The "flipping" of the input qubit, which realizes a ($1 \rightarrow 1$) Universal-NOT gate (U-NOT), can be implemented with a fidelity $F_{\mathrm{NOT}} = 2/3$ [5,6]. On the other hand, a peculiar and fundamental resource available in the field of quantum information is the quantum state teleportation (QST) protocol by which an unknown input qubit $|\phi\rangle_S = \alpha|0\rangle_S + \beta|1\rangle_S$ is destroyed at a sending place (Alice: $\mathcal{A}$) while its perfect replica appears at a remote place (Bob: $\mathcal{B}$) via dual quantum and classical channels [7]. Here we present a linear scheme that establishes a connection between these three quantum operations.

* Corresponding author.
E-mail address: fabio.sciarrino@uniroma1.it (F. Sciarrino).

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Previously the phenomena of quantum teleportation, quantum cloning and UNOT gate have been analyzed and experimentally implemented using different approaches. The quantum teleportation has been realized using a linear interaction to couple the qubit S to be teleported with an EPR pair [8]. On the other hand, it has been proposed to implement the optimal quantum cloning machine by a non-linear quantum optical "amplification" method, i.e., by associating the cloning effect with the QED "stimulated emission" process [9]. This proposal was quickly followed by some successful experimental demonstrations [10–12]. It has been argued by [12] that when the cloning process is realized in a subspace H of a larger Hilbert space $H \otimes K$ which is acted upon by a physical apparatus, the same apparatus performs contextually in the space K the "flipping" of the input injected qubit, then realizing the U-NOT gate [5,6]. As an example, a UOQCM can be realized on one output mode of a non-degenerate "quantum-injected" optical parametric amplifier (QIOPA), while the U-NOT transformation is realized on the other mode [11,12].

Very recently a novel and quite unexpected relation between the QST, UOQCM and U-NOT gate processes has been presented [13]. Indeed it has been shown that it is possible to implement the $1 \to 2$ universal optimal quantum cloning machine and $1 \to 1$ universal NOT gate by slightly modifying the protocol of quantum state teleportation. Let us now present this protocol. Alice possesses the starting qubit $|\phi\rangle_S$, and, in analogy to the QST, there is a second partner, Bob, who shares with her the entangled "singlet" state $|\Psi^-\rangle_{AB} = 2^{-1/2}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)$. By means of a projective measurement and of classical communication, Alice obtains two qubits S, A that are the optimal quantum clones of the starting qubit $|\phi\rangle_S$, while Bob receives its optimal flipped state. Alice performs a dichotomic projective measurement on the systems S and A able to identify $|\Psi^-\rangle_{SA}$ and its complementary space. With a probability $p = 1/4$ the state $|\Psi^-\rangle_{SA}$ is detected by $\mathcal{A}$ and the qubit $|\phi\rangle$ is teleported to Bob. With probability $p = 3/4$, the qubits S and A are projected in the subspace spanned by $\{|\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$ which is the symmetric space of A and S. The overall output state is obtained by projecting the initial state $|\Psi^-\rangle_{AB}|\phi\rangle_S$ onto the subspace orthogonal to $|\Psi^-\rangle_{SA}\langle\Psi^-|_{SA} \otimes \mathbb{I}_B$, i.e., by applying the projector:

$$P_{SAB} = \left(\mathbb{I}_{SA} - \left|\Psi^-\right\rangle_{SA}\left\langle\Psi^-\right|_{SA}\right) \otimes \mathbb{I}_B. \tag{1}$$

The reduced density matrices for the qubits S, A and B are then [13]:

$$\begin{aligned}\rho_S &\equiv \mathrm{Tr}_A\, \rho_{SA} \\ &= \frac{5}{6}|\phi\rangle\langle\phi| + \frac{1}{6}\left|\phi^\perp\right\rangle\left\langle\phi^\perp\right| \\ &= \rho_A \equiv \mathrm{Tr}_S\, \rho_{SA},\end{aligned} \tag{2}$$

$$\begin{aligned}\rho_B &\equiv \mathrm{Tr}_{SA}\, |\tilde{\Omega}\rangle\langle\tilde{\Omega}| \\ &= \frac{2}{3}\left|\phi^\perp\right\rangle\left\langle\phi^\perp\right| + \frac{1}{3}|\phi\rangle\langle\phi|,\end{aligned} \tag{3}$$

