

Sounding the Atmospheric Density at the Altitude of LARES and Ajisai During Solar Cycle 24

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The passive spherical satellites LARES and Ajisai, placed in nearly circular orbits with mean geodetic altitudes between 1450 and 1500 km, were used, during Solar Cycle 24, as powerful tools to probe the neutral atmosphere density and the performances of six thermospheric models in orbital regimes for which the role of dominant atomic species is contended by hydrogen and helium, and accurate satellite measurements are scarce. The starting point of the analysis was the accurate determination of the secular semi-major axis decay rate, leading to the estimation of drag coefficients for each satellite, thermospheric model and solar activity condition. The associated components of the neutral drag acceleration in a satellite-centered orbital system were computed as well. Following the estimation of the physical drag coefficients for LARES and Ajisai, it was then possible to derive the mean density biases of the models. None of them could be considered unconditionally the best, the specific outcome depending on solar activity and on the regions of the atmosphere crossed by the satellites. During solar maximum conditions, an additional density bias linked to the satellite orbit inclination was detected.

Key Words: Neutral Drag, Thermospheric Density Models, Solar Cycle 24, LARES, Ajisai.

Nomenclature

C_D	: drag coefficient
C_{DF}	: physical drag coefficient
C_{DH}	: drag coefficient for the H species
C_{DHe}	: drag coefficient for the He species
C_{DO}	: drag coefficient for the O species
$F_{10.7}$: solar radio flux at 10.7 cm, sfu
h	: satellite geodetic altitude, km
H	: hydrogen atom
He	: helium atom
i	: orbital inclination, °
M	: molar mass, g/mol
n_H	: H atmospheric number density, cm ⁻³
n_{He}	: He atmospheric number density, cm ⁻³
n_O	: O atmospheric number density, cm ⁻³
O	: oxygen atom
R	: gas constant, = 8.314 J mol ⁻¹ K ⁻¹
R_D	: drag radial component, m s ⁻²
sfu	: standard flux unit, 10 ⁻²² W m ⁻² Hz ⁻¹
Si	: silicon atom
T	: atmospheric temperature, K
T_D	: drag transverse component, m s ⁻²
T_{exo}	: exospheric temperature, K
V	: satellite speed w.r.t. the atmosphere, m/s
V_m	: most probable molecular speed, m/s
W	: tungsten atom
W_D	: drag normal component, m s ⁻²

α	: δ accommodation coefficient
δ	: C_D accommodation factor
φ	: geocentric latitude, °
μ	: ratio of the atmospheric constituent atomic mass to that of the material which makes up the surface of the satellite
ρ	: atmospheric density, g cm ⁻³
σ	: standard deviation
°	: degree
$\langle \dots \rangle$: mean of the parameter between brackets

1. Introduction

The primary objective of the Laser Relativity Satellite (LARES) mission is to improve the measurement accuracy of some General Relativity effects, in particular the Lense-Thirring one,¹⁻⁴⁾ providing as well significant contributions in the geodynamics and geodesy fields. In view of the main goal, this spherical satellite, made of tungsten alloy and uniformly hosting on its surface 92 fused silica (Suprasil®) corner cube laser retroreflectors, with a radius of 18.2 cm and a mass of 386.80 kg, was designed in order to minimize the effects of non-gravitational perturbations, making LARES the densest artificial object ever launched in space.⁵⁾ Put into a nearly circular orbit with an altitude of about 1454 km, inclined by 69.5°, and characterized by an extremely low area-to-mass-ratio (2.69×10^{-4} m²/kg), the satellite was not expected to be substantially affected by non-conservative forces.

Nevertheless, thanks to precise orbit determinations based on the laser ranging data of LARES provided by the International Laser Ranging Service (ILRS),⁶⁾ it was possible to detect a very small secular semi-major axis decay, which was consistent with a non-conservative net force acting nearly opposite to the velocity vector of the satellite, i.e. a drag-like effect.^{7,8)}

As such drag-like effect might be ascribable to neutral atmosphere drag, the accurate knowledge of the observed accelerations may be assumed as a starting point to probe, with LARES, the behavior of the neutral atmosphere under different environmental conditions, by exploring as well orbital regimes for which the dominant atomic species are helium and hydrogen, and accurate satellite measurements are scarce. The effect of the neutral drag perturbation in the first 3.7 years of the mission (from April 6, 2012, to December 25, 2015) had been investigated in a previous analysis,⁷⁾ indicating that nearly 99% of the observed secular semi-major axis decay was due to neutral atmosphere drag.

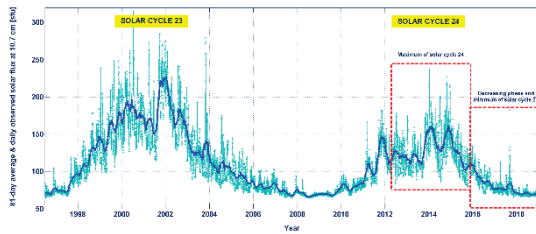


Fig. 1. Solar radio flux at 10.7 cm measured during the last two activity cycles of our star. The daily measurements and their running averages over three solar rotations (81 days) are reported.

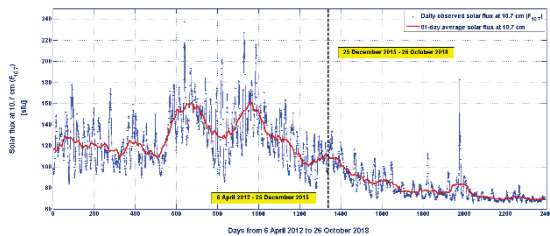


Fig. 2. Enlargement of the observed solar radio flux at 10.7 cm included in the two dashed boxes of Fig. 1. The analyses presented in this paper relate to the two time intervals separated by the vertical dashed line.

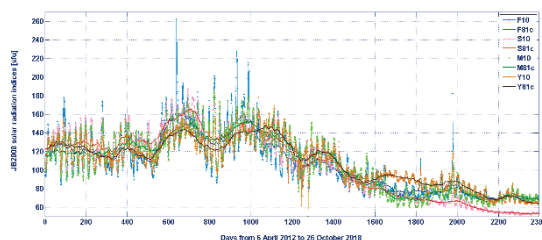


Fig. 3. Representation of the solar flux proxies used by the JB2008 thermospheric density model during the time span in which the study presented in this paper was carried out.

