# Generalizing Hypergraph Ego-Networks and their Temporal Stability

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**Abstract.** Ego-networks provide a local perspective on networked systems by focusing on a central entity and its immediate relational context. While extensively studied in static pairwise interaction models, their analysis in dynamic and higher-order interaction contexts remains limited. In this paper, we introduce a formal analytical framework to generalize ego-networks to temporal hypergraphs, capable of modeling complex, non-dyadic group interactions over time. Our framework consists of two main components, namely, the concept of Rooted Ego-Networks (RENs), and generalized similarity criterion. RENs extend traditional ego-networks by capturing temporal and structural characteristics of groups centered on one or more nodes. We define multiple inclusion functions and similarity criteria to compare RENs across time, and we introduce the notion of *stability* to identify persistent local structures. Through empirical evaluation on real-world temporal hypergraphs from the SocioPatterns database, we illustrate the expressiveness of our approach in capturing and analyzing the evolution of localized group dynamics. To the best of our knowledge, this is the first comprehensive framework for analyzing the temporal and non-monotonic evolution of ego structures in higher-order networks.

**Keywords:** ego-network, temporal hypergraphs, rooted ego-networks, stability

## 1 Introduction

Ego-networks are a foundational concept in network science, offering a localized view of a system by focusing on a central entity, the *ego*, and its immediate neighbors, the *alters*, along with their interactions. When using graphs to formalize connections between entities, an ego-centric perspective enables the analysis of the structural and functional properties of a network at a micro level, facilitating insights into the dynamics of social networks, communication networks, and genomics to cite a few [1, 23, 32, 30, 20]. Additionally, increasing attention has been devoted to the *temporal* dimension of egos, recognizing that the evolution of such structures over time reveals critical aspects of stability, adaptability, and behavioral dynamics [8, 28], shedding light on the complexity of individual-centric interaction patterns, and tie stability [21].

Network science has traditionally focused on models representing systems with pairwise interactions. Nevertheless, the dynamics of systems displaying higher-order interactions can not be entirely understood by simply decomposing them into pairwise relationships [5]. Hypergraphs, in particular, have emerged as a powerful representation to characterize interactions involving three or more entities simultaneously, thereby enabling the analysis of a collective dimension playing a key role in social, ecological, and biological contexts, among others [17, 6, 27, 24]. Extending the concept of ego networks to hypergraphs allows for the exploration of individual and group participation across overlapping and variably sized circles, thus broadening the analytical scope of egocentric methods. Despite the growing relevance of hypergraph models, the study of ego structures, and more generally, substructures, in higher-order networks remains comparatively underdeveloped. In particular, while some recent efforts have introduced definitions of hypergraph ego networks, the systematic study of their temporal evolution is still in its early stages [12, 10]. Questions concerning the persistence, transformation, and structural similarity of group-based ego structures over time are largely unexplored, for which the need for a consistent methodological framework arises.

This paper addresses this gap by introducing a novel analytical framework for studying ego-centric structures in *temporal hypergraphs.* We define and operationalize the novel concept of a *Rooted Ego-Network* (REN), which generalizes the notion of ego-networks to accommodate both higher-order and temporal aspects. Also, the framework incorporates customizable definitions of inclusion and structural similarity, enabling the comparative analysis of ego structures over time. These definitions include both classical measures, such as the direct overlap, and a novel one, called *node overlap-aware similarity.* Additionally, to describe how local structures around groups of nodes transform over time, we formally introduce the concept of the *stability* event. We evaluate our proposed framework in a series of experiments on six datasets from the SocioPatterns database[4], showing that the node overlap-aware similarity criterion better captures group persistence, and that a structural characterization of such emerging local, node-centric groups in terms of their average size and node composition can highlight dataset-specific trends in function of the REN size and time. To the best of our knowledge, this is the first comprehensive framework to study and analyze the temporal and non-monotonic evolution of substructures, particularly ego-networks, in temporal hypergraphs.

The structure of the paper is as follows. In Section 2, we outline an overview of related work. In Section 3, we provide the necessary background on the context we focus on. Then, in Section 4, we formally introduce the framework and its main components. In Section 5, we define the concept of stability for temporal

[4] http://www.sociopatterns.org/

ego structures, and in Section 6 we discuss the results obtained by the evaluation of the framework. Finally, in Section 7, we draw our conclusions.