by which the "optimal" values for the fidelities of the two "forbidden" processes $F_{\mathrm{CLON}} = 5/6$ and $F_{\mathrm{UNOT}} = 2/3$ are achieved for the qubits on the Alice and Bob sites respectively [11,12]. Bob identifies whether the U-NOT gate is realized at his site by reading the information (1 bit) received by Alice on the classical channel. For example, such bit can assume the value 1 if Alice identifies the Bell state $|\Psi^-\rangle_{SA}$ and 0 if not. At the end of the protocol the universal NOT gate applied to the input qubit has been teleported at Bob's site and hence this process has been called Tele-UNOT gate. Recently Gottesman and Chuang have proposed to exploit the teleportation protocol to transfer quantum gates in order to relax several experimental constraints to achieve fault-tolerant processing [14]. Indeed the linear optics quantum computation exploits gate teleportation in order to transform a probabilistic computation into a near-deterministic one [15].

The projector P_{SAB} in Eq. (1) leads to the realization of the UOQCM and Tele-UNOT gate, as said. Here we present, in analogy with the quantum circuit associated with quantum teleportation, a quantum circuit that achieves this purpose: the projection is obtained combining Hadamard gates, C-NOT gates, a Toffoli gate, and the projective measurement of an ancilla qubit.

The box "EPR preparation" prepares the singlet state while the "quantum machine" box achieves the projection into the symmetric subspace. The readout of the ancilla qubit $\tilde{a}$, initially in the state $|0\rangle_{\tilde{a}}$, ensures that the projection into the symmetric space has been obtained. Let us now analyze in more details the logic of the quantum circuit. After the EPR preparation, the state of the overall system is

$$|\phi\rangle_S \otimes |\Psi^-\rangle_{AB} \otimes |0\rangle_{\tilde{a}} = \frac{1}{2}\Big[-|\Psi^-\rangle_{SA}|\phi\rangle_B - |\Psi^+\rangle_{SA}(\sigma_Z|\phi\rangle_B) + |\Phi^-\rangle_{SA}(\sigma_X|\phi\rangle_B) + |\Phi^+\rangle_{SA}(\sigma_Z\sigma_X|\phi\rangle_B)\Big] \otimes |0\rangle_{\tilde{a}}. \quad (4)$$

The box labelled (1) transforms the state $|\Psi^-\rangle_{SA}$ into $|1\rangle_S|1\rangle_A$, while the other three Bell states $\{|\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$ are respectively transformed into $\{|0\rangle_S|1\rangle_A, |1\rangle_S|0\rangle_A, |0\rangle_S|0\rangle_A\}$. By means of a Toffoli gate, the state $|1\rangle_S|1\rangle_A$ induces the flipping of the state of the qubit $\tilde{a}$ from $|0\rangle_{\tilde{a}}$ to $|1\rangle_{\tilde{a}}$, whereas the other states leave the qubit $\tilde{a}$ unaltered. Then we get the following state

$$\frac{1}{2}\Big[-|1\rangle_S|1\rangle_A|\phi\rangle_B|1\rangle_{\tilde{a}} - |0\rangle_S|1\rangle_A(\sigma_Z|\phi\rangle_B)|0\rangle_{\tilde{a}} + |1\rangle_S|0\rangle_A(\sigma_X|\phi\rangle_B)|0\rangle_{\tilde{a}} + |0\rangle_S|0\rangle_A(\sigma_Z\sigma_X|\phi\rangle_B)|0\rangle_{\tilde{a}}\Big]. \quad (5)$$

Finally the action of the box labelled (2) restores the initial states of the qubits S and A leading to:

$$|\Sigma\rangle_{SAB\tilde{a}} = \frac{1}{2}\Big[-|\Psi^-\rangle_{SA}|\phi\rangle_B|1\rangle_{\tilde{a}} - |\Psi^+\rangle_{SA}(\sigma_Z|\phi\rangle_B)|0\rangle_{\tilde{a}} + |\Phi^-\rangle_{SA}(\sigma_X|\phi\rangle_B)|0\rangle_{\tilde{a}} + |\Phi^+\rangle_{SA}(\sigma_Z\sigma_X|\phi\rangle_B)|0\rangle_{\tilde{a}}\Big]. \quad (6)$$