In support of such a relevant contribution of the neutral drag at the altitude of LARES, the orbital decay of another passive spherical satellite, Ajisai, just 40 km higher than LARES but at an inclination of 50.0° and with an area-to-mass-ratio 19.70 times greater ($5.30 \times 10^{-3} \text{ m}^2/\text{kg}$), had been investigated as well

in the same interval of time.⁸⁾ With a diameter of 215 cm and a mass of 685.2 kg, this hollow sphere was much more sensitive to non-gravitational perturbations, like atmospheric drag and radiative forces. It therefore represented an optimal target for comparative investigations of drag-like perturbations near the altitude of LARES. The surface of Ajisai is basically silica (SiO₂) in composition, being completely covered with 318 mirrors for reflecting sunlight and 1436 quartz corner cube retroreflectors for reflecting laser beams.⁹⁾

This paper extends our previous analyses to the first 6.6 years of the LARES mission (from April 6, 2012, to October 26, 2018), highlighting in particular the impact of the space weather conditions on the results obtained, being the first interval, from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24, and the second one, from December 25, 2015, to October 26, 2018, corresponding to the declining phase and to the deep minimum of the same cycle. The neutral drag perturbation acting both on LARES and Ajisai was investigated using an updated version of the SATRAP orbit propagator,^{10,11)} accounting for the real evolution of the space weather conditions. Among the many atmospheric density models implemented in SATRAP, the following six were applied in this study: Jacchia-Roberts 1971 (JR-71),¹²⁾ the Mass Spectrometer and Incoherent Scatter Radar Model 1986 (MSIS-86),¹³⁾ the Mass Spectrometer and Incoherent Scatter Radar Extended Model 1990 (MSISE-90),¹⁴⁾ NRLMSISE-00, developed at the US Naval Research Laboratory (NRL),^{15,16)} GOST-2004, issued by the State Committee on Standardization and Metrology of the Russian Federation,¹⁷⁾ and Jacchia-Bowman 2008 (JB2008).¹⁸⁾

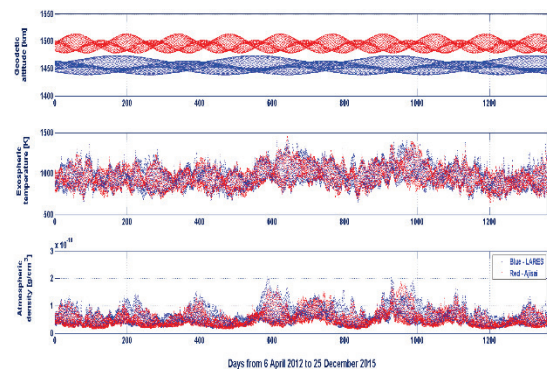


Fig. 4. Geodetic altitude of LARES and Ajisai, and corresponding exospheric temperature and atmospheric density, from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24.

2. Space Environment during Solar Cycle 24

Figure 1 shows the solar radio flux at 10.7 cm (2800 MHz) measured during the last two activity cycles of our star, i.e. 23 and 24. The daily values and the averages over three solar rotations (81 days) are reported in standard flux units (sfu). This radio flux is the most widely used proxy for solar activity and all the atmospheric models mentioned at the end of the previous section make use of it to properly represent the density variations of the thermosphere as a function of the varying emission of extreme ultraviolet radiation from the Sun. The

dashed boxes identify the two time intervals for which the analyses described in this papers were carried out. An enlargement of the observed flux is presented in Fig. 2. It should be remarked that the LARES mission, started at the beginning of 2012, has so far taken place completely within Solar Cycle 24, which was also the weakest one recorded since the beginning of 2800 MHz daily flux measurements, at Penticton, British Columbia, in 1947. Finally, Fig. 3 shows, during the same analyzed time span of Fig. 2, the full set of solar activity proxies used by the JB2008 atmospheric density model,^{19,20} in addition to the daily and averaged values of $F_{10.7}$.

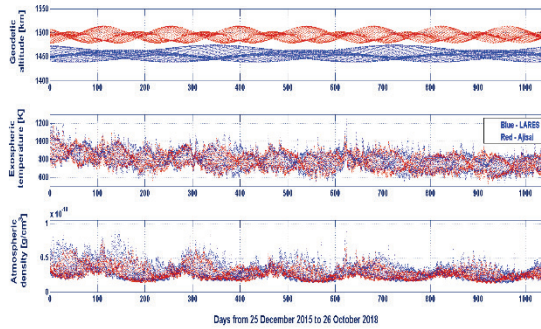


Fig. 5. Geodetic altitude of LARES and Ajisai, and corresponding exospheric temperature and atmospheric density, from December 25, 2015, to October 26, 2018, during the declining phase and the minimum of Solar Cycle 24.

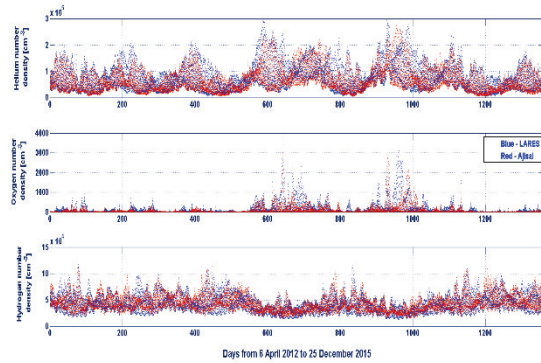


Fig. 6. Concentration of the three main atomic species, i.e. helium, hydrogen and oxygen, at the altitudes of LARES and Ajisai, from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24.

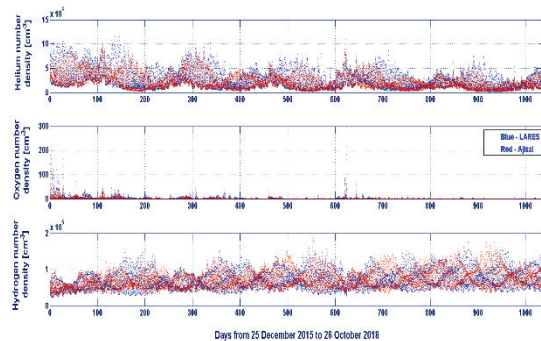


Fig. 7. Concentration of the three main atomic species, i.e. hydrogen, helium and oxygen, at the altitudes of LARES and Ajisai, from December 25, 2015, to October 26, 2018, during the declining phase and the minimum of Solar Cycle 24.

