Experimental investigation of two dimensional Anderson localization of light in the presence of a nonlocal nonlinearity

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GGI-Advances in Non Equilibrium Statistical Mechanics, 19 June 2014

Two directions

- **Dissipative (gain and losses)**:
  - Random lasers
- **Hamiltonian case**:
  - Transverse localization

•Introduction
•Effect of nonlinearity
•Disordered fiber experiments
•Action at a distance

- Dep Physics Sapienza Rome
    - Marco Leonetti (IPCF-CNR)
    - Viola Folli (IPCF-CNR)

- University of Wisconsin-Milwuakee
    - Salman Karbasi & Arash Mafi

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

- Above a certain amount of disorder no transport is possible „Anderson localization"

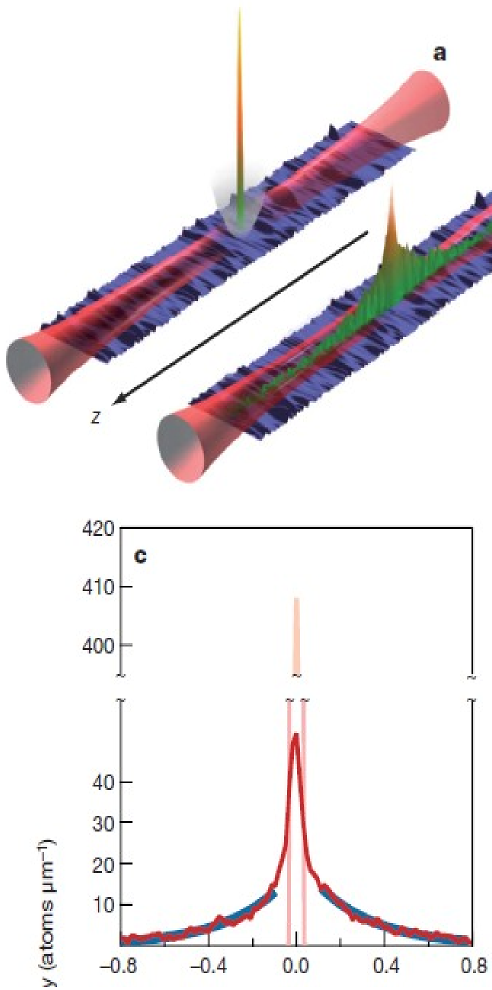
- The reason: localized states due to disorder

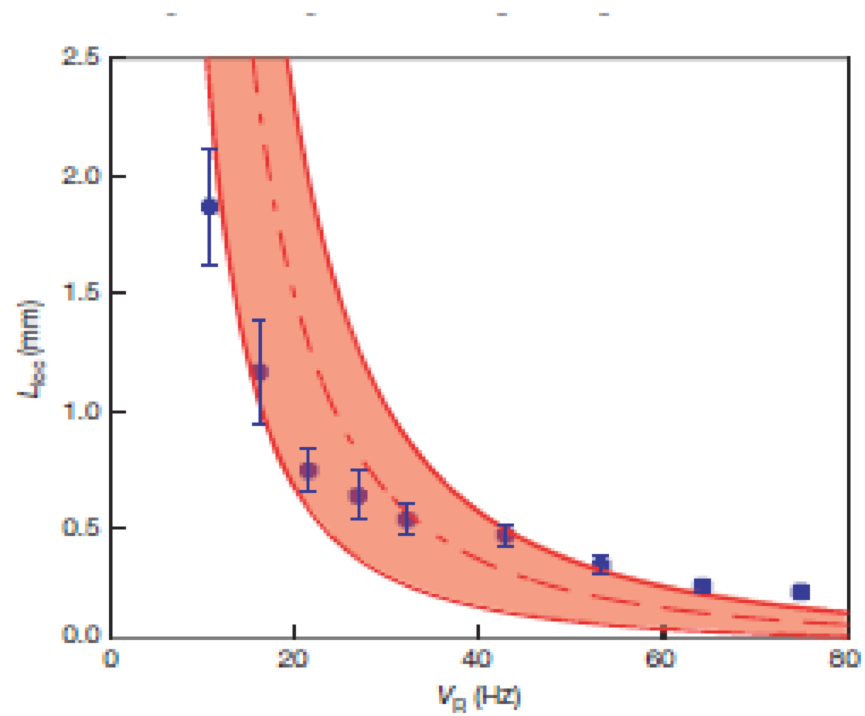
Literature

- Observation of Anderson localization in
  - Nonlinear Optics
    - Y. Lahini et al. PRL 100, 013806 (2008)
    - T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)
  - Bose-Einstein condensation
    - J. Billy et al. Nature 453, 891 (2008)
    - G. Roati et al. Nature 453, 895 (2008)
    - S. S. Kondov, Science 66, 334 (2011)
  - Linear disordered media (optics)
    - M. Storzer, P. Gross, C. M. Aegerter, G. Maret, PRL 96, 063904 (2006)
    - A. A. Chabanov, M. Stoytchev, A. Z. Genack, Nature 404, 850 (2000)
    - T. Sperling at al, Nature Photonics 7, 48 (2013)

1D Bosons (BEC)

- Billy et Nature 2008





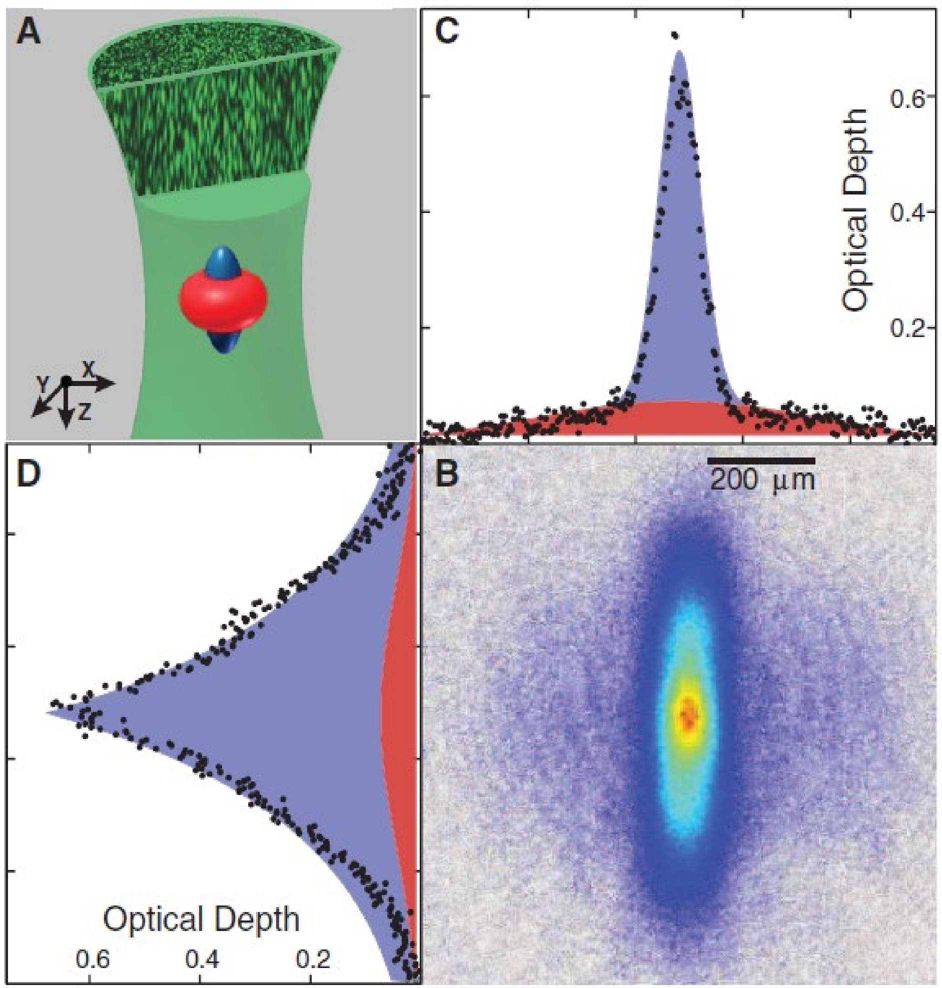
Localization length versus strenght of disorder

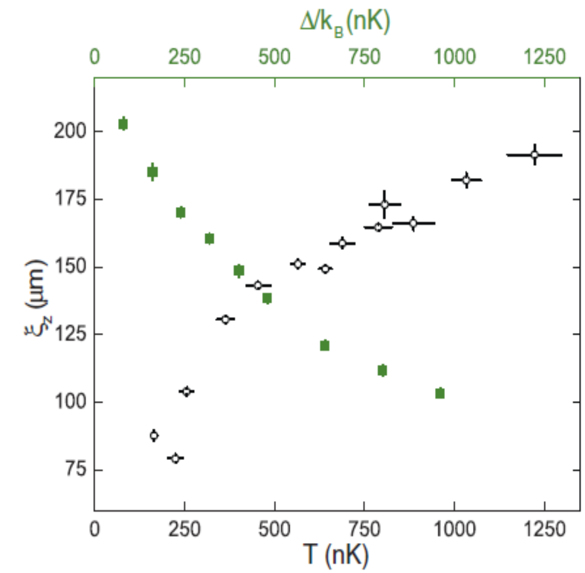
Also Roati et al Nature 2008

3D Fermions (BEC)

- Kondov et al. Science 2011

Fig. 1. (**A**) Ultracold gas expanding into an optical speckle field (green) and separating into localized (blue) and mobile (red) components. (**B**) The measured optical depth, proportional to the atomic density integrated through *y*, is shown in false color. The image depicts a 480-nK gas that has expanded for 20 ms through the disordered potential with $\Delta = k_B \times 240$ nK. All images shown in this manuscript are averaged over at least five experimental realizations. Slices are shown through the image along *x* (**C**) and *z* (**D**). The filled curves are fits to independent mobile (red) and localized (blue) components.





Localization length
Versus disorder

3D Photon

- Sperling et al.