If the projective measurement on the ancilla qubit $\tilde{a}$ gives as result 1 the qubits A and S end up in the state $|\Psi^-\rangle_{SA}$ while the qubit B is in the state $|\phi\rangle$, which has then been teleported from $\mathcal{A}$ to $\mathcal{B}$. On the contrary, if we obtain the result 0 the overall state becomes

$$-|\Psi^+\rangle_{SA}(\sigma_Z|\phi\rangle_B) + |\Phi^-\rangle_{SA}(\sigma_X|\phi\rangle_B) + |\Phi^+\rangle_{SA}(\sigma_Z\sigma_X|\phi\rangle_B) \quad (7)$$

that is equal to the state obtained applying the projector P_{SAB} (1) to $|\phi\rangle_S \otimes |\Psi^-\rangle_{AB}$. The result of the ancilla measurement is communicated to Bob and we realize the optimal quantum cloning machine and the Tele-UNOT gate of the input qubit $|\phi\rangle$.

Note that the presence of the entangled state $|\Psi^-\rangle_{AB}$ is not strictly necessary for the implementation of the solely quantum cloning process. Instead of sharing an EPR pair with Bob, Alice needs a qubit A prepared in a *fully mixed* state $\rho_A = \frac{\mathbb{I}_A}{2}$. Of course in this case only the Alice's apparatus is relevant and the U-NOT process is absent. In a previous experiment [13] we have reported the realization of the Tele-UNOT gate by adopting an entangled state. Here we want to demonstrate experimentally that the entanglement is not required to obtain the cloning process.

We have implemented the cloning protocol in the photonic framework, hence the qubits S and A have been codified in the polarization states of two single photons. The computational circuit of Fig. 1 could be realized adopting linear optical C-NOT gates, which have been recently experimentally demonstrated [16]. However, in the present Letter we have implemented the projector P_{SAB} by adopting the simplest configuration available, that is an Ou–Mandel interferometer, realized by the beamsplitter BS_A (Fig. 2). In this case we exploit the Bose mode coalescence (BMC) associated to the projection into the symmetric subspace Π_{sym}. Indeed the *symmetry* of the projected subspace by BMC is implied by the intrinsic *Bose symmetry* of the 2 photons Fock state realized at the output of BS_A.

The qubit to be cloned is: $|\phi\rangle_S = \alpha|H\rangle_S + \beta|V\rangle_S$ where $|H\rangle$ and $|V\rangle$ respectively correspond to the horizontal and vertical linear polarizations of a single photon injected in one input mode k_S of the 50 : 50 beamsplitter BS_A. A fully mixed state ρ_A is simultaneously injected on the other input mode k_A of BS_A where the two input modes are linearly superimposed. Consider the overall output state which is realized on the two output modes k_1 and k_2 of BS_A. It can be expressed as a linear combination of the Bell states $\{|\Psi^-\rangle_{SA}, |\Psi^+\rangle_{SA}, |\Phi^-\rangle_{SA}, |\Phi^+\rangle_{SA}\}$. As it is well known, the realization of the singlet $|\Psi_{SA}^-\rangle$ is unambiguously identified by the detection of a single photon on each one of the two output modes of BS_A while the realization of the set of the other three Bell states corresponds to the emission of photon pairs on either one of the output modes [17]. Hence the detection of two photons over either the mode k_1 or k_2, implies the projection by $P_{SA} = (\mathbb{I}_{SA} - |\Psi_{SA}^-\rangle\langle\Psi_{SA}^-|)$ of the input state into the space orthogonal to $|\Psi_{SA}^-\rangle$.

In the present experiment, a pair of non-entangled photons with wavelength $\lambda = 532$ nm and with a coherence-time $\tau_{\mathrm{coh}} = 80$ fs, were generated by a spon-