Table 1. Average atmospheric properties along the orbits of LARES, from April 6, 2012, to December 25, 2015.

Physical parameters	Mean values in the considered time span
Geodetic altitude	1454 km
Exospheric temperature	971 K
Atmospheric density	5.89×10^{-19} g/cm ³
H number density	4.17×10^4 cm ⁻³
He number density	7.36×10^4 cm ⁻³
O number density	9.16×10^1 cm ⁻³

Table 2. Average atmospheric properties along the orbits of Ajisai, from April 6, 2012, to December 25, 2015.

Physical parameters	Mean values in the considered time span
Geodetic altitude	1494 km
Exospheric temperature	960 K
Atmospheric density	5.21×10^{-19} g/cm ³
H number density	4.25×10^4 cm ⁻³
He number density	6.51×10^4 cm ⁻³
O number density	5.46×10^1 cm ⁻³

Table 3. Average atmospheric properties along the orbits of LARES, from December 25, 2015, to October 26, 2018.

Physical parameters	Mean values in the considered time span
Geodetic altitude	1454 km
Exospheric temperature	796 K
Atmospheric density	3.11×10^{-19} g/cm ³
H number density	7.10×10^4 cm ⁻³
He number density	2.64×10^4 cm ⁻³
O number density	1.96×10^0 cm ⁻³

Table 4. Average atmospheric properties along the orbits of Ajisai, from December 25, 2015, to October 26, 2018.

Physical parameters	Mean values in the considered time span
Geodetic altitude	1494 km
Exospheric temperature	784 K
Atmospheric density	2.78×10^{-19} g/cm ³
H number density	7.17×10^4 cm ⁻³
He number density	2.19×10^4 cm ⁻³
O number density	9.77×10^{-1} cm ⁻³

For the two time intervals for which the neutral drag modeling was carried out, i.e. from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24, and from December 25, 2015, to October 26, 2018, during the declining phase and the low minimum of the same cycle, Figs. 4 and 5 plot the geodetic altitude of LARES and Ajisai, and the corresponding exospheric temperature and atmospheric density for the two satellites. Figures 6 and 7 show instead the concentration of the three main atomic species, i.e. helium, hydrogen and oxygen. All these figures were obtained with the NRLMSISE-00 model. The average values found are listed in Tables 1 (LARES, around the solar cycle maximum), 2 (Ajisai, around the solar cycle maximum), 3 (LARES, during the solar cycle decrease and minimum), and 4 (Ajisai, during the solar cycle decrease and minimum).

In the time interval centered around the solar maximum, the dominant atmospheric atomic species at the altitudes of the two

satellites was He (>60%), while when the exospheric temperature decreased (by about 175 K, on average) during the declining and minimum phases of the solar cycle, the dominant atomic species was H (around 75%) and the overall mean atmospheric density diminished by 47%. The total period considered for the analysis was then well representative of the varying environmental conditions encountered during a full solar activity cycle, even though very high activity levels were never experienced for more than a few days, being Cycle 24 particularly weak.

3. Semi-Major Axis Decay

The semi-major axis decay analysis for the two satellites, presented elsewhere for the first 3.7 years,^{7,8)} was repeated and extended here over 6.6 years. The decay rate of LARES was obtained by processing the laser ranging information (normal points) provided by the ILRS with the NASA/GSFC software package GEODYN II.^{21,22)} The observables were fitted over 7-day orbit arcs and the precise orbit determination process uncovered the details of the semi-major axis secular decrease (Fig. 8). Since the launch, the mean semi-major axis of the satellite diminished by little more than 5 m. During the period (1358 days, 3.72 years) from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24, the average secular decay rate was 2.74 mm/d (i.e. 1.00 m per year), revealing the action of a non-conservative net force on the satellite, with a mean along-track acceleration component of -1.444×10^{-11} m/s². In the period (1036 days, 2.84 years) from December 25, 2015, to October 26, 2018, during the declining phase and the low minimum of the same cycle, the average secular decay rate was 1.50 mm/d (i.e. 0.55 m per year), corresponding to a mean along-track acceleration component of -7.900×10^{-12} m/s².

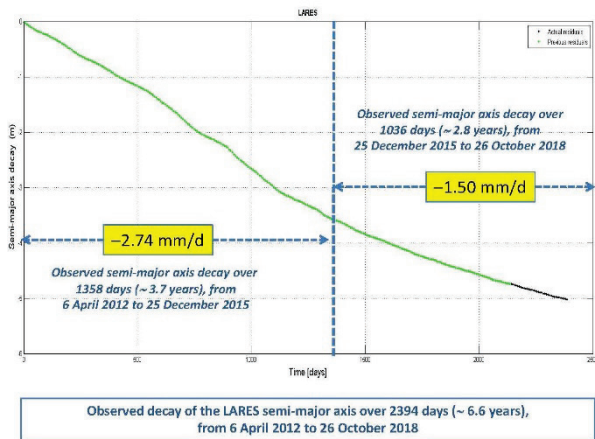


Fig. 8. Semi-major axis decay of LARES determined over 2394 days, from April 6, 2012, to October 26, 2018, processing the laser ranging data with GEODYN II.

The orbital decay of Ajisai due to the drag of neutral atmosphere is known since its launch and has been used over the years to check the predictions of several thermospheric density models under various conditions of solar and

geomagnetic activity.²³⁻²⁵⁾ Concerning the present study, the average secular decrease of the semi-major axis of the satellite was obtained by analyzing with SATRAP the two-line elements sets determined by the US Strategic Command and issued by the Space Track Organization.^{26,27)} During the period from April 6, 2012, to December 25, 2015, centered around the maximum of Solar Cycle 24, the average secular decay rate was 38.44 mm/d (i.e. 14.04 m per year), corresponding to a mean along-track acceleration component of -2.013×10^{-10} m/s². From December 25, 2015, to October 26, 2018, during the declining phase and the low minimum of the same cycle, the average secular decay rate was 24.48 mm/d (i.e. 8.94 m per year), corresponding to a mean along-track acceleration component of -1.282×10^{-10} m/s².