  Nature Photonics 2013

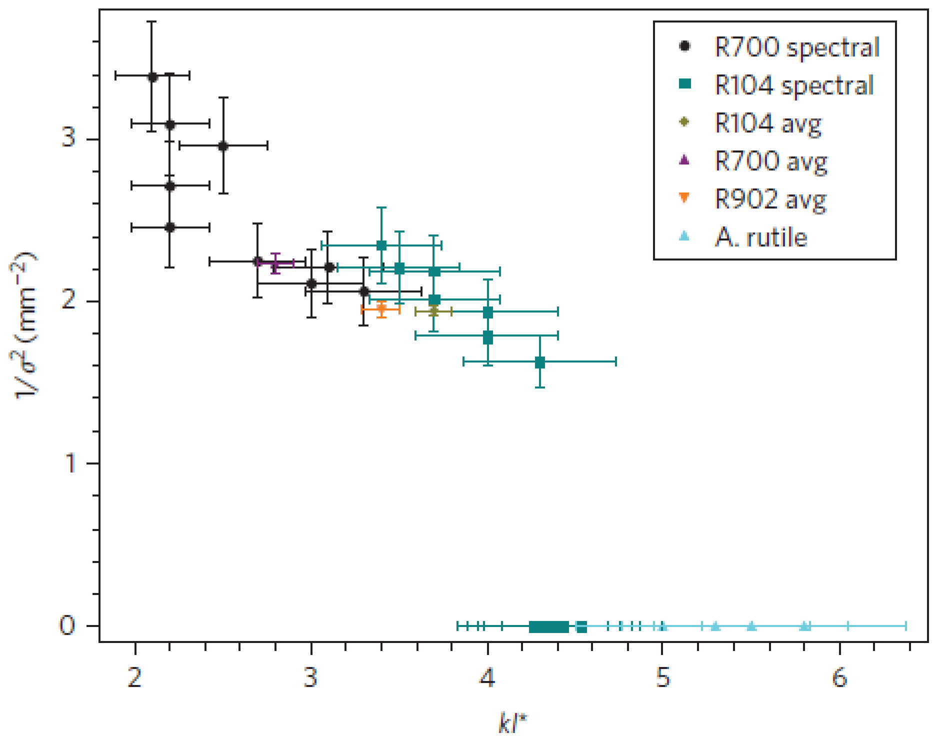


Figure 4 | Inverse of the mean-square width σ^2_∞ of the plateau versus kl^* for different samples. As can be seen, the width (corresponding to the localization length) diverges at $l^* \approx 4.5$, indicating the transition from a localized to a non-localized state. The increase of the localization length approaching the critical turbidity can also be used to estimate the critical exponent. All error bars correspond to systematic errors.

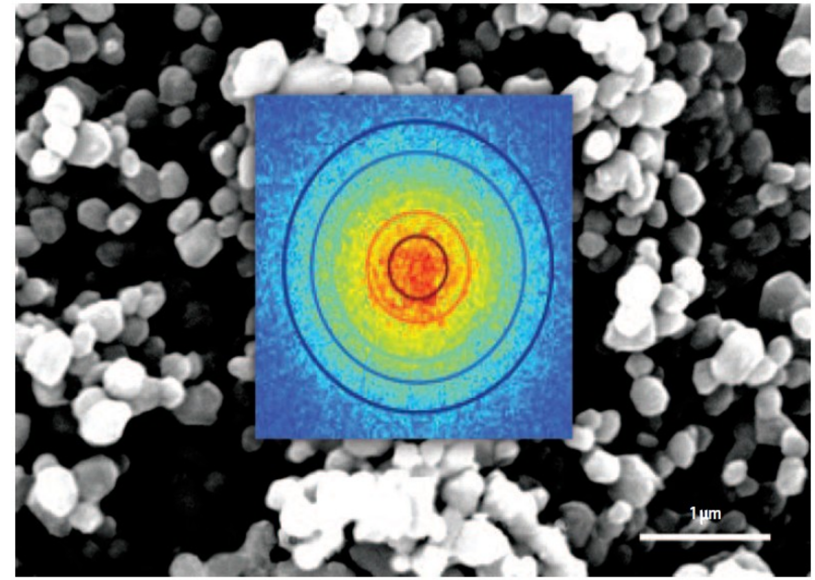
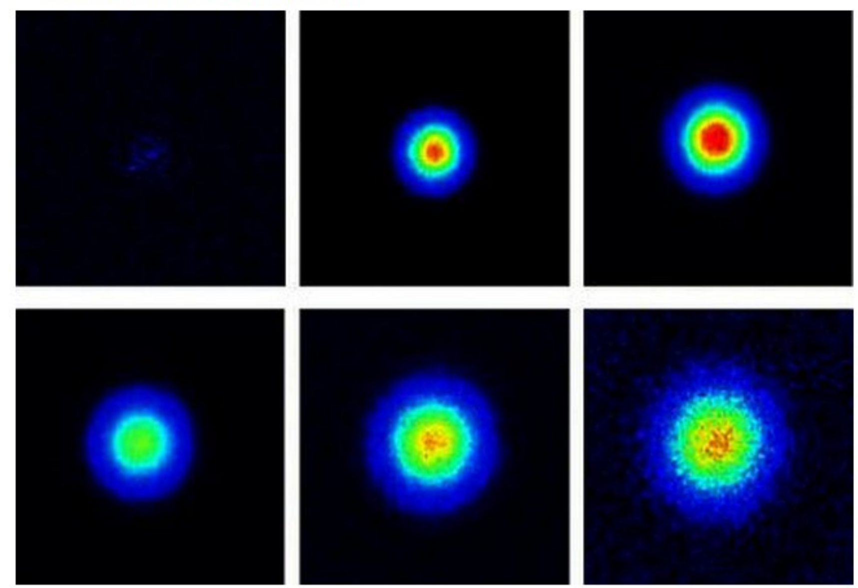


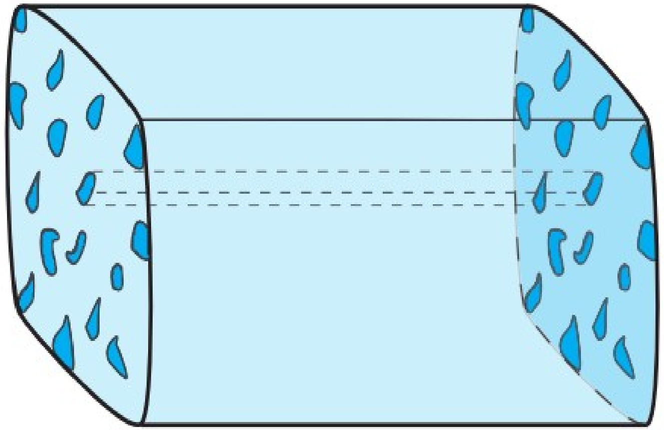
Figure 1 | Light at the onset of the Anderson localization superimposed over a scanning electron microscopy image of a disordered sample.



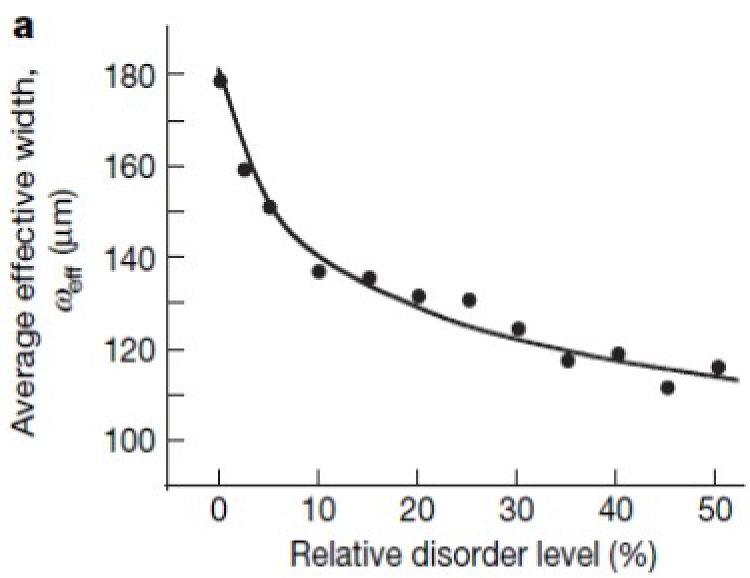
Diffusion of light in a disordered, cloudy medium at intervals of 1 ns. After about 4 ns, the light stops spreading any further. (*Courtesy of the University of Zurich*)

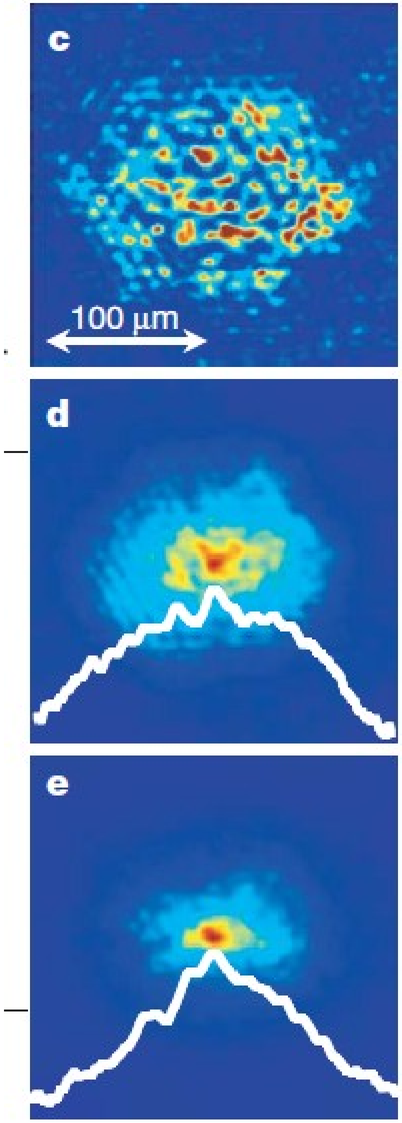
TRANSVERSE Anderson Loc

T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)