## 2 Related Work

The micro-level structural properties of a network can be analyzed by focusing on *ego networks*. In the context of graphs, an ego network is a subgraph induced on a focal node (called *ego*) and its immediate neighbors (the *alters*). The assumption underlying ego-network characterization is trivial: if network topology can be used to extract knowledge on an actor, then this knowledge shall be contained in its ego network. Taking this micro-level perspective is crucial for research tasks in social networks [23, 30], firm alliances [22], science of science [4], biology [32], cognition [11], and healthcare [33]. Previous research has already focused on studying the topological properties of ego networks. Gupta and colleagues, for instance, investigate the relation between local and global degree distribution [19], while Everett and Borgatti show that the betweenness centrality of a node in a network depends on the betweenness within the ego network [14]. Egocentric analysis is particularly prevalent in online social media, where ego networks are often enriched with edge weights that proxy the strength of a connection through frequency of interaction [25]. This allows distinguishing relations at different degrees of intimacy [3], with individuals typically having few strong ties and many weak ones. The heterogeneity in the strength of ties also impacts the way information is spread and opinions are formed [2].

Another line of research focuses on the temporal dynamics of ego networks, examining how they evolve over time. Studies of temporal egocentric networks have noted a recurring duality in ego network dynamics. On the one hand, the identity of intimate alters remains stable; on the other hand, lower-ranked ties tend to churn over time, with defined interaction patterns (frequency bursts, gaps, etc.) [20]. Identifying *temporal motifs*, i.e., frequently occurring, time-ordered patterns, is an other important trend of research [21]. For instance, an ego might frequently interact with alter $b$ after interacting with alter $a$ [20].

Recently, some works have investigated ego networks on hypergraphs. Hypergraphs bring a new perspective on egocentric network analysis, with egos now included in differently-sized groups. Ref. [12] defines three types of higher-order egocentric structures, each capturing different nuances: (i) *star ego-network*, composed of all edges that contain the ego; (ii) *radial ego-network*, composed of all edges where every node in the edge is either the ego or an alter of the ego; (iii) *contracted ego-network*, composed of all edges containing at least the ego or an alter. Combining temporal and higher-order approaches, the authors study how these structures evolve and address the task of reconstructing such an evolution using local structural features. Ref. [10] studied correlations between higher-order egocentric structures and bursty behaviors in physical contact networks, showing that interactions that are close in time are also close in topology, often overlapping on core nodes. Moreover, hypergraph ego networks can further be enriched with node attributes to study both individual and group homophilic tendencies. In [16], for instance, a temporal hypergraph with attributes on nodes is used to study micro- and meso-level homophilic mixing patterns in Reddit political discussions, highlighting different behaviors across political orientations. In [11], information aggregated in node-attributed hypergraph ego structures has been used as a predictor of cognitive foundations of semantic memory in human cognition.

Finally, this work is also inspired by previous research on the temporal evolution of groups/communities [28]. Dynamic Community Detection (DCD) focuses on identifying well-connected groups of nodes and tracking their evolution across time. This tracking is typically formalized through *events*, which represent structural transformations such as splitting, merging, growth, or contraction. Numerous approaches exist to model this evolution (see [8] for a review), each defining and quantifying *stability* in its own way. A common and straightforward strategy involves comparing group configurations at successive time steps using set similarity functions, such as the intersection size or Jaccard index. More sophisticated methods consider node-level dynamics by quantifying how individual nodes shift between groups over time [15].

## 3 Background

In this section, we provide the necessary background to introduce our proposed framework. We start by defining the concept of hypergraph and several of its properties, and we follow this definition by the one of temporal hypergraph.

A *hypergraph* is a generalization of a graph where an edge, here called *hyperedge*, can connect any number of nodes. Formally, a hypergraph is defined as a pair $H = (V, E)$ where $V$ is the set of nodes, and $E$ is the set of hyperedges. A hyperedge $e \in E$ is a non-empty subset of nodes $v \subseteq V$. Given a hypergraph $H$, two valuable properties are its order and size. Particularly, the *order* of a hypergraph $H$ is the number $n$ of its nodes, e.g., $n = |V|$; similarly, the *size* of a hypergraph $H$ is the number $m$ of its hyperedges, e.g., $m = |E|$. Given a node $v \in V$, we say that $v$ belongs to $e \in E$ if and only if $v \in e$, and we also say that $e$ contains $v$. The *degree* of $v$, denoted as $deg(v)$, is the number of hyperedges in which $v$ appears, that is, $deg(v) = |\{e \in E \mid v \in e\}|$. Similarly, the *hyperedgree* of $v$, denoted as $hdeg(v)$, is the number of neighbors of $v$, where a node $u \in V$ is the neighbor of node $v$ if and only if there exists at least one hyperedge $e$ which $v$ and $u$ both belong to.