Table 5. Drag coefficients able to reproduce the secular semi-major axis decay of LARES, from April 6, 2012, to December 25, 2015.

Atmospheric model	Drag coefficient C_D
JR-71	3.955
MSIS-86	3.713
MSISE-90	3.730
NRLMSISE-00	3.783
GOST2004	4.207
JB2008	3.051

Table 6. Drag coefficients able to reproduce the secular semi-major axis decay of Ajisai, from April 6, 2012, to December 25, 2015.

Atmospheric model	Drag coefficient C_D
JR-71	3.420
MSIS-86	3.139
MSISE-90	3.131
NRLMSISE-00	3.194
GOST2004	3.340
JB2008	2.482

Table 7. Drag coefficients able to reproduce the secular semi-major axis decay of LARES, from December 25, 2015, to October 26, 2018.

Atmospheric model	Drag coefficient C_D
JR-71	3.951
MSIS-86	4.003
MSISE-90	4.018
NRLMSISE-00	3.923
GOST2004	3.397
JB2008	2.635

Table 8. Drag coefficients able to reproduce the secular semi-major axis decay of Ajisai, from December 25, 2015, to October 26, 2018.

Atmospheric model	Drag coefficient C_D
JR-71	3.846
MSIS-86	3.940
MSISE-90	3.928
NRLMSISE-00	3.820
GOST2004	3.086
JB2008	2.445

Therefore, even though Ajisai was 40 km higher than LARES, its greater area-to-mass ratio, by a factor of nearly 20,

led to significantly larger mean along-track drag-like accelerations, by almost 14 times during the first period, and by more the 16 times during the second one. As a consequence, Ajisai was much more sensitive than LARES to thermospheric neutral drag, and also to other non-gravitational perturbations, but the orbit determinations of the latter were so accurate to make both satellites powerful and complementary probes to investigate the properties of the atmosphere at those heights.

4. Neutral Atmosphere Drag Modeling

Having detected and measured the secular semi-major axis decay of LARES and Ajisai during the time spans of interest, the corresponding mean along-track accelerations were modeled with SATRAP, using the six thermospheric density models listed at the end of the introduction and taking into account the real evolution of the space weather conditions. All the models were able to reproduce the observed secular decay of the semi-major axes, but such outcome was obtained converging to different drag coefficients, reflecting the density biases among the models themselves. The results obtained are summarized in Tables 5, 6, 7 and 8.

For LARES, around the maximum of Solar Cycle 24, with $\langle T_{exo} \rangle = 971$ K, $\langle C_D \rangle = 3.740 \pm 0.385$ (1σ , $\approx 10\%$), while during the declining phase and the minimum of the cycle, with $\langle T_{exo} \rangle = 796$ K, $\langle C_D \rangle = 3.655 \pm 0.551$ (1σ , $\approx 15\%$). The highest average atmospheric density, and therefore the lowest C_D (-18.4% with respect to $\langle C_D \rangle$ around the maximum and -27.9% during the declining phase and the minimum of the cycle), were predicted by JB2008, while the lowest $\langle \rho \rangle$ and highest C_D were predicted by GOST2004 around the maximum ($+12.5\%$ with respect to $\langle C_D \rangle$) and by MSISE-90 during the declining phase and the minimum of the cycle ($+10.0\%$ with respect to $\langle C_D \rangle$). Regarding the change of C_D resulting from the diminishing value of $\langle T_{exo} \rangle$, from 971 K to 796 K, it should be remarked that with JR-71 it basically displayed no variation (-0.1%), with GOST2004 and JB2008 it dropped by about 19% and 14%, respectively, while it increased by nearly 4% with NRLMSISE-00, and by almost 8% with MSIS-86 and MSISE-90.

Concerning Ajisai, around the maximum of Solar Cycle 24, with $\langle T_{exo} \rangle = 960$ K, $\langle C_D \rangle = 3.118 \pm 0.332$ (1σ , $\approx 11\%$), while during the declining phase and the minimum of the cycle, with $\langle T_{exo} \rangle = 784$ K, $\langle C_D \rangle = 3.511 \pm 0.614$ (1σ , $\approx 17\%$). Again, the highest average atmospheric density, and therefore the lowest C_D (-20.4% with respect to $\langle C_D \rangle$ around the maximum and -30.4% during the declining phase and the minimum of the cycle), were predicted by JB2008, while the lowest $\langle \rho \rangle$ and highest C_D were predicted by JR-71 around the maximum ($+9.7\%$ with respect to $\langle C_D \rangle$) and by MSIS-86 during the declining phase and the minimum of the cycle ($+12.2\%$ with respect to $\langle C_D \rangle$). Regarding the change of C_D resulting from the diminishing value of $\langle T_{exo} \rangle$, from 960 K to 784 K, it should be highlighted that with JB2008 it displayed a very small variation (-1.5%), with GOST2004 it decreased by less than 8%, while it increased by almost 12% with JR-71, by nearly 20% with NRLMSISE-00, and by more than 25% with MSIS-86 and MSISE-90.

Comparing the C_D estimates obtained for LARES and Ajisai, both passive spherical satellites at nearly the same altitude, but with quite different orbital inclinations (about 70° and 50° , respectively), led to the following main conclusions:

1. All the C_D found for Ajisai were systematically smaller than those of LARES, with the same time intervals, space weather conditions and thermospheric density models;
2. Around the maximum of Solar Cycle 24, $\langle C_D \rangle$ for Ajisai was smaller by 16.6% compared with that of LARES;
3. During the decreasing phase and the minimum of the cycle, $\langle C_D \rangle$ for Ajisai was smaller by 3.9% compared with that of LARES;
4. When $\langle T_{exo} \rangle$ passed from ≈ 965 K to ≈ 790 K, for LARES $\langle C_D \rangle$ decreased by 2.3%, while for Ajisai $\langle C_D \rangle$ increased by 12.6%, leading to a much better agreement between the mean drag coefficients of the two satellites;
5. The standard deviations of the C_D distributions were similar for the two satellites, depending on the environmental conditions: 10-11% around the solar maximum, 15-17% during the declining phase and the minimum;
6. For LARES, when the solar activity diminished, the C_D of JR-71 remained basically the same, those of MSIS-86, MSISE-90 and NRLMSISE-00 increased, and those of GOST2004 and JB2008 decreased;
7. For Ajisai, when the solar activity diminished, the C_D of JB2008 remained substantially stable, those of JR-71, MSIS-86, MSISE-90 and NRLMSISE-00 increased, and that of GOST2004 decreased;
8. The JB2008 model predicted for both satellites and space weather conditions the highest atmospheric densities and lowest C_D ;
9. In all the cases, MSIS-86, MSISE-90 and NRLMSISE-00 produced very similar results, with maximum discrepancies of 3%, or less; this was expected, due to the common origin and heritage of the three models.