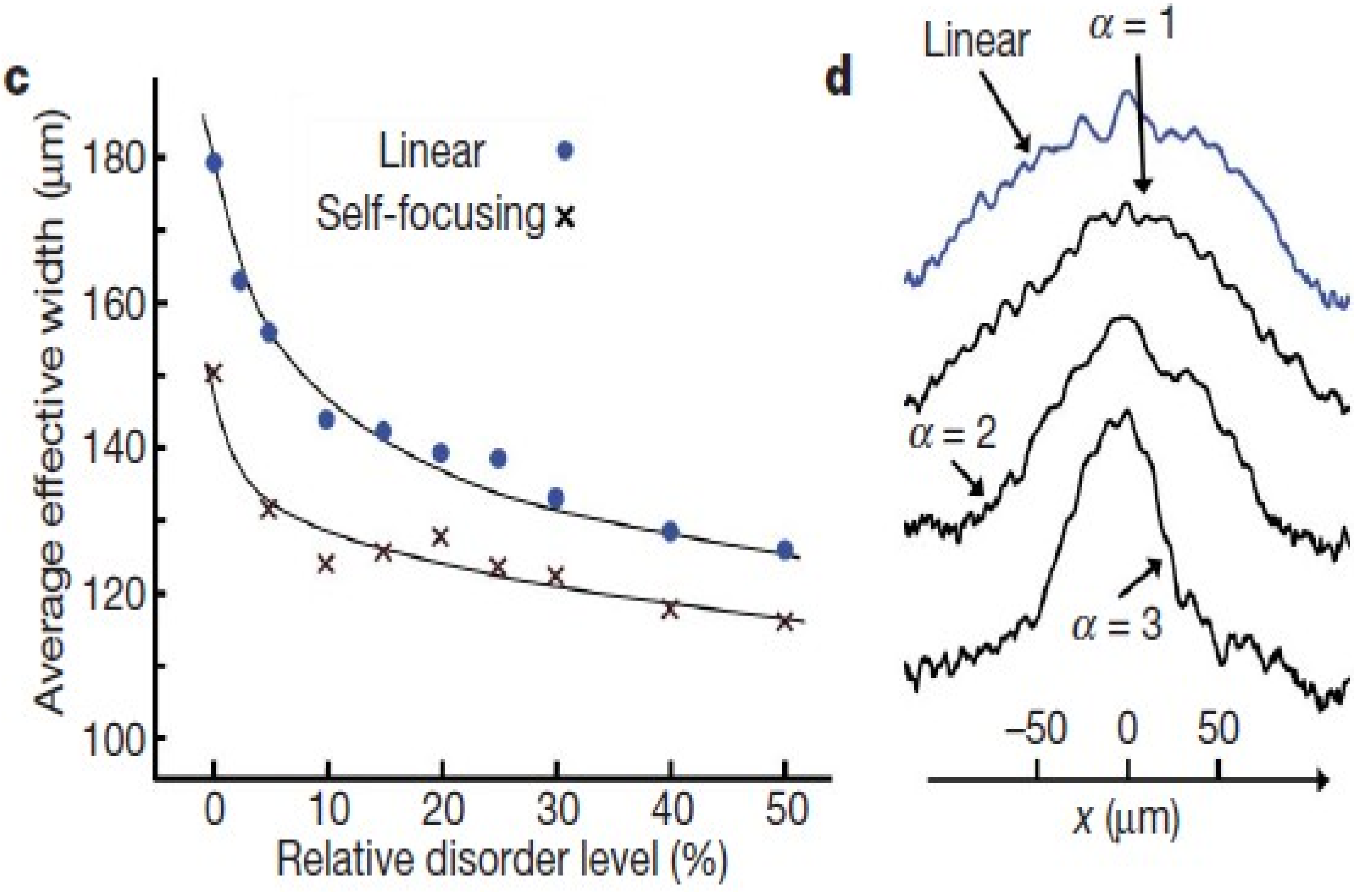
INDEX CONTRAST 0.0001
PROPAGATION 1cm

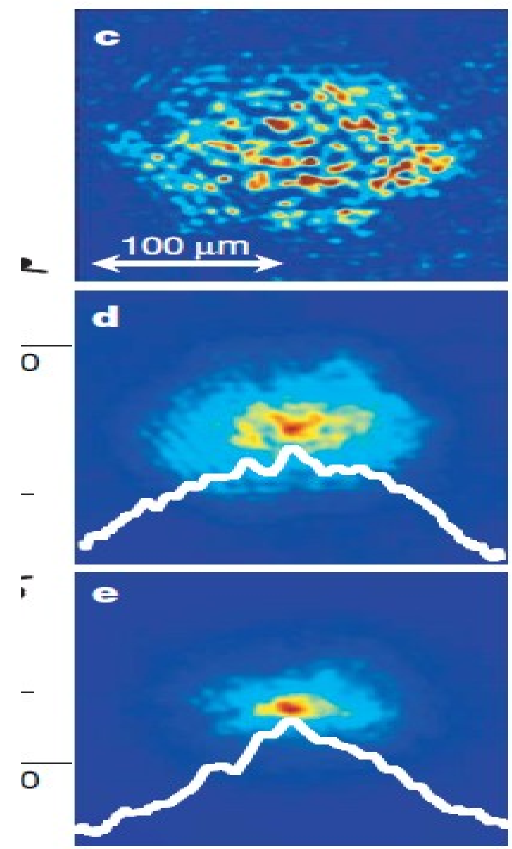




The effect of nonlinearity on the 2D Anderson localization profile

- T. Schwartz, G. Bartal, S. Fishman, M. Segev, Nature 446, 52 (2007)





The simplest model

The model

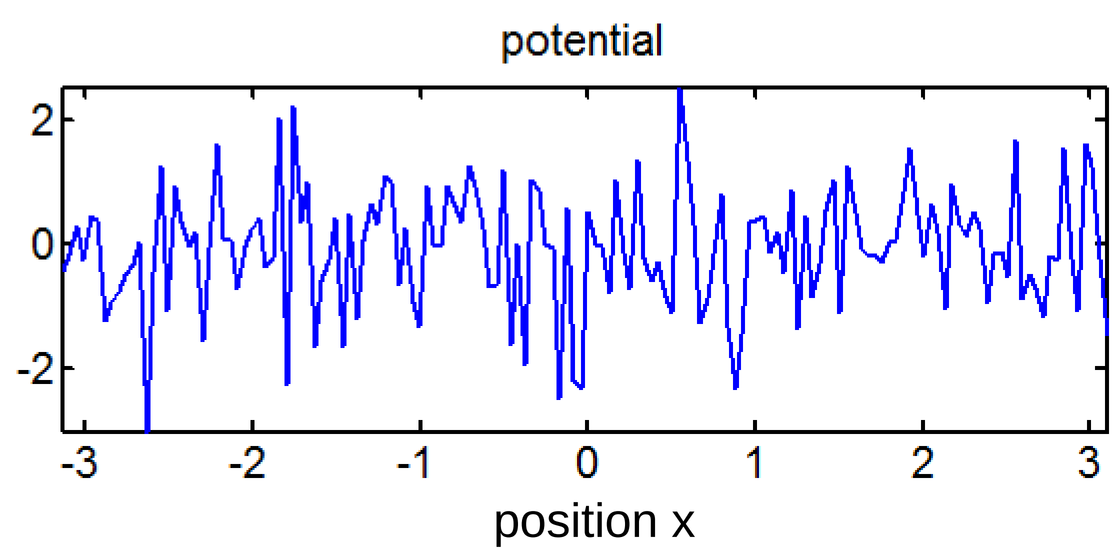
- One-dimensional NLS with a random potential

$$i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi$$

$\chi = 0$	Linear
$\chi = +1$	Focusing
$\chi = -1$	Defocusing

$$P = \int |\psi|^2 dx$$

$$\langle V(x)V(x')\rangle = V_0^2\delta(x - x')$$



CC, PRA 86, 061801R (2012)

Nonlinear Anderson localization

- Bound state equation

$$\psi = \varphi \exp(-iEt)$$

$$-\varphi_{xx} + V(x)\varphi - \chi\varphi^3 = E\varphi,$$

- This is solved numerically by a pseudo-spectral Newton-Raphson algorithm

The simplest Anderson localization

$$-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,$$

$$\chi = 0$$

- One dimensional LINEAR Schroedinger equation with random potential
  - Specific case:
    - a Gaussianly distributed random potential
  - Known issues:
    - Existence of exponentially localized states (negative eigenvalues)
    - Distribution of eigenvalues
    - Localization length

Linearly localized states

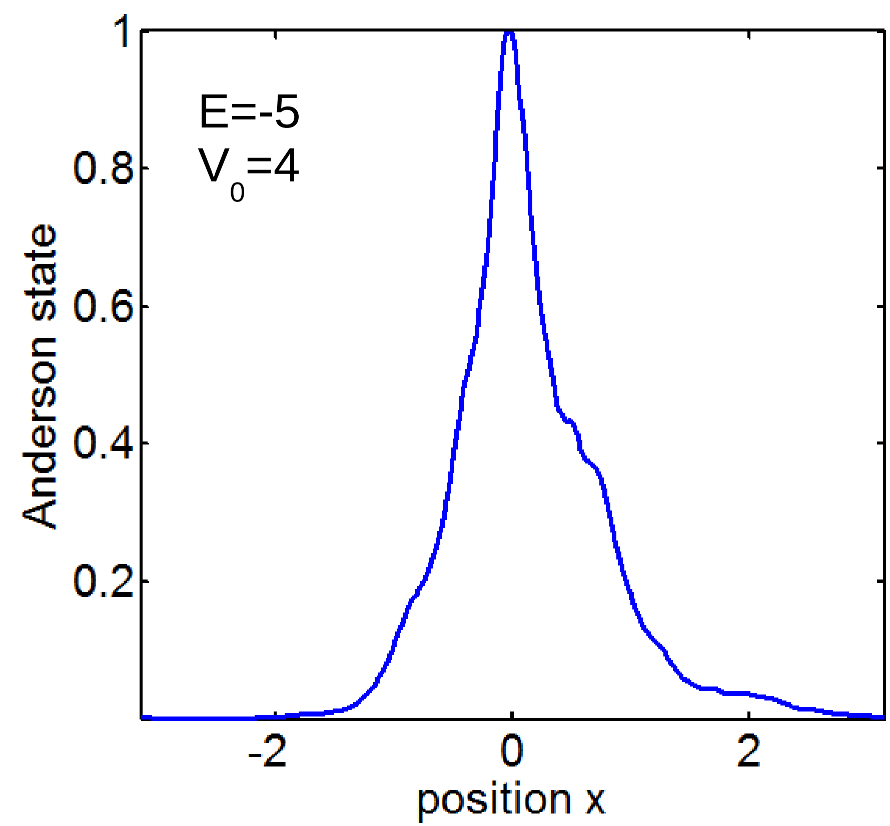
- Gaussian potential
- Negative eigenvalues
- Decays as $\exp(-\sqrt{-E}|x|)$
- **Link between**

