

An Innovative Two-Step Method for MR-Based Electrical Properties Tomography

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Abstract—Magnetic resonance electrical properties tomography is a noninvasive imaging technique aimed at estimating the electromagnetic properties of biological tissues from magnetic resonance data. The underlying task involves a nonlinear and ill-posed inverse problem. In this work, a two-step method is proposed to reformulate the problem as a cascade of linear subproblems. In the first step, a linear ill-posed inverse problem is solved to estimate the distribution of contrast sources and the internal electric field within the region of interest. In the second step, the electrical properties are retrieved through a least-squares fitting based on the quantities estimated in the first step. A numerical example involving a simple phantom is presented to provide an initial validation of the proposed approach.

Index Terms—Magnetic Resonance Imaging (MRI), Electrical Properties Tomography (EPT), Inverse Problem, Contrast source inversion

I. INTRODUCTION

In the last years, electrical property tomography (EPT) has emerged as a powerful non-invasive imaging technique for quantifying the electrical properties (EP) of biological tissues using magnetic resonance imaging (MRI) [1]. The technique relies on reconstructing the complex permittivity and conductivity distributions within the imaging domain by leveraging the relationship between these properties and the measured radiofrequency (RF) magnetic field data.

Traditional EPT reconstruction methods face significant challenges due to the inherently nonlinear and ill-posed nature of the inverse problem, noise sensitivity, and computational complexity [1]. In this framework, the contrast source inversion EPT approach (CSI-EPT) has shown promise in addressing these limitations by reformulating the inverse problem in terms of an optimization task, where the contrast source (i.e., the current induced in the target by the MRI coil) is exploited as an auxiliary variable to enforce physical consistency [2]. The conventional CSI-EPT approach optimizes a single cost function that enforces the discrepancy between measured and computed RF magnetic fields. However, this formulation often suffers from convergence issues and may not fully exploit all available physical constraints. Moreover, it is computationally intensive.

This work introduces a novel two-step CSI-EPT algorithm that addresses these limitations through a reformulated problem structure. The key concept lies in decoupling the inverse problem into two sequential optimization phases, each

targeting specific aspects of the reconstruction process. The first phase employs a joint optimization strategy that simultaneously enforces RF magnetic field data consistency and electric field relationships, while the second phase leverages tissue segmentation to efficiently retrieve electrical properties. Thanks to the decoupling of the problem, the proposed methodology offers several advantages over existing approaches in terms of improved numerical stability and enhanced convergence properties.

In the following, the proposed method is described, and an initial numerical validation is presented.

II. THE EPT INVERSE PROBLEM

When a body is present in the MRI scanner, the RF magnetic field generated by the birdcage coil is perturbed, giving rise to a magnetic scattered field [2,3]:

$$\mathbf{H}_s(\mathbf{r}) = \mathcal{A}_h[\mathbf{w}] \quad \mathbf{r} \in D \quad (1)$$

where \mathbf{w} is the contrast source, i.e., the current induced in the target by the incident field, and \mathcal{A}_h is the short notation for the magnetic radiation operator linking \mathbf{w} to the scattered magnetic field in the imaging domain.

The scattered field \mathbf{H}_s is related to the transmit magnetic field (B_1^+) measured by the MRI system as [1,2]:

$$B_1^+ = \mu_0 \frac{\mathbf{H}_{s,x} + j\mathbf{H}_{s,y}}{2} \quad (2)$$

where μ_0 is the magnetic permeability of the vacuum.

By putting together (1) and (2), one gets:

$$B_1^+ = \mu_0 \frac{\mathcal{A}_h[\mathbf{w}]_x + j\mathcal{A}_h[\mathbf{w}]_y}{2} \quad (3)$$

This equation relates the B_1^+ data to the EP, because the contrast source is given by [3]:

$$\mathbf{w} = \chi \mathbf{E}_{tot} \quad (4)$$

where χ represents the contrast function encoding the target's electrical properties and \mathbf{E}_{tot} is given by:

$$\mathbf{E}_{tot}(\mathbf{r}) = \mathbf{E}_{inc}(\mathbf{r}) - \mathcal{A}_e[\mathbf{w}] \quad (5)$$

\mathbf{E}_{inc} being the incident field and \mathcal{A}_e denoting the electric radiation operator that establishes the relationship between \mathbf{w} and the electric scattered field in the imaging domain.