In any case, all the changes and differences recorded in the models and among them are perfectly compatible with their known uncertainties and biases, even at the much lower altitudes (< 1000 km) for which they were originally developed. Rather, their performances around 1500 km, i.e. at the altitude of LARES and Ajisai, turned out to be surprisingly good and above expectations.

5. Neutral Drag Acceleration Components

The six thermospheric density models were used within SATRAP to compute the components of the neutral drag acceleration on both satellites in the reference system $R_D T_D W_D$,²⁸⁾ having the origin in the center of mass of the satellites and with three orthogonal axes aligned, respectively, along the radial direction, from the center of the Earth to the satellite, normal to the orbit plane, in the direction of the osculating orbital angular momentum, and in the transverse direction, lying on the orbit plane 90° from the radial direction, nearly aligned with the satellite velocity vector. In each case, the drag coefficients were rescaled according to the results presented in Tables 5, 6, 7 and 8 in order to reproduce, with every atmospheric density model, the observed secular semi-

major axis decay and the corresponding drag-like perturbing acceleration. The results of the simulations are summarized in Figs. 9-20, where, for each satellite and time interval, every neutral drag acceleration component is plotted for all the six atmospheric density models.

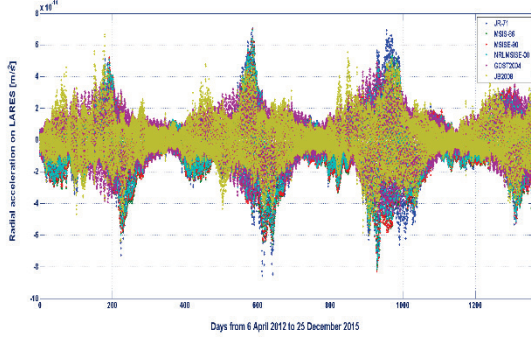


Fig. 9. Radial component (R_D) of the neutral drag acceleration on LARES, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

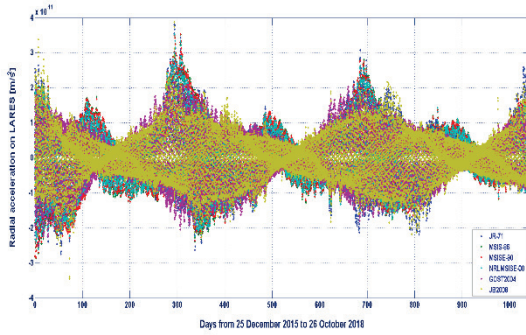


Fig. 10. Radial component (R_D) of the neutral drag acceleration on LARES, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

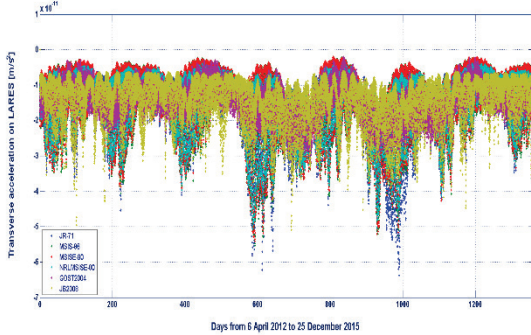


Fig. 11. Transverse component (T_D) of the neutral drag acceleration on LARES, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

Concerning R_D , the agreement among the models was quite good, both for LARES and Ajisai, and for “high” and “low” solar activity. $\langle R_D \rangle$ was very close to zero ($< 2 \times 10^{-15} \text{ m/s}^2$) in any situation. For LARES, the greatest excursion of R_D was $\pm 8 \times 10^{-14} \text{ m/s}^2$ (Fig. 9) around the maximum of Solar Cycle 24, and $\pm 4 \times 10^{-14} \text{ m/s}^2$ (Fig. 10) during the declining phase and the minimum of the same cycle. For Ajisai, the corresponding excursions of R_D were $\pm 1.5 \times 10^{-12} \text{ m/s}^2$ (Fig. 15) and $\pm 6 \times 10^{-13} \text{ m/s}^2$ (Fig. 16), respectively.

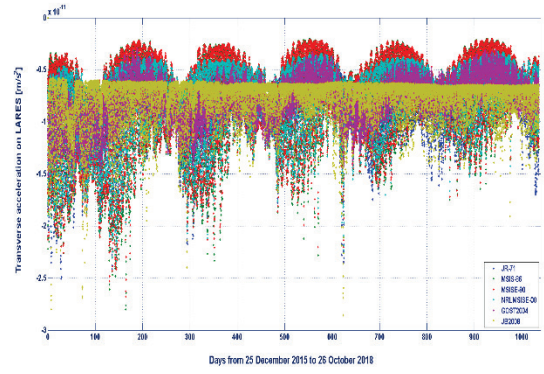


Fig. 12. Transverse component (T_D) of the neutral drag acceleration on LARES, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

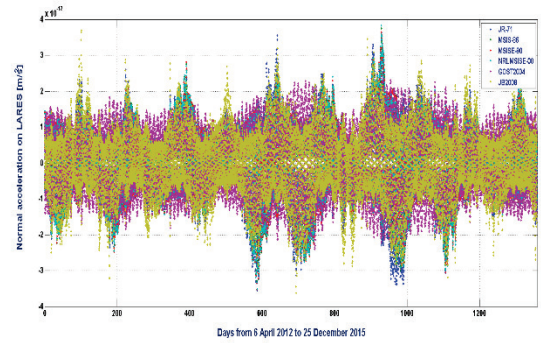


Fig. 13. Normal component (W_D) of the neutral drag acceleration on LARES, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