  localization length

  and

  eigenvalue

$$-\varphi_{xx} + V(x)\varphi = \mathcal{L}\varphi = E\varphi,$$



The statistical distribution of eigenvalues

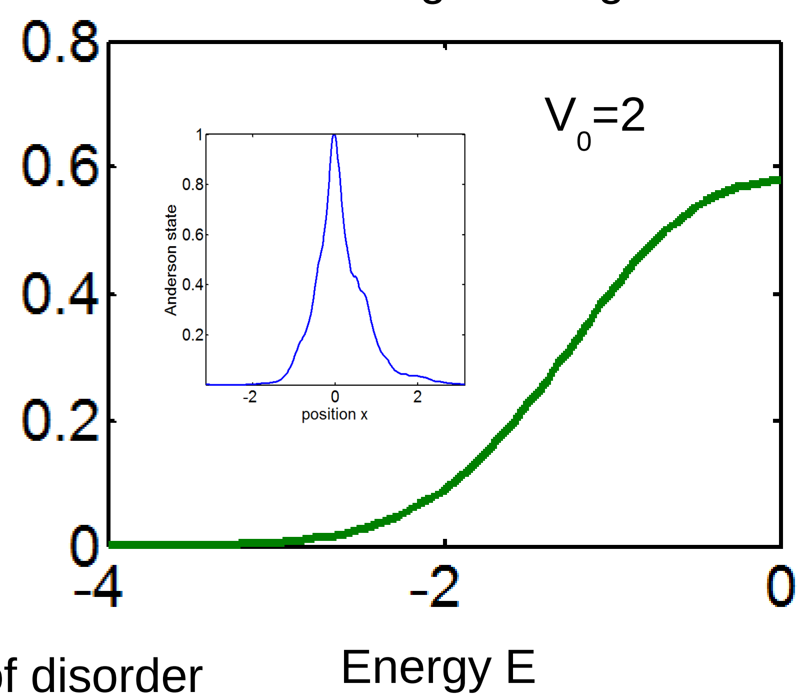
- There is a tail of negative energies corresponding to

  exponentially highly localized states

$$\langle V(x)V(x')\rangle = V_0^2\delta(x-x')$$

$$\overline{E}_L \cong -V_0^{4/3}/3$$

Distribution of negative eigenvalues



The localization length decreases as the Inverse square root of the |energy|, hence the localization length decreases with the amount of disorder (as observed experimentally)

Localization length l

- It is calculated by the inverse participation ratio

$$l = \frac{|\int \varphi^2 dx|^2}{\int \varphi^4 dx} = \frac{P^2}{\int \varphi^4 dx}$$

$$P = \int |\psi|^2 dx$$

- For an exponentially localized state

$$\varphi_e = \frac{e^{-2|x|/l}}{\sqrt{l/2}}$$

Link between localization length and eigenvalue in the LINEAR case

- The localization length scales as inverse squares root of the eigenvalue

$$l = \frac{3}{\sqrt{-E}}$$

- The lower the negative energy,

  the more localized

Parameters for the nonlinear case

- INPUT POWER $P = \int |\psi|^2 dx$
  - Controls the amount of nonlinearity
  - What happens when increasing nonlinearity ?

- In the presence of nonlinearity we have
  - POWER DEPENDENT EIGENVALUE $E = E(P)$
  - POWER DEPENDENT LOCALIZATION $l = l(P)$

Two regimes

- Strong pertubation regime (soliton for focusing)

HIGH POWER, LARGE P

$$i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi$$

- Weak perturbation regime (Anderson localization)

LOW POWER, SMALL P

$$i\psi_t = -\psi_{xx} + V(x)\psi - \chi|\psi|^2\psi$$

STRONG PERTURBATION (SOLITON)

Strong perturbation theory

- A simple multiple scale approach on the NLS shows that the random potential becomes negligible when increasing power

$$\varphi = P\eta(Px)$$
$$x_P \equiv Px$$

High P expansion

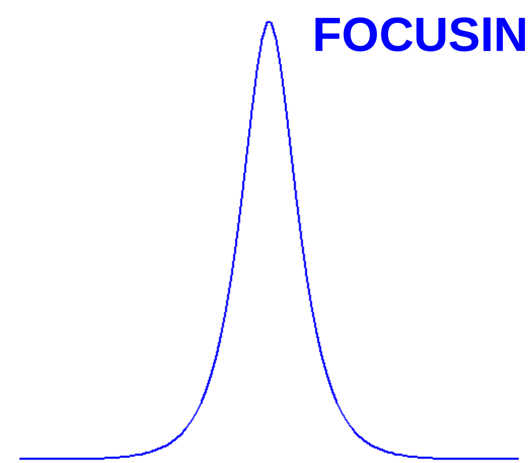
$$\frac{d\eta^2}{dx_P^2} + \chi\eta^3 = E_P\eta,$$

In this regime the only supported localization is the bright soliton

$$\varphi = \sqrt{-2E}/\cosh(\sqrt{-E}x)$$

$$E = E_S = -P^2/16$$ Negative !

$$l = l_S = 12/P$$



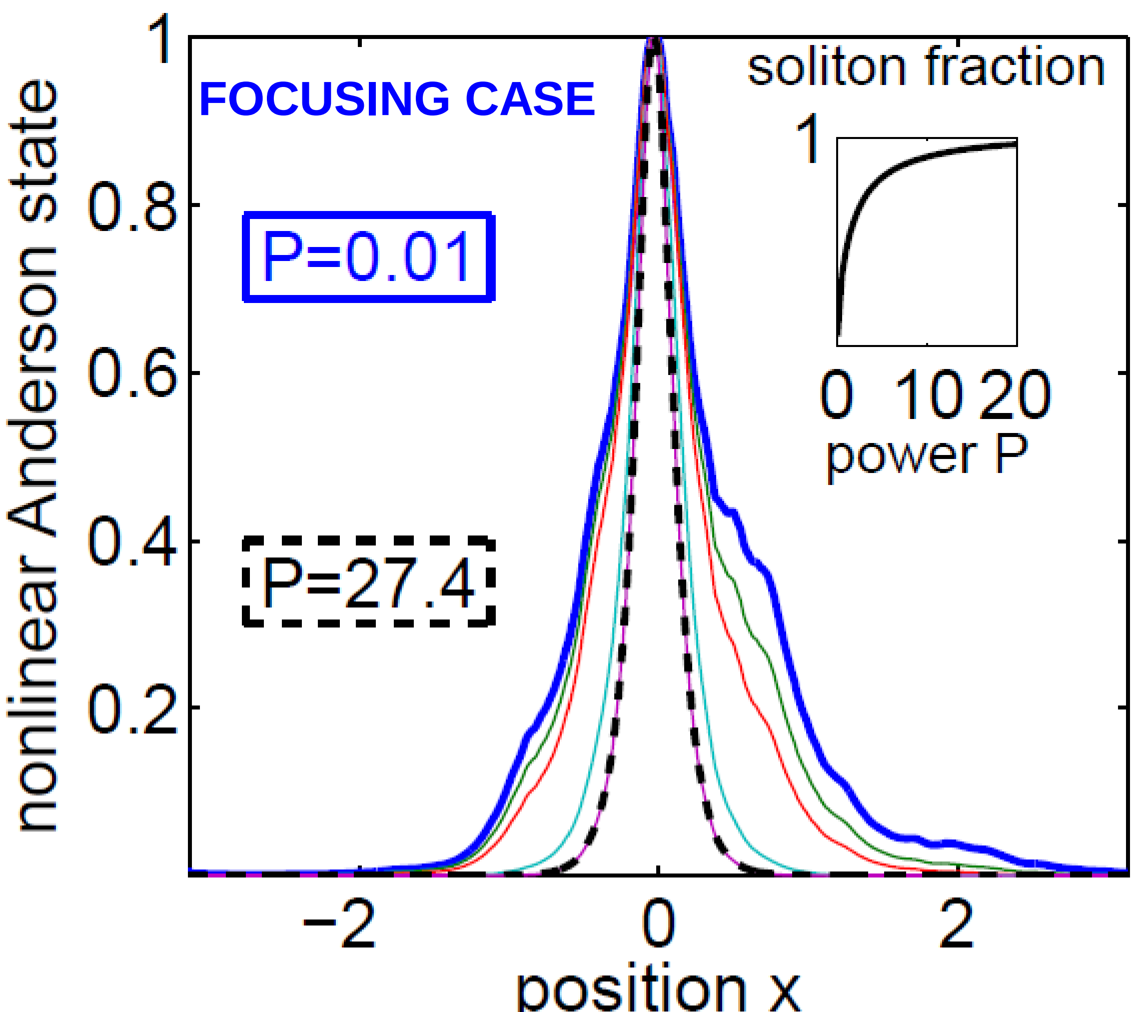
Solitons

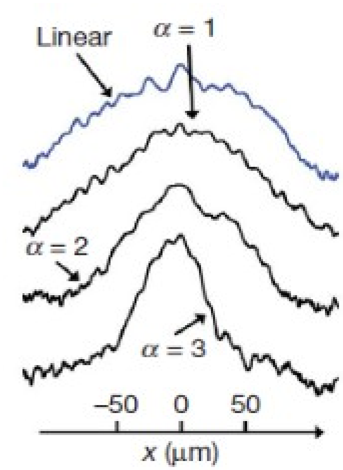
- Features in common with Anderson localization
  - Location (they can be located anywhere in space)
  - Exponential localization
  - Negative (nonlinear) eigenvalue
  - Link between localization length and the eigenvalue

$$l = \frac{3}{\sqrt{-E}}$$

Calculated exact profiles

- The linear fundamental state is numerically prolongated to high power
- Profiles for different powers $-\varphi_{xx} + V(x)\varphi - \chi\varphi^3 = E\varphi,$