In our context, we are interested in dealing with temporal hypergraphs, that is, hypergraphs whose nodes and hyperedges may change during time. A *temporal hypergraph* is a sequence of hypergraphs $\mathcal{H} = (H_1, H_2, \ldots, H_t)$, where each $H_r = (V_r, E_r)$, $1 \leq r \leq t$, represents the state of the temporal hypergraph at time $r$. We also refer to $t$ as the length of the temporal hypergraph. Furthermore, when dealing with a temporal hypergraph, it may be convenient to have a representation of nodes and hyperedges across all time instances. Therefore, we define the universe of nodes of $\mathcal{H}$ as $\mathcal{U} = \bigcup_{r=1}^{t} V_r$; similarly, we define the universe of hyperedges of $\mathcal{H}$ as $\mathcal{E} = \bigcup_{r=1}^{t} E_r$.

## 4 The Proposed Framework

We are now able to introduce our framework to generalize hypergraph ego-networks. Our framework is analytical in the sense that its components can be used to study group structures in temporal hypergraphs, thus allowing one to study the evolution of these groups. We formally define our framework as a tuple

$$\mathfrak{E} = (\mathcal{H}, \phi, \delta)$$

where *(i)* $\mathcal{H} = (H_1, H_2, \ldots, H_t)$ is a temporal hypergraph of length $t$, *(ii)* $\phi$ is a *rooted ego-network* function, and *(iii)* $\delta$ is a function defining a similarity criterion to assess whether two rooted ego-networks are similar.

Essentially, the two main components of $\mathfrak{E}$, $\phi$ and $\delta$, allow one to exploit the framework to study the temporal evolution of groups, and specifically of rooted ego-networks, in a temporal hypergraph. In what follows, we first provide a formal definition of rooted ego-networks, which allows us to define $\phi$, and then we illustrate different ways to define their similarity, which is the core aspect of $\delta$.

### 4.1 Rooted Hypergraph Ego-Networks

Given a hypergraph $H = (V, E)$, and a subset of nodes $U \subseteq V$, a rooted hypergraph ego-network (REN from now on) is the resulting collection of hyperedges $\phi(U) \subseteq E$ in $H$ rooted on the set of nodes $U$. Here, rooted indicates that nodes in $U$ have certain relationships with the hyperedges obtained by $\phi(U)$, e.g., the nodes are contained in the hyperedges. A REN intuitively captures the local neighborhood of the node set $U$ according to a chosen definition of proximity of inclusion. Indeed, different choices of $\phi$, and thus of the chosen definition of proximity of inclusion, yield different types of RENs.

In the following, we give some possible definitions of $\phi$; also, some of them will be studied in our experiments, whose results will be discussed in Section 6.

**Definition 1 (Simple REN).** *Given a hypergraph $H = (V, E)$, and a subset of nodes $U \subseteq V$, a simple REN, denoted as $\phi_{sim}(U)$, is the collection of hyperedges of $H$ where each hyperedge include at least one node from $U$. Formally, $\phi_{sim}(U) = \{e \in E : |e \cap U| > 0\}$.*

**Definition 2 ($\alpha$-fractured REN).** *Given a hypergraph $H = (V, E)$, a subset of nodes $U \subseteq V$, and a value $\alpha \in [0, 1]$, an $\alpha$-fractured REN, denoted as $\phi_{frac_\alpha}(U)$, is the collection of hyperedges of $H$ where each hyperedge contains at least a fraction $\alpha$ of the nodes in $U$. Formally, $\phi_{frac_\alpha}(U) = \{e \in E : |e \cap U| \geq \alpha|U|\}$.*

**Definition 3 ($\beta$-core REN).** *Given a hypergraph $H = (V, E)$, a subset of nodes $U \subseteq V$, and a value $\beta \in [0, 1]$, a $\beta$-core REN, denoted as $\phi_{core_\beta}(U)$, is the collection of hyperedges of $H$ where each hyperedge belongs to the rooted ego-network only if a proportion $\beta$ of its nodes belong to $U$. Formally, $\phi_{core_\beta}(U) = \{e \in E : |e \cap U| \geq \beta|e|\}$.*

An important notation arises from the fact that within $\mathfrak{E}$, we are interested in studying the evolution of group structures, particularly RENs, in a temporal hypergraph $\mathcal{H} = (H_1, H_2, \ldots, H_t)$. Therefore, given a specific time instant $1 \leq r \leq t$, indicating the hypergraph $H_r = (V_r, E_r)$, we refer to the REN rooted on a subset of nodes $U \subseteq V_r$ as $\phi^r(U)$.