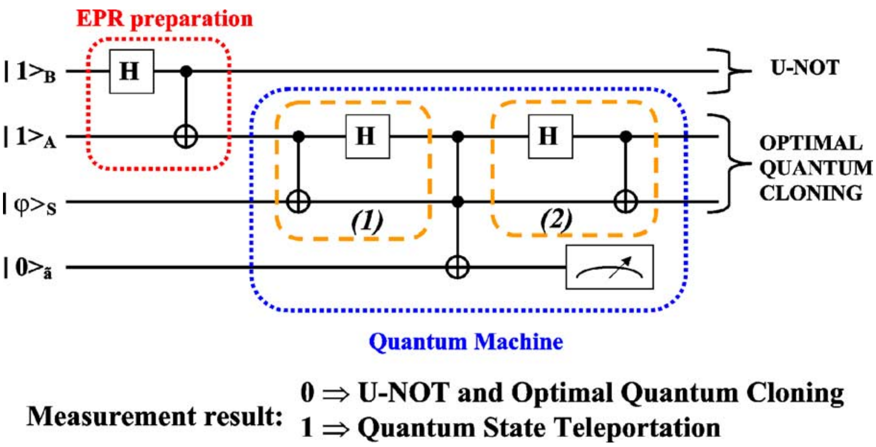

Fig. 1. Realization of UOQCM and Tele-UNOT gate by means of a quantum circuit. The preparation box contains the circuit that generates the state $|\Psi^-\rangle_{AB}$ while the "quantum machine" box represents the apparatus performing the projective measurement over the antisymmetric and symmetric (Π_{sym}) subspaces of the two qubits A and S. The Toffoli gate can be realized by means of single qubit gates and C-NOT gates.

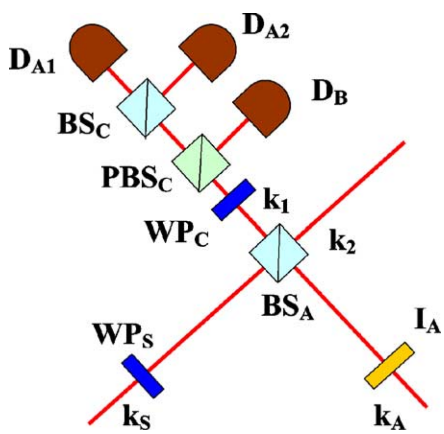

Fig. 2. Experimental setup for the optical implementation of the UOQCM by a modified teleportation protocol.

taneous parametric down conversion (SPDC) process in a Type I BBO crystal in the initial polarization product state $|H\rangle_S|H\rangle_A$. The input qubit $|\phi\rangle_S$, associated with mode k_S was polarization encoded by means of a waveplate (wp) WP_S. The transformation used to map the state $|H\rangle_A$ into $\rho_A = \frac{\mathbb{I}_A}{2}$ was achieved by stochastically rotating a $\lambda/2$ waveplate inserted on the mode k_A during the experiment. In this way the statistical evolution of $|H\rangle_A$ into two orthogonal states with equal probability was achieved. The photons S and A were injected in the two input arms of BS_A with a mutual delay Δt micrometrically adjustable by a translation stage with position settings $Z = 2c\Delta t$. The setting value $Z = 0$ was assumed to correspond to the full overlapping of the photon pulses injected into BS_A, i.e., to the maximum photon interference.

For the sake of simplicity, we only analyzed the measurements performed on the BS_A output mode k_1: Fig. 2. The polarization state on this mode was analyzed by the combination of the wp WP_C and of the polarization beam splitter PBS_C. For each input polarization state $|\phi\rangle_S$, WP_C was set in order to make PBS_C to transmit $|\phi\rangle_S$ and reflect $|\phi^\perp\rangle_S$. The "cloned" state $|\phi\phi\rangle_S$ could be detected by a two-photon counter, realized in our case by first separating the two photons by a 50 : 50 beam splitter BS_C and then detecting the coincidence $[D_{A1}, D_{A2}]$ between the output detectors D_{A1} and D_{A2}: Fig. 2. Any coincidence between detectors D_B and D_{A2} corresponded to the realization of the state $|\phi\phi^\perp\rangle_S$. Thus, by the present scheme, the cloning process is experimentally demonstrated in the coincidence basis. First consider the cloning machine switched off, by setting: $\Delta t > \tau_{\text{coh}}$, i.e., by making S and A not interfering on BS_A. In this case, since the states $|\phi\phi\rangle_S$ and $|\phi\phi^\perp\rangle_S$ were realized with the same probability on mode k_1, the rate of coincidences detected by $[D_{A1}, D_{A2}]$ and $[D_{A2}, D_B]$ were expected to be equal. By turning on the cloning machine, i.e., by setting $\Delta t \ll \tau_{\text{coh}}$, on mode k_1 the output density matrix ρ_{SA} was realized implying an enhancement by a factor $R = 2$ of the counting rate $[D_{A1}, D_{A2}]$ and no rate enhancement of $[D_{A2}, D_B]$. The measurement