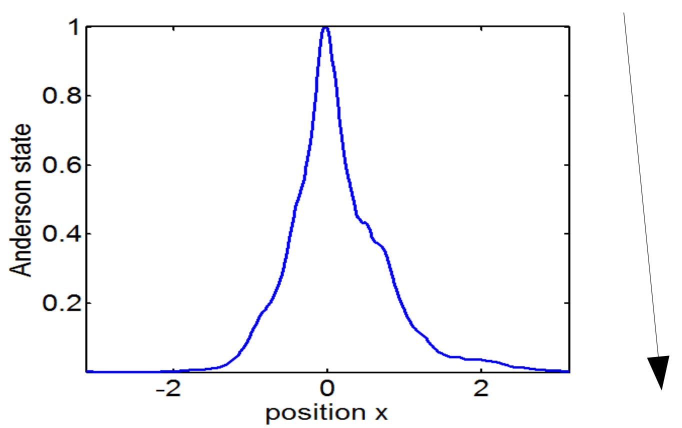
Swartz et al Nature 08

WEAK PERTURBATION
(Anderson states)

Perturbation of the Anderson state

- It is possible to develop a perturbation theory in terms of the power P
- We derive expressions for the localization length and for the eigenvalue valid at small P

$$\psi = \sqrt{P}(\psi_0 + P\psi^{(1)} + P^2\psi^{(2)} + \ldots)$$



The lowest order term is the Anderson state with the smallest negative energy

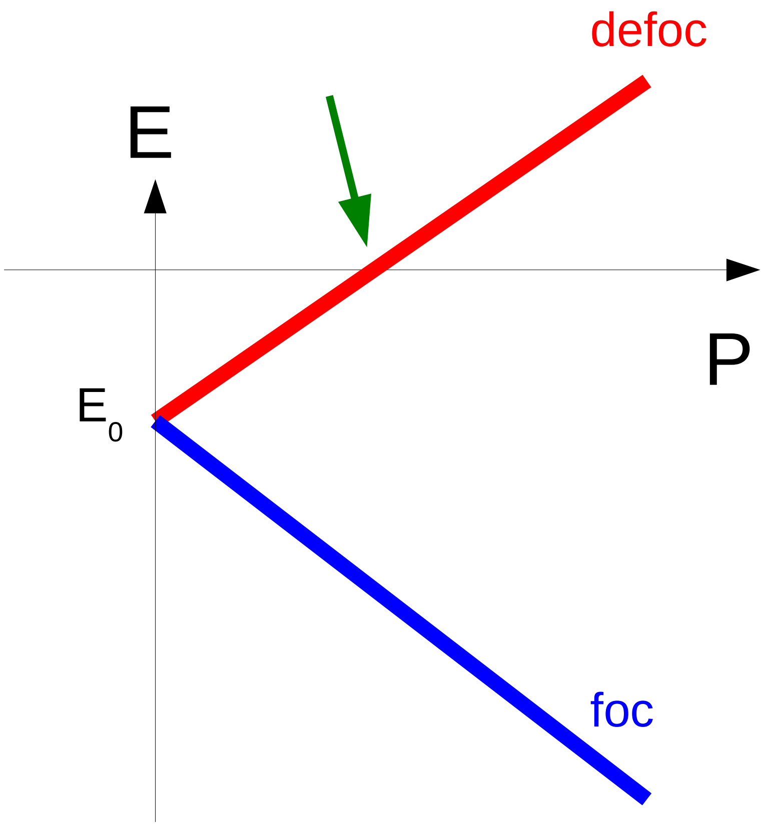
Perturbation of the Anderson state

- Eigenvalue ($E<0$)

$$E = E_0 - \chi \frac{P}{l_0}$$

Linear negative energy

Linear localization length



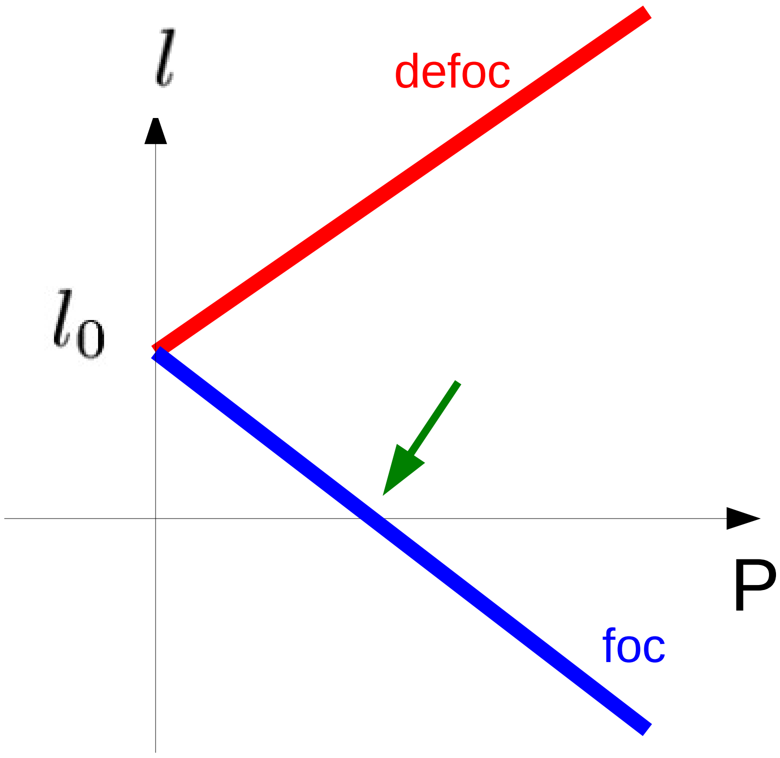
- In the **DEFOCUSING CASE** there is a power at which the eigenvalue becomes positive

Perturbation of the Anderson state

- Localization length

$$l = l_0 \left(1 - \chi \frac{P}{P_0}\right)$$

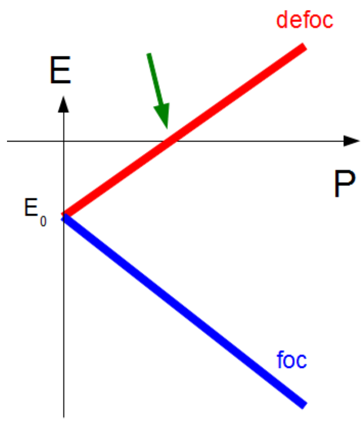
$$\frac{1}{P_0} = 4 l_0 \sum_{n>0} \frac{\left(\varphi_n, \varphi_0^3\right)^2}{E_n - E_0}$$



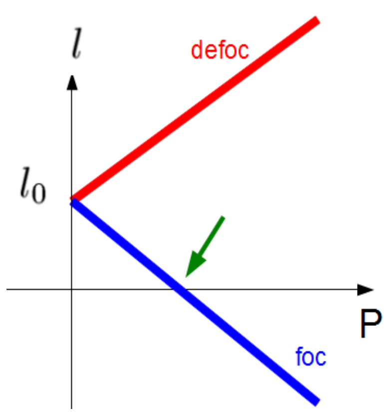
- In the **FOCUSING CASE** there is power at which the localization length becomes negative

Focusing Vs Defocusing case
(weak perturbation theory results)

- In the defocusing case the energy increases
  - The wave delocalizes with P
  - There is a power at which the eigenvalue changes sign $P = |E_0|\ l_0$



- In the focusing case the energy decreases
  - $|E|$ increases with P
  - The wave becomes more localized
  - There is a power at which the localization length becomes zero ($P=P_0$)



TWO critical powers !

- In the defocusing case for delocalization

$$P_{defocusing} = l_0 |E_0|$$

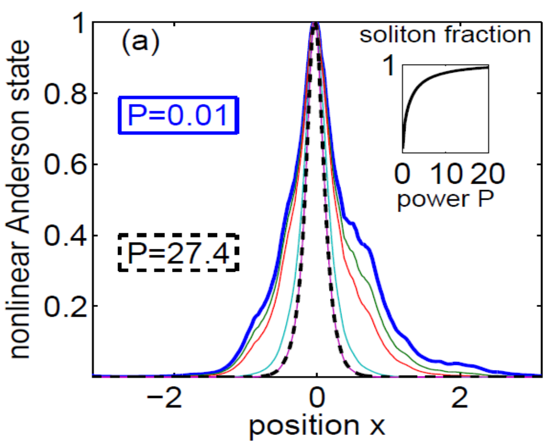
- In the focusing case for solitonization