Obviously, the aforementioned definitions are not exhaustive: additional variants can be defined by composing conditions, weighting nodes or hyperedges, or including temporal constraints. Indeed, each definition also has a direct impact on the resulting collection of hyperedges. For instance, it is evident that that the adopted definition of proximity of inclusion for $\phi_{sim}$ is less stringent than the one of $\phi_{frac_\alpha}$ when $\alpha \leq \frac{1}{|U|}$. Furthermore, the generality of $\phi$ allows the framework to be adaptable to a wide range of scenarios involving local structure in temporal hypergraphs. In light of this consideration, some remarks are in order. First off, consider that in the classical setting, i.e., when dealing with graphs, an ego-network is defined as a set of nodes (the ego node and its alters) and edges (the dyadic ties between them) while, in our setting, a REN is essentially represented by a collection of hyperedges (which in turn can encompass non-dyadic relationships). Secondly, while our definition of REN is novel, it is also coherent with the classical one. Specifically, when $|U| = 1$, that is, we define a REN on a single node only, our definition of $\phi_{sim}$ corresponds to the classical definition of ego-networks used in the literature considering hypergraph scenarios [12]. Additionally, consider the various definitions of hypergraph ego-networks provided in [12]. It is evident that $\phi$ is general enough to encompass these, simply by aligning its definition with the constraints specified in those formulations. Moreover, there are cases, especially when dealing with graphs, in which ego-networks do not consider the ego nodes within them [13]; in our case, each $\phi$ definition can be seamlessly extended not to include ego nodes. For instance, $\phi_{sim}$ could be extended as $\phi_{sim}(U) = \{e \setminus U \mid e \in E \wedge |e \cap U| > 0\}$.

### 4.2 Similarity Criterion

The second main component of the framework is the similarity criterion $\delta$ that qualifies the similarity between two given RENs. As it is clear from before, our framework can be exploited to conduct analysis on the evolution of group structures in a temporal hypergraph; therefore, the temporal dimension is particularly important, and thus in the definition of the similarity criterion $\delta$, we strive to take into account this aspect. In what follows, we formally define a generalization of the similarity criterion $\delta$, and we then specialize it with some classical measures and a novel one.

Given a temporal hypergraph $\mathcal{H} = (H_1, H_2, \ldots, H_t)$, let $\phi^r(U)$ and $\phi^s(W)$ be two RENs rooted on the node set $U \subseteq V_r$ and $W \subseteq W_s$, respectively, with $1 \leq r, s \leq t$. Then, we define the similarity criterion as the function

$$\delta : 2^{E_r} \times 2^{E_s} \to [0, 1]$$

Here, higher values of $\delta(\phi^r(U), \phi^s(W))$ indicate greater similarity between the two RENs $\phi^r(U)$ and $\phi^s(W)$. Note that this definition is deliberately general, in the sense that there are no constraints on the set of nodes $U$ and $W$ which one can choose among all hypergraphs in $\mathcal{H}$. Indeed, in our setting, we are interested in studying the evolution of a REN by taking into account the set of nodes on which it originates. Therefore, as we will also show in our experiments (see Section 6), given a set of nodes $U$, such that $U \subseteq V_r$ and $U \subseteq V_s$, we will analyze the evolution of a defined REN by studying the similarity $\delta(\phi^r(U), \phi^s(U))$. Also, in case the set of nodes $U$ is clear from the context, to not burden the notation we simply write $\phi^r$ and $\phi^s$, respectively.

Given the general definition of the similarity criterion, it is possible to specialize it by one of several measures. In what follows, we first present some classical measures $\delta$ can be specialized by; then, we propose a novel one and we give the rationale behind it. Let $\phi^r(U)$ and $\phi^s(U)$ be two RENs. Some classical measures are:

- *Direct overlap* — Formally defined as $\delta_{do}(\phi^r, \phi^s) = \frac{|\phi^r \cap \phi^s|}{|\phi^r \cup \phi^s|}$, it measures the direct overlap between the hyperedge sets whose RENs are composed of, and coincides with the classical Jaccard similarity;
- *Minimum overlap* — Formally defined as $\delta_{mo}(\phi^r, \phi^s) = \frac{|\phi^r \cap \phi^s|}{min(|\phi^r|, |\phi^s|)}$. Similarly to the previous one, it takes into account the relative preservation with respect to the smaller REN;
- *Harmonic mean* — Formally defined as $\delta_{hm}(\phi^r, \phi^s) = \frac{2|\phi^r \cap \phi^s|}{|\phi^r| + |\phi^s|}$, it is meant to offer a balanced measure sensitive to changes in the size of a REN;
- *Weighted overlap* — Formally defined as $\delta_{wo}(\phi^r, \phi^s) = \frac{\sum_{e_i \in \phi^r \cap \phi^s} |e_i|}{\sum_{e_j \in \phi^r} |e_j| + \sum_{e_k \in \phi^s} |e_k|}$.