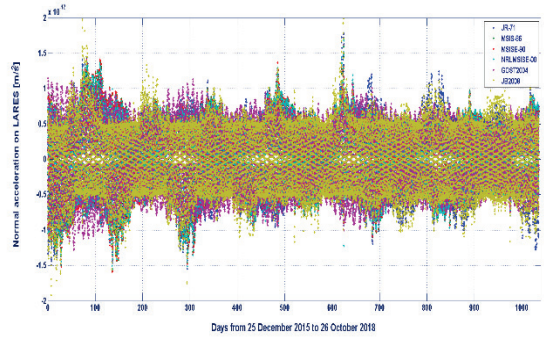


Fig. 14. Normal component (W_D) of the neutral drag acceleration on LARES, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

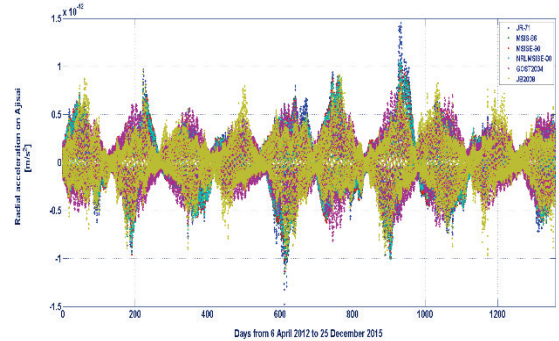


Fig. 15. Radial component (R_D) of the neutral drag acceleration on Ajisai, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

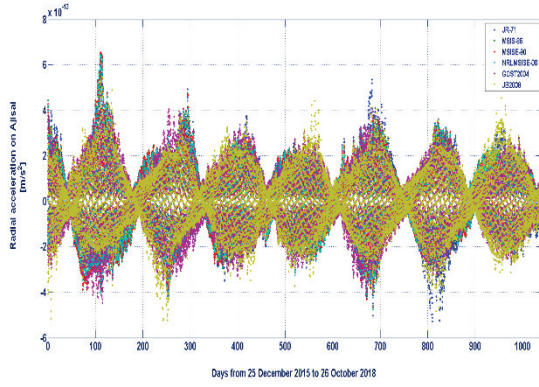


Fig. 16. Radial component (R_D) of the neutral drag acceleration on Ajisai, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

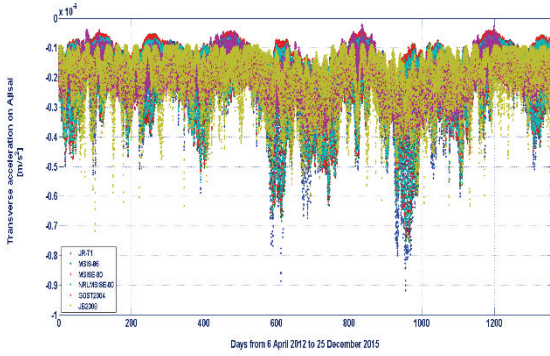


Fig. 17. Transverse component (T_D) of the neutral drag acceleration on Ajisai, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

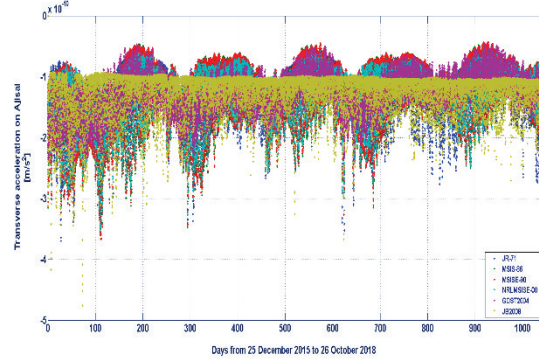


Fig. 18. Transverse component (T_D) of the neutral drag acceleration on Ajisai, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

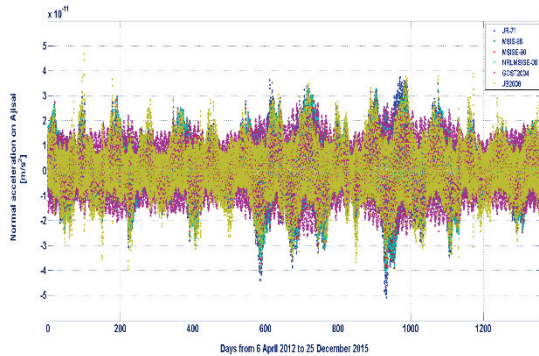


Fig. 19. Normal component (W_D) of the neutral drag acceleration on Ajisai, from April 6, 2012, to December 25, 2015, computed with the six atmospheric density models.

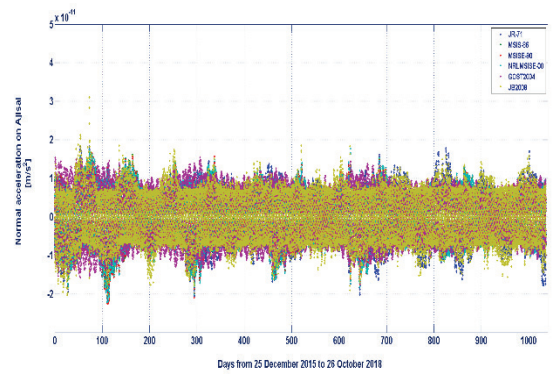


Fig. 20. Normal component (W_D) of the neutral drag acceleration on Ajisai, from December 25, 2015, to October 26, 2018, computed with the six atmospheric density models.

Regarding the widely prevalent acceleration component T_D , the agreement among the density models, apart from the same value of $\langle T_D \rangle$ needed for reproducing the observed secular decay rate, was much less good, due to the presence of different periodic terms, intrinsic to the definition of the models themselves.^{7,8)} Due to their common heritage and development, MSIS-86, MSISE-90 and NRLMSISE-90 generally exhibited a very good agreement, and in most cases also JR-71 was not far from them. However, both GOST2004 and JB2008 displayed a quite distinctive behavior (see Figs. 11, 12, 17 and 18), further confirming the results already obtained around the maximum of Solar Cycle 24 and anticipated elsewhere.^{7,8)} For LARES, the greatest excursion of T_D with respect to the averaged secular value $\langle T_D \rangle$ was $-5 \times 10^{-11} \text{ m/s}^2$ (Fig. 11) around the solar activity maximum and $-2 \times 10^{-11} \text{ m/s}^2$ (Fig. 12) during the declining phase and the minimum of the same cycle. For Ajisai, the corresponding greatest excursions of T_D were $-7 \times 10^{-10} \text{ m/s}^2$ (Fig. 17) and $-3 \times 10^{-10} \text{ m/s}^2$ (Fig. 18), respectively. However, GOST2004 always exhibited much smaller fluctuations, by factors from 3 to 7, and relatively symmetrical long-period oscillations, a pattern not shared by any of the other models.