$$P_{soliton} = P_0$$

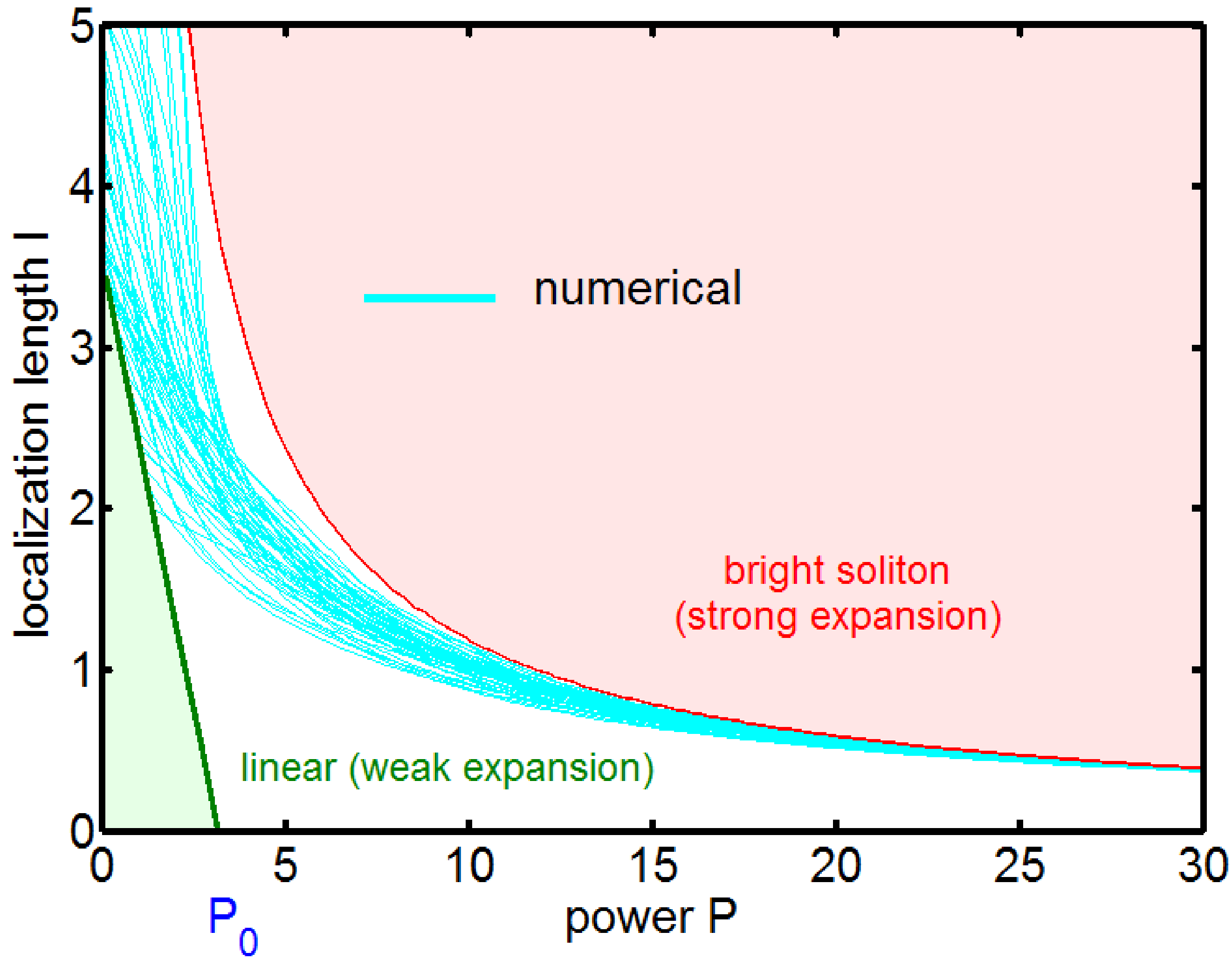
CC PRA 86, 061801 (2012)

Comparing the weak expansion with the numerical results

- Localization length l(P)



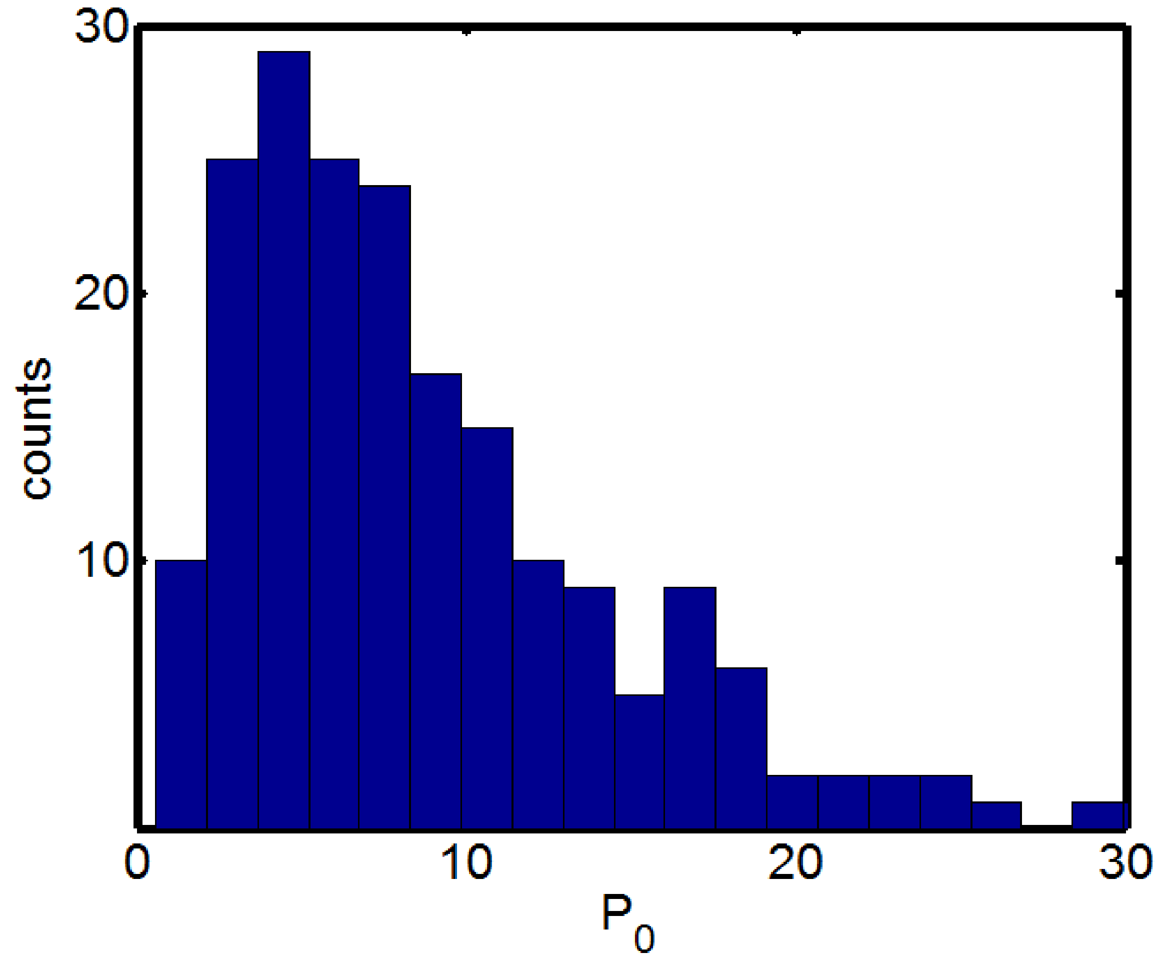
FOCUSING



Statistical distribution of the critical power in the focusing case

- Critical power to become a soliton

$$\frac{1}{P_0} = 4l_0 \sum_{n>0} \frac{\left(\varphi_n, \varphi_0^3\right)^2}{E_n - E_0}$$



NON PERTURBATIVE APPROACH
(disorder averaged variational ansatz)

Results from the variational approach

- Final exact expression for the nonlinear Anderson state features

$$E = E(P)$$

$$E_C = -\frac{P^2}{16}\left(1 + \frac{P_C}{P}\right)^2$$

Nonlinear eigenvalue

$$l = l(P)$$

$$l_C = \frac{12/P}{(1 + P_C/P)}$$

Localization length

$$l = \frac{3}{\sqrt{-E}}$$

One single parameter $P_C = 4V_0^{2/3}/\sqrt{3}.$

Strong and weak limits

- As P grows

$$l_C = \frac{12/P}{(1 + P_C/P)} \longrightarrow l_S = 12/P$$

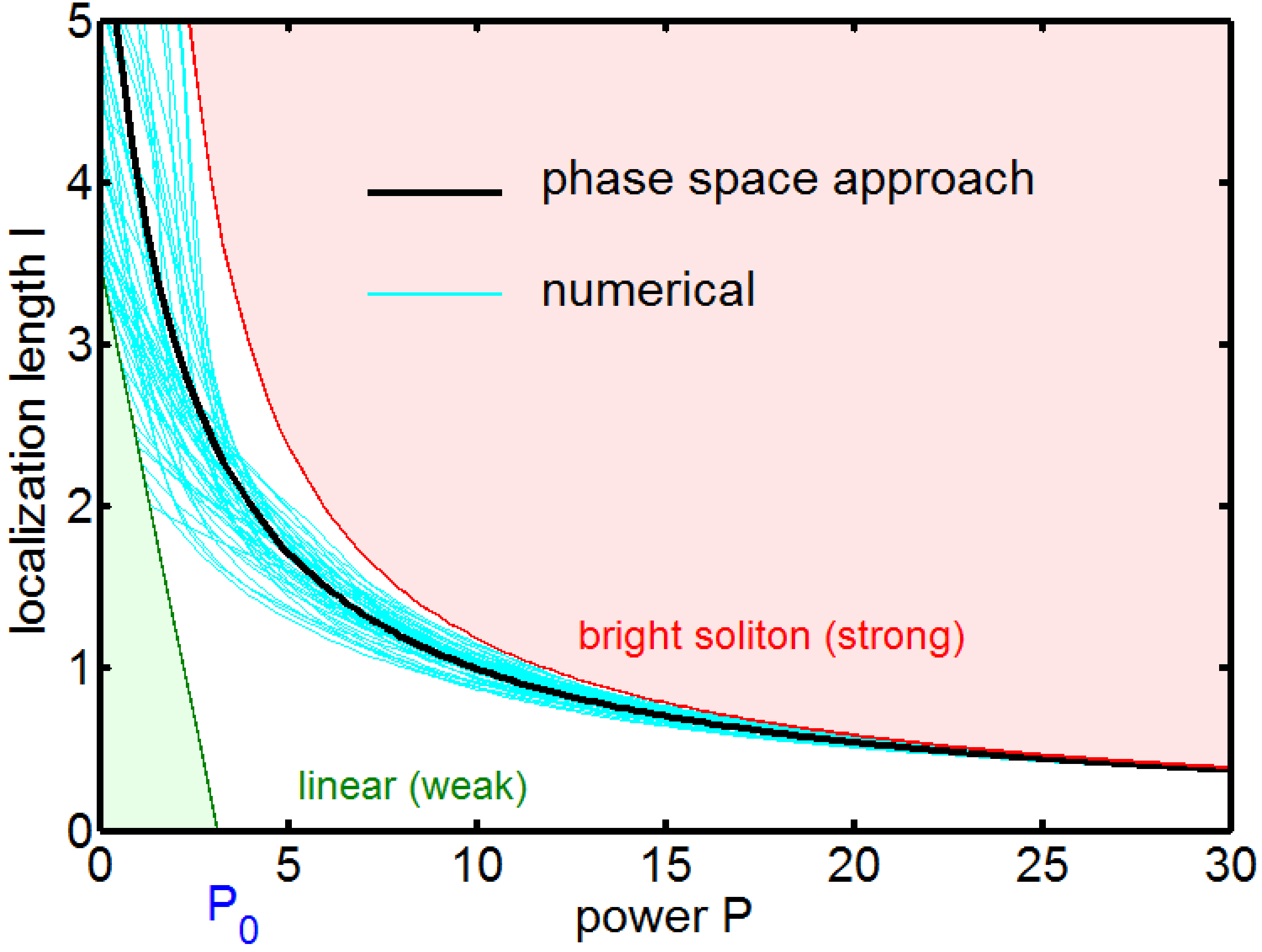
- As P grows

$$E_C = -\frac{P^2}{16}\left(1 + \frac{P_C}{P}\right)^2 \longrightarrow E_S = -P^2/16$$

- Also the weak limit provides the correct result, and P_C turns out to be a good approx for P_0
- The found expressions correctly reproduce the two perturbative limits (strong and weak) !