Furthermore, similarly to $\phi$, whenever it is clear from the context, we drop the subscript and simply write $\delta$.

**Node overlap-aware similarity** The aforementioned measures used to specialize $\delta$ focus on the set-level identity of the hyperedges composing a REN. However, in many practical scenarios, especially those involving non-monotonic temporal changes, it is valuable to consider partial structural similarity between hyperedges. This leads to a node-overlap-aware similarity criterion that compares hyperedges based on their internal node composition.

Let us denote with $J(e_1, e_2) = \frac{|e_1 \cap e_2|}{|e_1 \cup e_2|}$ the Jaccard similarity between two hyperedges $e_1, e_2 \in \mathcal{E}$. Then, we define the node overlap-aware similarity between two RENs $\phi^r$ and $\phi^s$ as

$$\delta_{noa}(\phi^r, \phi^s) = \frac{1}{|\phi^r| + |\phi^s|} \left( \sum_{e_i \in \phi^r} \max_{e_j \in \phi^s} J(e_i, e_j) + \sum_{e_j \in \phi^s} \max_{e_i \in \phi^r} J(e_j, e_i) \right)$$

This measure performs a "best match" operation, assigning each hyperedge from one set its most similar counterpart in the other. The rationale is that even if hyperedges do not remain exactly identical over time, their structural similarity may persist due to overlapping node memberships. By capturing partial overlaps, this measure is particularly suited for temporal hypergraphs where hyperedges may evolve incrementally or fragment across time. This specialization of $\delta$ can be interpreted as a soft structural matching and complements the previous strict set-based criteria, allowing for more expressive modeling of continuity and structural evolution in temporal hypergraphs.

## 5 Defining and Studying Stability

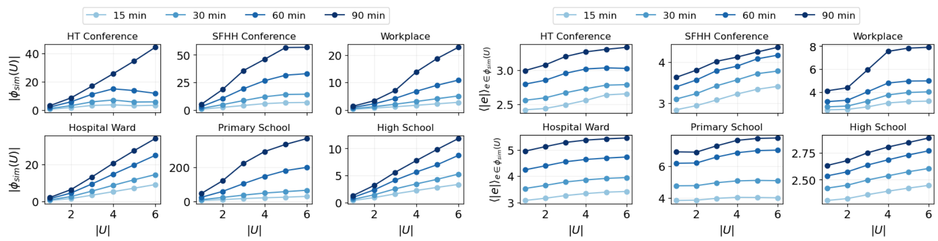
We observe that $\mathfrak{C}$ can be used to support a broad range of analytical tasks over temporal hypergraphs, and especially ones revolving around the study of RENs, such as clustering and structural comparison. Indeed, one of our objectives for this framework is to support the temporal evolution analysis of RENs, thus characterizing how local structures around groups of nodes transform over time. To pursue this objective, we focus on the concept of *stability* — i.e., the ability to determine whether a REN maintains its structural properties over time.

We can now formally define the concept of stability, and we refer to it as the STABILITY event. It describes whether a REN remains sufficiently similar across two consecutive time instants, thus allowing one to study the persistence of structure over time.

**Definition 4 (STABILITY event).** *Let $\mathcal{H} = (H_1, H_2, \ldots, H_t)$ be a temporal hypergraph, and let $\phi^r$ and $\phi^{r+1}$, $1 \leq r < t$, be two RENs rooted on the same set of nodes $U$ at time $r$ and $r+1$, respectively. Given a similarity criterion $\delta$ and a threshold $\gamma \in [0,1]$, we define the occurrence of a STABILITY event if the following holds:*