Concerning W_D , the agreement among the models was quite better, both for LARES and Ajisai, and for “high” and “low” solar activity, but JB2008 and, in particular, GOST2004, exhibited again rather distinct features. $\langle W_D \rangle$ was very close to zero ($< 8 \times 10^{-14} \text{ m/s}^2$ for Ajisai and $< 8 \times 10^{-15} \text{ m/s}^2$ for LARES) in any space environment condition. For LARES, the greatest excursion of W_D was $\pm 4 \times 10^{-12} \text{ m/s}^2$ (Fig. 13) around the maximum of Solar Cycle 24, and $\pm 2 \times 10^{-12} \text{ m/s}^2$ (Fig. 14) during the declining phase and the minimum of the same cycle. For Ajisai, the equivalent excursions of W_D were $\pm 5 \times 10^{-11} \text{ m/s}^2$ (Fig. 19) and $\pm 2 \times 10^{-11} \text{ m/s}^2$ (Fig. 20), respectively.

6. Biases of the Density Models

In order to further investigate the neutral atmosphere at the altitude of LARES and Ajisai and evaluate the performances of the six atmospheric models used in the present analysis, an attempt was made to estimate the “physical” drag coefficient C_{DF} of the two satellites. When $V > V_m$, as in the cases considered here, C_{DF} can be computed as follows:²⁹⁻³¹⁾

$$C_{DF} = \delta \left[2 + \frac{4}{3} \left(\frac{V_m}{v} \right)^2 - \frac{2}{15} \left(\frac{V_m}{v} \right)^4 \right], \quad (1)$$

where

$$V_m = \sqrt{\frac{2RT}{10^3 M}}, \quad (2)$$

$$\delta = 1 + \frac{4}{9} \sqrt{1 - \alpha}, \quad (3)$$

and

$$\alpha = \frac{3.8\mu}{(1+\mu)^2}. \quad (4)$$

The computation was independently carried out for each of the relevant atomic species of the atmosphere at the altitude of interest, i.e. H, He and O. For the numerical constant appearing in the numerator of Eq. (4), the value proposed by Jacchia, i.e. 3.8, was adopted.³¹⁾ Moreover, the chemical composition of the surface of LARES was supposed to be 26% silicon dioxide (SiO₂) and 74% tungsten (W). Finally, the comprehensive physical drag coefficient applicable to the neutral atmosphere as a whole, for each satellite and time interval considered, was obtained as follows:

$$C_{DF} = \frac{C_{DH}n_H + C_{DHe}n_{He} + C_{DO}n_O}{n_H + n_{He} + n_O}. \quad (5)$$

Table 9 shows the attained mean physical drag coefficients. Under the same space weather conditions, they turned out to be very similar, with a difference between LARES and Ajisai of about 0.5% around the maximum of Solar Cycle 24, and a difference of just 0.05% during the declining phase and the low minimum of the same cycle. The $\langle C_{DF} \rangle$ of both satellites increased by more or less 3% in response to the atmospheric changes, from a He dominated to a H dominated composition, when $\langle T_{exo} \rangle$ diminished from ≈ 965 K to ≈ 790 K.

Assuming that the theoretical values of $\langle C_{DF} \rangle$ thus obtained correctly represented the real mean drag coefficients of the two satellites, comparing them with the “observed” values, listed in Tables 5, 6, 7 and 8, allowed a detailed evaluation of the overall performances of the atmospheric density models, at the altitude of 1450-1500 km, as a function of solar activity and satellite orbit inclination. The further relevance of the latter parameter derives from the fact that LARES, with its greater inclination, probes medium to high latitude atmospheric regions, with $50^\circ < |\varphi| < 70^\circ$, which are inaccessible to Ajisai and can respond differently to space weather and solar activity disturbances. The comparison between “observed” and “theoretical” mean drag coefficients led to the estimation of the average density biases displayed in Tables 10, 11, 12 and 13. A positive value implies that the model overestimates the actual average density, while a negative value means that the model underestimates it.

Around the maximum of Solar Cycle 24, along the orbits of LARES (see Table 10), characterized by $0^\circ \leq |\varphi| < 70^\circ$, five atmospheric models out of six significantly underestimated the average atmospheric density. MSIS-86, MSISE-90 and NRLMSISE-00, as expected, provided very similar results, with a density underestimation by 17-19%. For JR-71 it neared 25%, while for GOST2004 exceeded 30%. The best model, as far as the average atmospheric density was concerned, resulted to be JB2008, which overestimated the density by only 4%.

Table 9. Mean physical drag coefficients estimated for LARES ($\langle h \rangle = 1454$ km, $i = 69.5^\circ$) and Ajisai ($\langle h \rangle = 1494$ km, $i = 50.0^\circ$) as a function of the mean exospheric temperature.

Satellite	$\langle T_{exo} \rangle$	$\langle C_{DF} \rangle$
LARES	971 K	3.181
Ajisai	960 K	3.164
LARES	796 K	3.276
Ajisai	784 K	3.277

Table 10. Estimated mean density $\langle \rho \rangle$ biases of the atmospheric models for $\langle h \rangle = 1454$ km, $i = 69.5^\circ$ and $\langle T_{exo} \rangle = 971$ K (LARES, from April 6, 2012, to December 25, 2015).

Atmospheric model	Mean density $\langle \rho \rangle$ bias (%)
JR-71	-24.3
MSIS-86	-16.7
MSISE-90	-17.3
NRLMSISE-00	-18.9
GOST2004	-32.2
JB2008	+4.09

Table 11. Estimated mean density $\langle \rho \rangle$ biases of the atmospheric models for $\langle h \rangle = 1494$ km, $i = 50.0^\circ$ and $\langle T_{exo} \rangle = 960$ K (Ajisai, from April 6, 2012, to December 25, 2015).