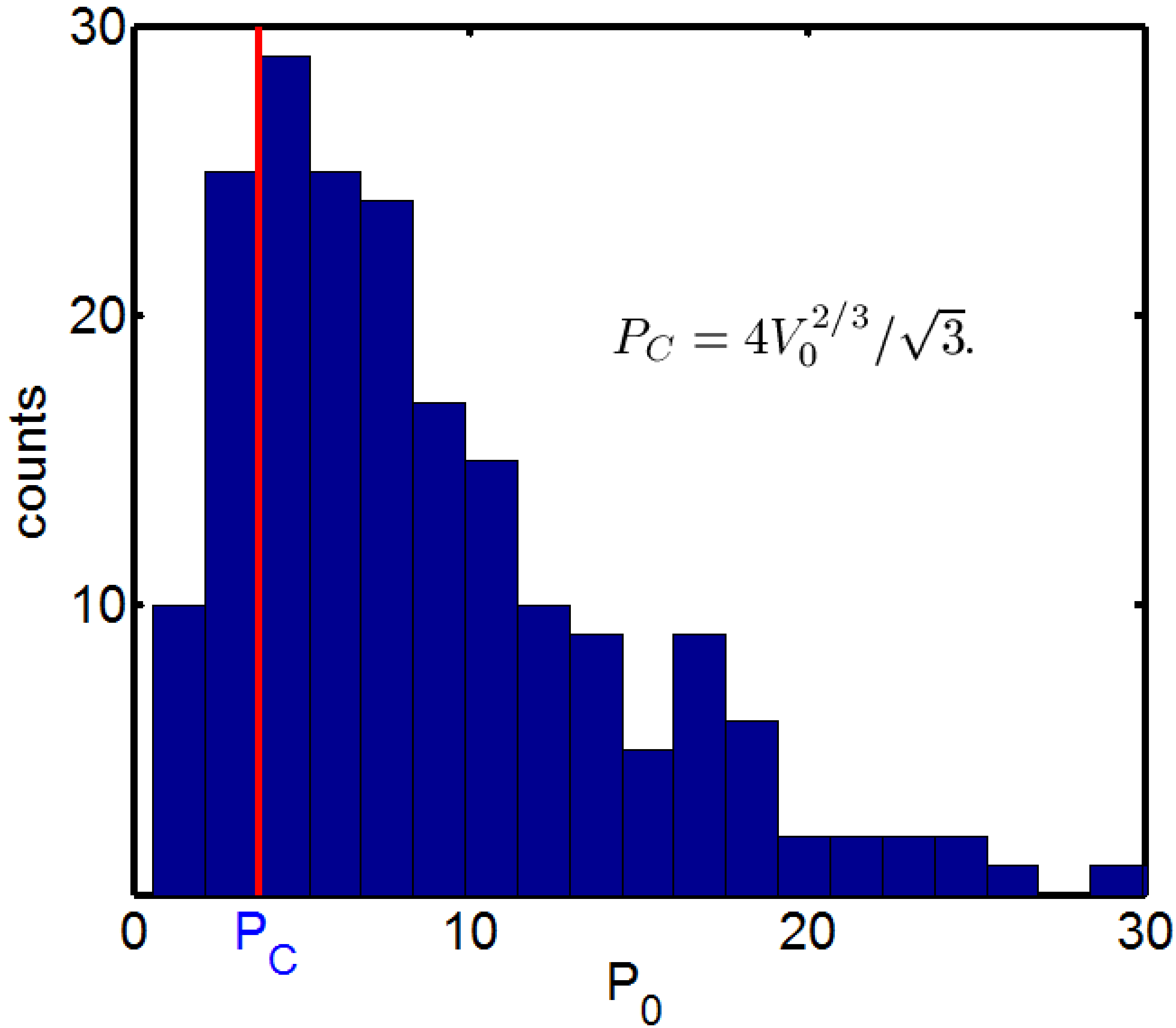
Numerical localization length

- compare

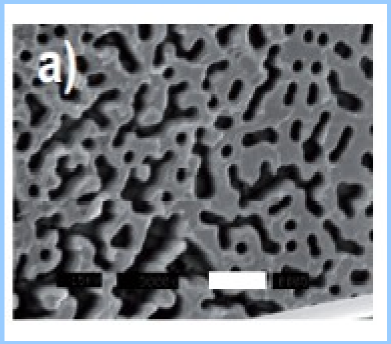


Distribution of critical power

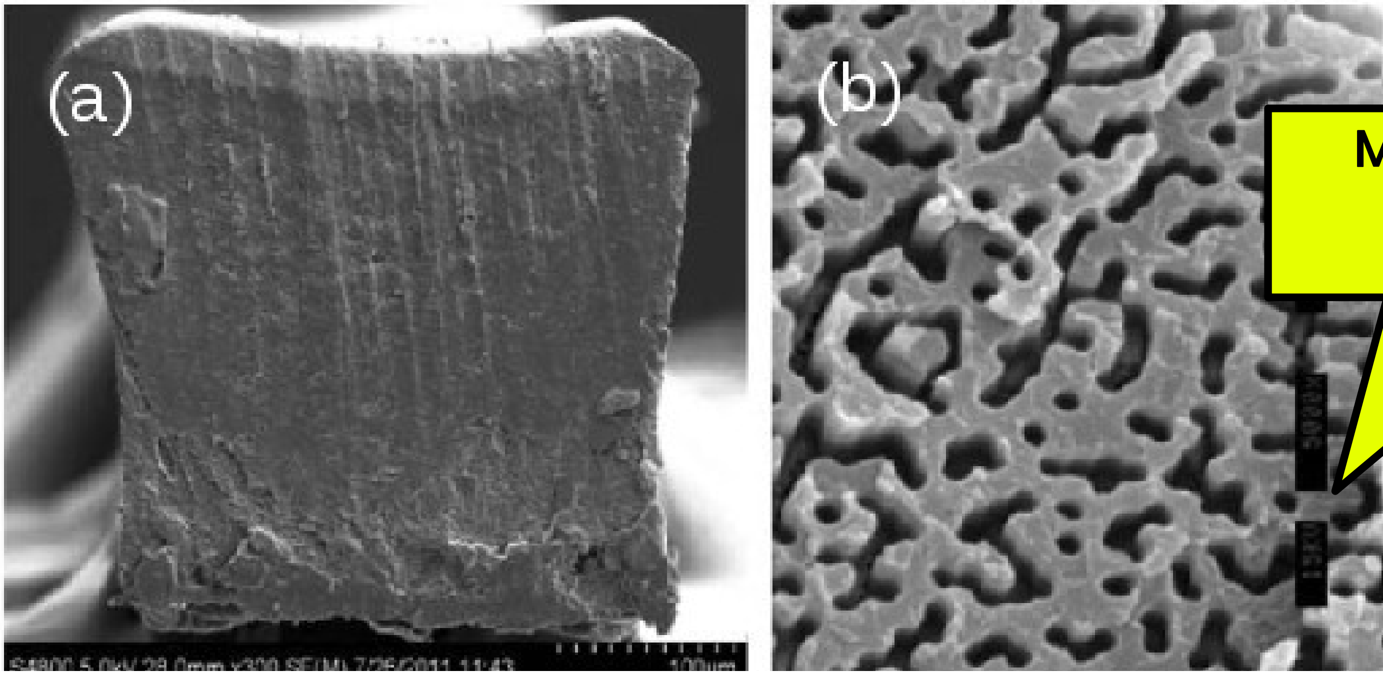
- P_c gives the peak of the distribution