The inverse EPT problem cast through (3) is nonlinear and ill-posed. Nonlinearity descends from the dependency of \mathbf{E}_{tot} on the contrast function, see (4), (5). Ill-posedness is a consequence of the indeterminacy arising from the fact that B_1^+ is only related to the transverse components of \mathbf{H}_s , and thus encodes only a partial information.

III. THE PROPOSED APPROACH

To alleviate the difficulties of the EPT inverse problem, the proposed approach splits the inversion task into two distinct phases. The first phase is aimed at simultaneously retrieving both the contrast source and the total field. To this end, an optimization problem is formulated as the minimization of the objective function:

$$\phi_s(\mathbf{w}, \mathbf{E}_{tot}) = \frac{\|B_1^+ - \mathcal{P}\mathcal{A}_h\mathbf{w}\|^2}{\|B_1^+\|^2} + \frac{\|\mathbf{E}_{tot} - \mathbf{E}_{inc} - \mathcal{A}_e[\mathbf{w}]\|^2}{\|\mathbf{E}_{inc}\|^2} \quad (6)$$

where \mathcal{P} denotes the operator that extracts the positively rotating component of the magnetic field, see (3).

The solution to this minimization yields the pair $(\mathbf{w}, \mathbf{E}_{tot})$ that best fits both Equations (3) and (5). The process is carried out iteratively, stopping when a predefined tolerance is reached or the maximum number of iterations is exceeded. Notably, since the dependency on the contrast is implicit, the cost functional (6) is quadratic in the unknowns and the underlying inverse problem is linear. Nonetheless, the problem remains ill-posed due to data incompleteness. Regularization is enforced through a proper initialization.

In the second phase, from the estimated $(\mathbf{w}, \mathbf{E}_{tot})$, the EP are retrieved in terms of the contrast function χ . In particular, the estimated contrast $\bar{\chi}$ is achieved via least-squares fitting the constitutive relation (4):

$$\bar{\chi} = \operatorname{argmin} \|\mathbf{w} - \chi \mathbf{E}_{tot}\| \quad (7)$$

This second step is again free from false solutions, although errors in the first step will obviously affect the accuracy of the final estimation.

IV. NUMERICAL ANALYSIS

To evaluate the performance of the proposed EPT method, a numerical simulation was conducted using a simplified three-dimensional cubic phantom. The outermost layer models the skin, followed by an intermediate layer that mimics adipose tissue, characterized by low conductivity and permittivity. The core region simulates liver tissue, which exhibits significantly higher dielectric properties. This configuration allows for a controlled yet sufficiently realistic testing environment that includes both geometric complexity

and property heterogeneity, thereby offering a meaningful validation scenario for the reconstruction algorithm.

The input data for the simulation were generated by computing the incident electromagnetic field within an empty MRI scanner using CST Studio Suite® 2021 (Dassault Systèmes, Vélizy-Villacoublay, France). The scanner model consists of a cylindrical RF shield and a birdcage coil, both taken from CST's component library. The MRI system operates with a 1.5 T static magnetic field, and nuclear resonance is excited at the proton Larmor frequency of 64 MHz, corresponding to a rotating magnetic field B_1^+ around the static z-axis. The birdcage coil is tuned to 64 MHz using lumped capacitors and is excited by two discrete ports positioned 90 degrees apart, enabling the generation of a circularly polarized field. The incident field distribution in the region of interest (ROI), as computed by CST, was then used to evaluate the B_1^+ field by solving the scattering equations using a proprietary forward solver based on the Method of Moments.

To evaluate the robustness of the proposed algorithm under more realistic conditions, inversion tests with perturbed data were performed also. These perturbations were modelled as an additive gaussian noise (SNR = 25) to simulate the presence of measurement noise and modelling inaccuracies, thereby enabling a more comprehensive assessment of the algorithm's stability and performance.

The initial guess for the inversion in the first step was designed to reflect uncertainty in prior knowledge. Specifically, the dielectric contrast χ was initialized by applying a random perturbation around the true value for each voxel in the imaging region, according to the following formulation:

$$\chi^0 = \chi_{true} \cdot (1 + \beta \cdot (\operatorname{rand} - 0.5)) \quad (8)$$

where $\beta = 0.7$ is a scaling factor that defines the amplitude of the zero-mean random perturbation.