Atmospheric model	Mean density $\langle \rho \rangle$ bias (%)
JR-71	-8.08
MSIS-86	+0.80
MSISE-90	+1.05
NRLMSISE-00	-0.94
GOST2004	-5.55
JB2008	+21.6

Table 12. Estimated mean density $\langle \rho \rangle$ biases of the atmospheric models for $\langle h \rangle = 1454$ km, $i = 69.5^\circ$ and $\langle T_{exo} \rangle = 796$ K (LARES, from December 25, 2015, to October 26, 2018).

Atmospheric model	Mean density $\langle \rho \rangle$ bias (%)
JR-71	-20.6
MSIS-86	-22.2
MSISE-90	-22.7
NRLMSISE-00	-19.8
GOST2004	-3.71
JB2008	+19.6

Table 13. Estimated mean density $\langle \rho \rangle$ biases of the atmospheric models for $\langle h \rangle = 1494$ km, $i = 50.0^\circ$ and $\langle T_{exo} \rangle = 784$ K (Ajisai, from December 25, 2015, to October 26, 2018).

Atmospheric model	Mean density $\langle \rho \rangle$ bias (%)
JR-71	-17.4
MSIS-86	-20.2
MSISE-90	-19.9
NRLMSISE-00	-16.6
GOST2004	+5.83
JB2008	+25.4

Along around the maximum of Solar Cycle 24, but along the orbits of Ajisai (see Table 11), characterized by $0^\circ \leq |\varphi| \leq 50^\circ$, the situation was completely different. In fact, JB2008 became

the worst model, overestimating the mean density by nearly 22%. GOST2004 and JR-71 underestimated the actual atmospheric density by a relatively small amount, around 6% and 8%, respectively, while the three models sharing a common origin and development, i.e. MSIS-86, MSISE-90 and NRLMSISE-00, resulted quite accurate, with biases of more or less 1%. The significant differences between the results obtained for LARES and Ajisai around the solar maximum might suggest that all the density models considered in the present analysis are not able to accurately describe latitude dependent effects, probably more pronounced at high ϕ values during periods of high solar activity.⁸⁾

During the declining phase and the minimum of Solar Cycle 24, these latitude dependent density biases mostly disappeared and all the models provided rather similar estimates, with the possible exception of GOST2004, for both LARES and Ajisai (see Tables 12 and 13). In both cases, GOST2004 was by far the best model, as far as the average atmospheric density was concerned, underestimating the density by less than 4% along the trajectory of LARES and overestimating it by less than 6% along the trajectory of Ajisai. JR-71, MSIS-86, MSISE-90 and NRLMSISE-00, on the other hand, underestimated the mean density by amounts between more than 16% and less than 23% for both satellites. JB2008, finally, overestimated the average density by about 20-25%.

7. Conclusions and Future Work

The extensive set of analyses carried out allowed to use the passive spherical satellites LARES and Ajisai as powerful tools to probe the neutral atmosphere properties at the altitude of 1450-1500 km during Solar Cycle 24, both around the maximum and during the declining phase and the deep minimum. The six thermospheric models used in this study, i.e. JR-71, MSIS-86, MSISE-90, NRLMSISE-00, GOST2004 and JB2008, were developed for $h < 1000$ km, but exhibited anyhow a quite satisfactory level of performances and variability as a function of the varying space weather conditions and orbit geometry with respect to the atmosphere. In other words, the comparative analyses and the estimated average density biases were fully compatible with the known uncertainties and discrepancies of the models, and the overall picture was not significantly worse than observed at the lower altitudes for which those same models were developed, and mostly tested, so far.³²⁻³⁵⁾ Moreover, these results alone are not enough to suggest the action, at the altitude of LARES and Ajisai, of other dissipative non-gravitational perturbations capable of producing a significant fraction of the secular decay of the semi-major axis observed for the two satellites.^{7,8)}

It should be also pointed out that the results obtained for Ajisai with the three oldest models, i.e. JR-71, MSIS-86 and MSISE-90, were in qualitative agreement with those obtained during Solar Cycle 22, from April 24, 1988, to June 30, 1997,^{23,24)} even if that cycle was much more intense than the current one and the periods considered are then not directly comparable.

Among the six thermospheric models used, none could be considered unconditionally the best, confirming a situation

already familiar at lower altitudes. The outcome, in fact, depended on the specifics of the circumterrestrial environment determined by solar activity and space weather, as well as by the regions of the atmosphere crossed by the satellites. Looking at the mean density biases estimated in this study, JB2008 seemed the most accurate around the solar maximum for orbits reaching a geocentric latitude close to 70° , while MSIS-86, MSISE-90 and NRLMSISE-00 resulted the best if the ceiling latitude was 50° . During the diminishing phase and the minimum of the solar cycle, on the other hand, the most accurate model was GOST2004, irrespective of the latitudes crossed by the satellites.

A further result of the analysis was that all the models displayed, around the solar maximum, a significant and roughly comparable latitude dependent bias.⁸⁾ In other words, maintaining the same nearly circular orbit at the same altitude, but passing from $i \approx 50^\circ$ to $i \approx 70^\circ$, would have led to an average deficit in the computed mean atmospheric density by about 19% compared with the real value.

According to the predictions issued in April 2019 by an international panel of experts gathered by the National Oceanic and Atmospheric Administration (NOAA), Solar Cycle 24, already one of the feeblest on record, will reach its lowest point sometime between July 2019 and September 2020, followed by a slow recovery toward solar maximum in 2023-2026.³⁶⁾ Solar Cycle 25 is expected to be very similar to Cycle 24, with a further really weak maximum, preceded by a long, and very deep, minimum.³⁶⁾

In the coming decade there will be therefore the possibility to additionally extend and possibly confirm the analysis outlined in this study, using again LARES and Ajisai as atmospheric density probes. Of particular scientific relevance will be the investigation of the environmental conditions during the long and deep minimum between Cycles 24 and 25, and the repetition of the analysis during the maximum of Cycle 25, in order to confirm the latitude dependent density bias of the models. A supplementary improvement could be represented by the inclusion in the study of additional state-of-the-art thermospheric density models.

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