$$\delta(\phi^r, \phi^{r+1}) \geq \gamma.$$

The choice of $\gamma$ controls the tolerance for structural change: a high threshold enforces stricter persistence, while lower values allow for looser continuity. By evaluating stability over time, we can trace how the neighborhood of a group of nodes evolves, potentially identifying abrupt changes, recurring structures, or long-term cohesion. Furthermore, we believe it is worth noting that our definition borrows some aspects from the well-studied field of overlapping community discovery [8, 28]. Indeed, here we use the term event to indicate a temporal transformation occurring between two RENs, both rooted on the same node set, observed at different time points. This notion surely aligns with prior work in the study of overlapping communities, where events capture the evolution of group memberships; similarly, in our setting, events represent structural dynamics within the ego neighborhoods of (possibly multi-node) root sets. Interestingly, by coupling RENs and temporal events, such as STABILITY, our framework enables the study of temporal evolution whose behavior could be non-monotone, in contrast with previous work in literature [12]. Also, it is clear that other events, such as the ones commonly studied in literature, can be easily defined within our framework [15].




(a) (b)

Fig. 1: Number of hyperedges (a) and average size of hyperedges (b) for different root set sizes and aggregation windows in simple RENs.

# 6 Results

## 6.1 Datasets

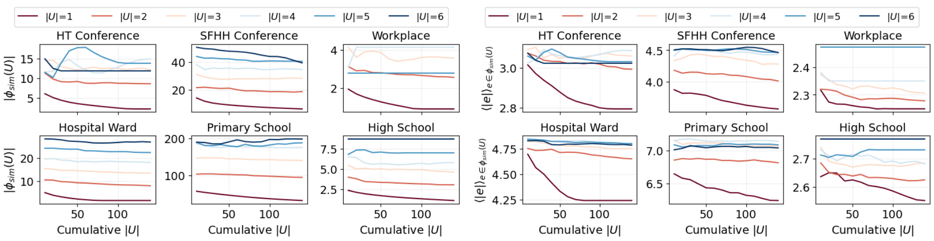
SocioPatterns[5] is a database collecting a variety of physical proximity and face-to-face interactions across several contexts. The datasets have typically been used to represent pairwise temporal networks at different granularities, but recently they have become a benchmark for higher-order networks as well [9]. The strategy we used for modeling hypergraphs is as follows: if at time t there are $n(n+1)/2$ dyads among the members of a set of $n$ nodes such that they form a fully connected clique, those links are promoted to form an $n$-hyperedge. Naturally, different aggregations correspond to different volumes of hyperedges. We choose 6 datasets to guarantee the heterogeneity of contexts: the HyperText and the SFHH Conferences [7], the Workplace [18], the Hospital Ward [31], and the Primary and High Schools [29, 26].

## 6.2 Structural Properties of Simple RENs

To assess the structural properties of RENs, we analyze both the number of hyperedges, $|\phi(U)|$, and the average size of the hyperedges, $\langle |e| \rangle_{e\in\phi}$, involved in RENs across different settings, focusing on the impact of root set size $|U|$ and time window length on the composition of the RENs.

Figure 1(a-b) shows these numbers for different values of $|U|$ and aggregation windows (15, 30, 60, 90 minutes). Both increase, across all datasets, on average, as the root set size and aggregation window length grow. This is expected, as longer time windows and larger root sets naturally aggregate more hyperedges. However, the extent is dataset-specific. For instance, the two conferences and the primary school show higher volumes (see Figure 1(a)) which can reflect the structured type of interactions occurring within these contexts, characterized by regular gatherings in common areas, such as parallel sessions, coffee breaks,

[5] http://www.sociopatterns.org/



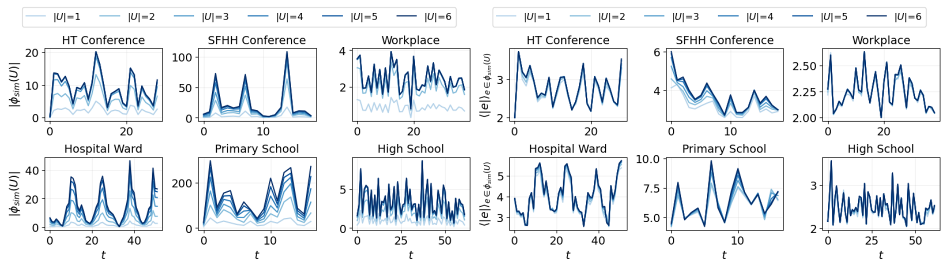

(a) (b)

Fig. 2: Cumulative distribution for the 150-top high-degree simple RENs of the number of hyperedges (a) and average size of hyperedges (b) for different root set sizes and aggregation windows.

lessons, and lunch time, often involving many people in a single room. In contrast, the high school, the workplace, and the hospital ward display relatively lower volumes, which can reflect more spontaneous and less structured group interactions (see Figure 1(a)). Regarding the average size of hyperedges in Figure 1(b), this number is considerably smaller. For instance, larger aggregation windows (60 and 90 minutes) result in the primary school dataset having an average of 6 elements per context: these are also the highest values across all datasets; their dynamics may be explained by the limited mobility and teacher supervision of children, which constrain their interactions. In contrast, the groups formed during conferences exhibit lower average sizes despite their relatively high number. This suggests that interactions in these contexts tend to occur within a larger number of smaller groups.