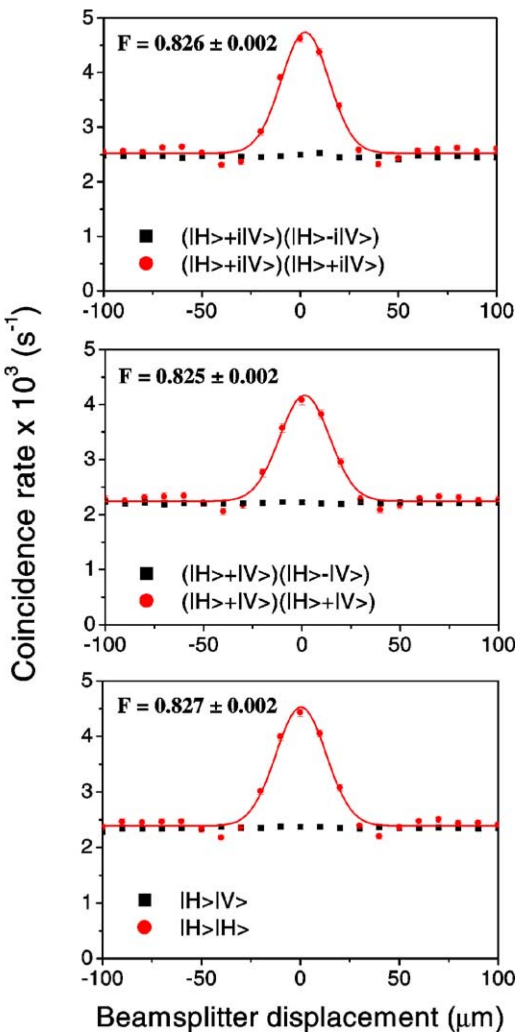

Fig. 3. Experimental result of the universal cloning process for different input qubits corresponding to the encoded polarizations: $|H\rangle$, $2^{-1/2}(|H\rangle + |V\rangle)$ and $2^{-1/2}(|H\rangle + i|V\rangle)$.

of R was carried out by coincidence measurements involving simultaneously $[D_{A1}, D_{A2}]$ and $[D_{A2}, D_B]$. The experimental data are reported in Fig. 3 for three different input states $|\phi\rangle_S = |H\rangle$, $2^{-1/2}(|H\rangle + |V\rangle)$, $2^{-1/2}(|H\rangle + i|V\rangle)$. There circle and square markers refer respectively to the $[D_{A1}, D_{A2}]$ and $[D_{A2}, D_B]$ coincidences versus the position setting Z. We may check that the cloning process only affects the $|\phi\phi\rangle_S$ component, as expected, and R is determined as the ratio between the peak values (cloning machine switched on) and the basis values (cloning machine switched off). The corresponding experimental values of the *cloning fidelity* $F = \frac{2R+1}{2R+2}$ are: $F_H = 0.827 \pm 0.002$; $F_{H+V} = 0.825 \pm 0.002$; $F_{H+iV} = 0.826 \pm 0.002$. These ones are in good agreement with the optimal value $F_{\text{th}} = 5/6 \approx 0.833$ which corresponds to the limit value of the amplification ratio $R = 2$. We note that, while our experiment did not adopt an ancilla qubit, the measurement of the number of photons over the output modes allowed us to identify whether the projection on the symmetric space Π_{sym} had been obtained.

In conclusion we presented how it is possible to implement the $1 \to 2$ universal optimal quantum cloning machine and $1 \to 1$ Tele-Universal NOT gate by means of a quantum network that can be realized for all kind of qubits. We report the highest experimental fidelity attained so far for the $1 \to 2$ UOQCM adopting single photon polarization encoded qubits. Furthermore the simplicity of our setup renders the cloning technique practically available for some optimal attacks to cryptographic communications [18]. For instance, in the incoherent eavesdropping on the six-state protocol, the optimal strategy coincides to the universal quantum cloning machine for a fixed value of Bob's disturbance [18].

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