The accuracy of the reconstructed maps is quantitatively assessed using the Mean Squared Error (MSE), defined as:

$$\text{MSE} = \frac{\sum |u_{true} - u|^2}{\sum |u_{true}|^2} \times 100 \quad (9)$$

where u_{true} denotes the true distribution (of the contrast source in the first step and of the EP in the second step) and u refers to either the initial guess u_{init} or the reconstructed distribution u_{rec} of the considered quantity.

Table I reports the MSE for the reconstructed components of the contrast source in the first step of the procedure, for both the ideal case of noiseless data and the case of noisy data. As can be seen, in both cases the procedure is able to reduce the error with respect to the initial condition, achieving MSE value which are comparable in both cases.

TABLE I
MSE FOR THE RECONSTRUCTED CONTRAST SOURCES

	w_x	w_y	w_z
Initial	11.8%	11.7%	12.1%
Ideal scenario	5.2%	5.1%	5%
Noisy scenario	6.4%	5.8%	6.1%

TABLE II
COMPARISON OF TRUE AND RECONSTRUCTED EP VALUES

	Ideal scenario		Noisy scenario	
	MSE (ϵ)	MSE (σ)	MSE (ϵ)	MSE (σ)
Skin	0.11%	0.03%	0.40%	0.31%
Fat	0.30%	0.17%	0.42%	0.82%
liver	0.14%	0.01%	0.43%	0.25%

In the second step, considering that in many cases the retrieval of an average value for the EP in each tissue is sufficient, a prior-informed representation of the contrast function was employed, based on MRI tissue segmentation [4]. This approach enables the fitting of (7) to be performed in a reduced-dimensional space by employing basis functions specific to each tissue type, estimating a single parameter value per tissue, with an improvement in both computational efficiency and reliability of the results.

The reconstruction results are illustrated in Fig. 1 in a central section of the geometric model used. The figure shows the EP maps for both the true model and the reconstructed estimates. The top row shows the true values of relative permittivity (ϵ_r , left) and electrical conductivity (σ , right), which were used as the reference ground truth in the simulation. The bottom row shows the reconstructed values obtained using the proposed two-step inversion method.

The quantitative assessment of the reconstruction is provided in Table II, which reports the MSE for both permittivity and conductivity across the different tissue regions. The error values remain consistently low, confirming the effectiveness of the proposed algorithm in accurately recovering the dielectric profiles. This indicates that the two-phases minimization process effectively manages both geometric and dielectric heterogeneity, even under realistic simulation conditions.

The introduction of random noise, both in the dielectric contrast (initial guess) and in the simulated measurement data, does not significantly degrade the quality of the reconstruction. The error values remain stable, further confirming the algorithm's capability to recover the correct dielectric properties despite uncertainties in the input. These results clearly show that the method is strong and reliable, even when there are small changes or noise in the data.

V. CONCLUSIONS

In this study, we presented a novel EPT inversion approach for the reconstruction of the dielectric properties of biological tissues from MRI data. The proposed method reformulates the original nonlinear inverse problem into two sequential linear subproblems. The first step aims to estimate the distribution of induced currents and electric fields from the acquired B_1^+ data, while the second step exploits these

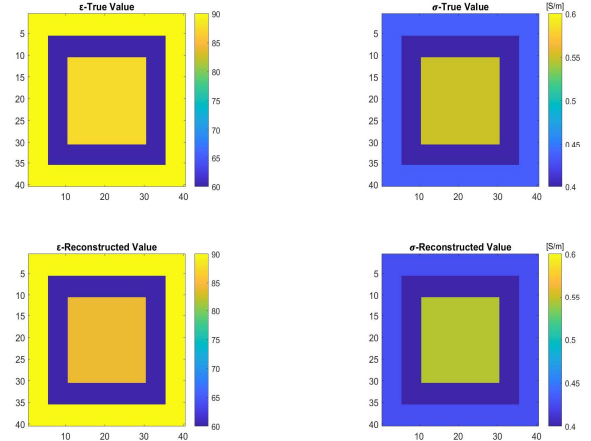


Figure 1 Comparison of True and Reconstructed Values for Relative Permittivity (ϵ) and Electrical Conductivity (σ)

quantities to retrieve the dielectric properties of the tissues, based on the physical relationship between them.

The numerical simulations carried out on ideal phantom models demonstrated that the proposed approach is capable of accurately reconstructing both permittivity and conductivity, even when starting from noisy data or inaccurate initial estimates. Further examples with more complex – anthropomorphic – phantoms, as well as a comparison with conventional CSI-EPT will be shown at the conference.

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