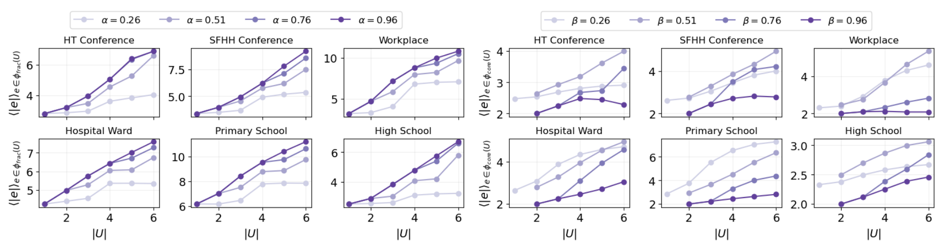
To further understand the structural characteristics of the interaction patterns within RENs, it is important to consider the role of high-degree nodes. Figure 2(a-b) shows the cumulative distribution of $|E(U)|$ and $\langle |e| \rangle_{e \in E(U)}$ for different values of $|U|$, given a fixed aggregation of 60 minutes. Specifically, for $|U| = 1$, we examine the distribution of the 150 nodes with the highest degrees. For $|U| = 2$, we consider the 150 highest-degree pairs of nodes, for $|U| = 3$, the top 150 highest-degree triples, etc. The degree of each node, pair, triple (and so on) is defined as the number of hyperedges containing the corresponding nodes. Beyond the expected trend that larger root set sizes let the descriptors increase, top-degree nodes appear relatively unaffected by variations in the number of hyperedges, but they seem to influence the average number of elements per hyperedge in certain datasets, e.g., for $|U| = 1$, in the HyperText conference and the hospital ward.

Finally, we are also interested in observing the temporal stability/evolution of such descriptors. Fiven a fixed aggregation window of 60 minutes, Figure 3(a-b) displays the time series of $|\phi(U)|$ and $\langle |e| \rangle_{e \in \phi(U)}$, again grouped by $|U|$, and highlights the temporal variability of hyperedge composition in simple RENs. Temporal fluctuations are dataset-dependent, even if we observe clear periodic




(a) (b)

Fig. 3: Time series of the number of hyperedges (a) and average size of hyperedges (b) for different root set sizes and a fixed aggregation window of 60 minutes in simple RENs.




(a) (b)

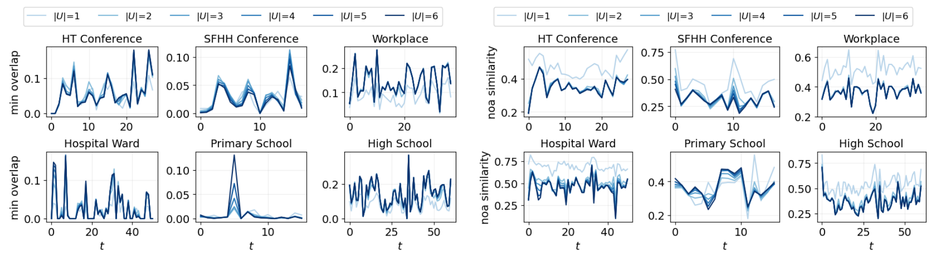
Fig. 4: Average hyperedge size as a function of root set size, given a fixed aggregation of 60 minutes, for $\alpha$-fractured (a) and $\beta$-core (b) RENs.

patterns in several datasets, particularly pronounced in the hospital ward and the SFHH conference, regarding the number of hyperedges over time (a). These recurring patterns may align with shift changes, rounds, or visiting hours, in the hospital ward, and with conference sessions.

### 6.3 Modifying "root" functions

Figure 4(a-b) illustrates in detail the differences between the other two RENs variants, corresponding to Definitions 2 and 3. Both panels report the average hyperedge size $\langle |e| \rangle$ as a function of the size of the root set $|U|$, under a fixed temporal aggregation window of 60 minutes, and for different values of the controlling parameters $\alpha$ and $\beta$.

In Figure 4(a), we focus on $\alpha$-fractured RENs, where $\phi_{\text{frac}_\alpha}(U)$ selects hyperedges that include at least a fraction $\alpha$ of the root nodes. Across all datasets, we observe that $\langle |e| \rangle$ generally increases with $|U|$ for a fixed $\alpha$, confirming that larger root sets tend to induce the selection of larger hyperedges. Moreover, for a fixed $|U|$, increasing $\alpha$ also leads to larger average hyperedge sizes: this is because




(a) (b)

Fig. 5: Minimum overlap (a) and node overlap-aware (b) similarities between consecutive timestamps for different root set sizes and a fixed aggregation window of 60 minutes in simple RENs.

higher $\alpha$ values require more node roots to be present in each hyperedge, favoring the selection of larger — and might be overlapping — group interactions.

In Figure 4(b), we analyze $\beta$-core RENs, where $\phi_{\text{core}_\beta}(U)$ selects hyperedges such that at least a fraction $\beta$ of their nodes belong to $U$. Here, the opposite trend emerges: for a fixed $|U|$, increasing $\beta$ systematically leads to a decrease in $\langle|e|\rangle$. This is expected as well, as larger $\beta$ values impose stricter dominance of the roots within the hyperedges, favoring the selection of smaller hyperedges where a few roots can constitute a large proportion.

### 6.4 Similarities and stability

Figure 5 highlights the similarity of groups across consecutive timestamps for two different definitions, minimum overlap (a) and node overlap-aware (b). Recall that minimum overlap focuses on the exact match of hyperedges, whereas the other measure allows for partial structural matches, with the aim of capturing continuity even when group boundaries shift or slightly reconfigure. These constraints, indeed, make the similarity values particularly different: They are extremely low when considering an exact match definition (a), and higher considering partial structural matches (b). The minimum overlap similarity values rarely exceed 0.2 across all datasets, highlighting that exact preservation of hyperedges between consecutive time windows is uncommon, even given an aggregation window of 60 minutes. Interestingly, we find the highest values in the high school and the workplace datasets, reflecting more selective peer and coworker relationships that maintain similar compositions across the observed consecutive time periods. The node overlap-aware similarity reveals much higher values, generally within the range of $0.3 - 0.8$, reflecting its ability to recognize partial continuity in these local and node-centric group structures. It is important to consider the case of the primary school dataset, for instance, where the similarity increases due to lunchtime, and the node overlap-aware similarity can capture this pattern. Analyzing similarities provides insight into the notion of *stability*; for instance, setting the right threshold, we can observe stability in the primary dataset only when the children are gathered during lunchtime. Moreover, as expected, larger root sets ($|U| > 2$) exhibit lower stability compared to smaller ones ($|U| = 1$ or 2), with the HyperText conference, the workplace, and the hospital ward exhibiting the larger gaps.

## 7 Conclusion

In this paper, we introduced a novel analytical framework for studying egocentric structures in temporal hypergraphs. We defined Rooted Ego-Networks (RENs), which generalize traditional ego-networks to accommodate both higher-order and temporal aspects. By integrating a suite of flexible similarity criteria, including both classical measures and a novel one, the node overlap-aware similarity, our framework enables the analysis of localized group dynamics beyond dyadic interactions and supports the investigation of their evolution and stability over time. Through a series of experiments on real-world datasets from the SocioPatterns project, we demonstrated the expressiveness of the framework, highlighting how the choice of REN definitions and similarity measures impacts the characterization of group persistence. Notably, our node overlap-aware similarity measure proved particularly effective in capturing the continuity of evolving local structures, even in non-monotonic temporal settings. We believe this work represents a first step toward systematically analyzing the (potentially non-monotonic) temporal evolution of ego-networks in higher-order settings. Several future research directions emerge from our study. For instance, RENs could be extended to incorporate node and hyperedge attributes, enabling the application of the framework to attributed temporal hypergraphs [16]. Furthermore, different types of temporal events, beyond stability, could be defined and analyzed to deepen the understanding of group dynamics over time.

## Availability of Materials and Code

The data used are available on the SocioPatterns website at the following link: http://www.sociopatterns.org/. The code used for generating figures is available at the following repository: https://github.com/dsalvaz/RENs. Tools to handle RENs and their temporal stability are implemented in the ASH python library for temporal hypergraphs: https://github.com/giuliorossetti/ASH .

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