Transverse localization in 2D fibers



Our experiments on transverse localization in two dimensional fibers



Mixture of PS and PPMA
Index contrast 0.1
Propagation >7 cm

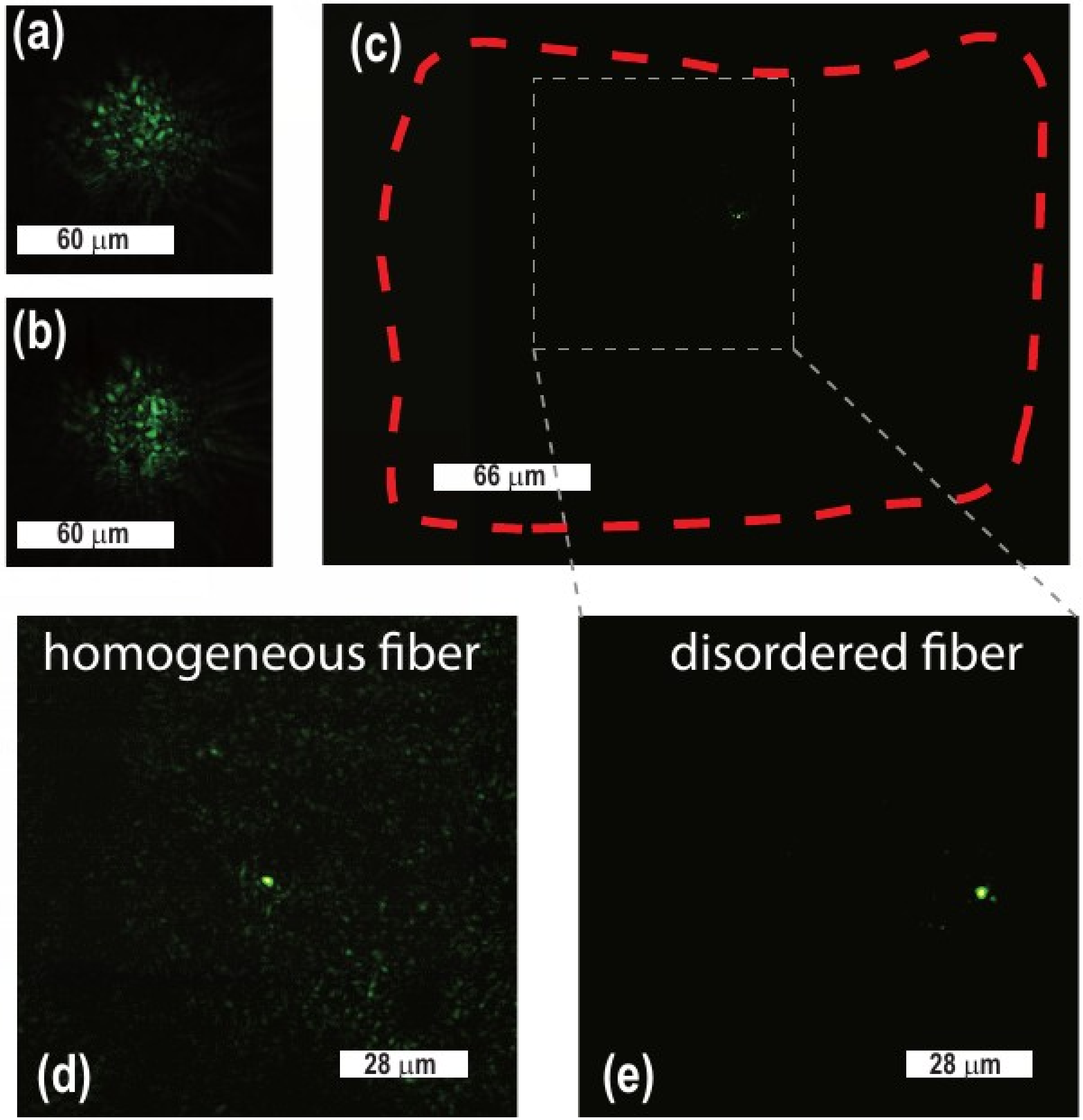
40000 pieces of PMMA and 40000 pieces of PS randomly mixed and fused together
n(PS)=1.59
n(PMMA)=1.49

2304 OPTICS LETTERS / Vol. 37, No. 12 / June 15, 2012

Observation of transverse Anderson localization in an optical fiber

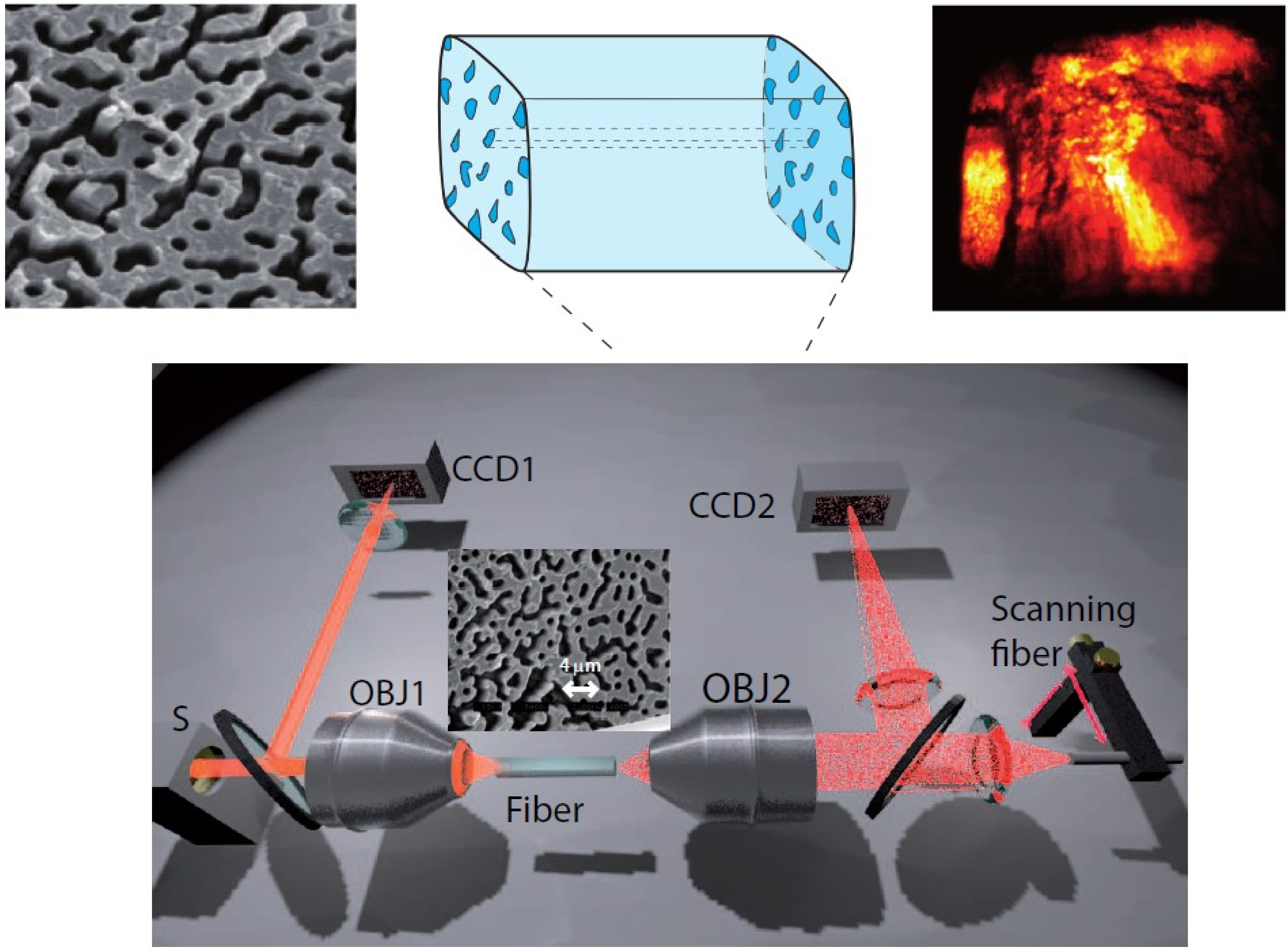
Salman Karbasi,[1] Craig R. Mirr,[1] Parisa Gandomkar Yarandi,[1] Ryan J. Frazier,[1] Karl W. Koch,[2] and Arash Mafi[1,*]

Absence of diffusion

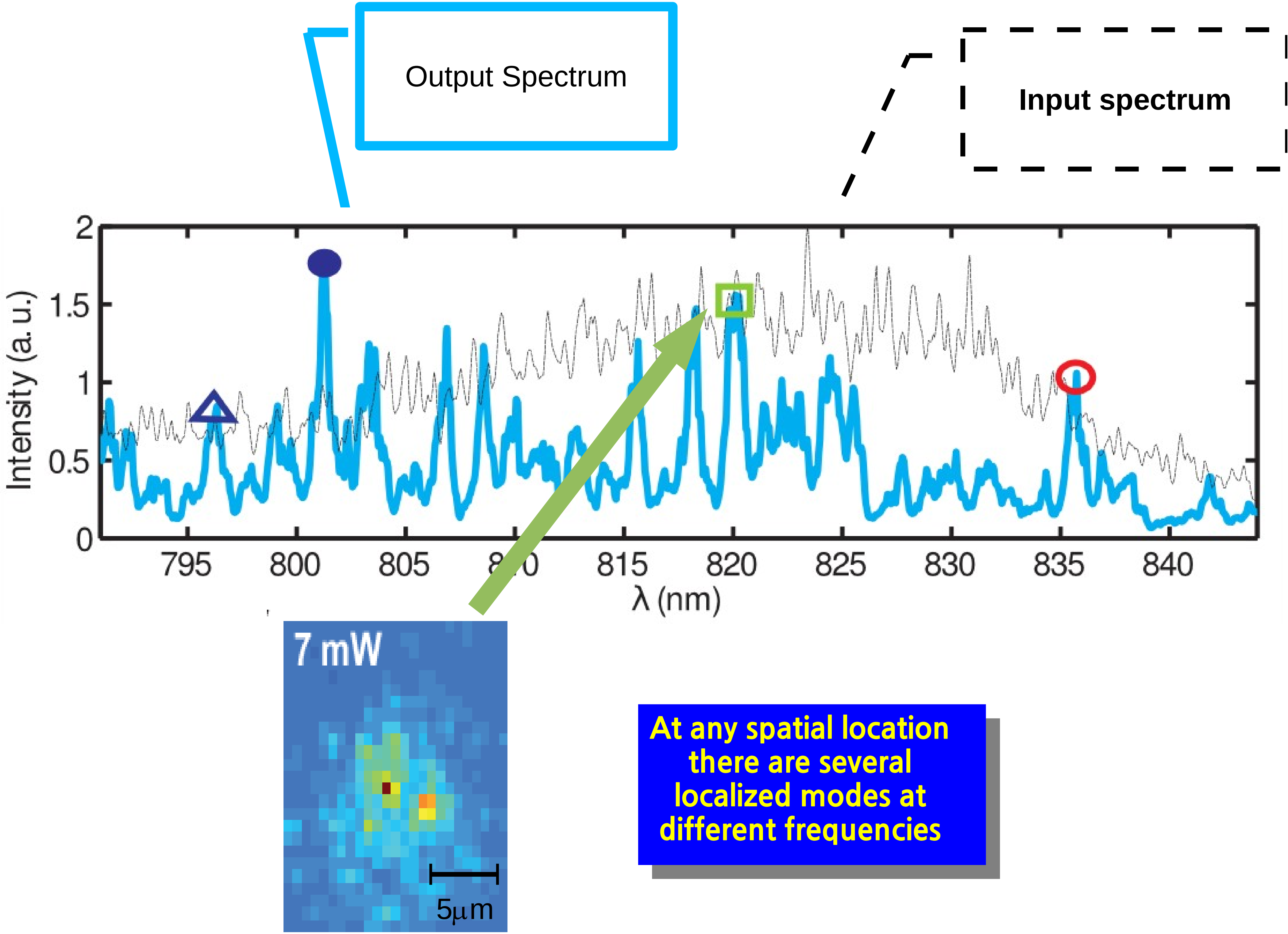


Multicolor transverse Anderson-localization

- we excite several localizations at different wavelengths simultaneously



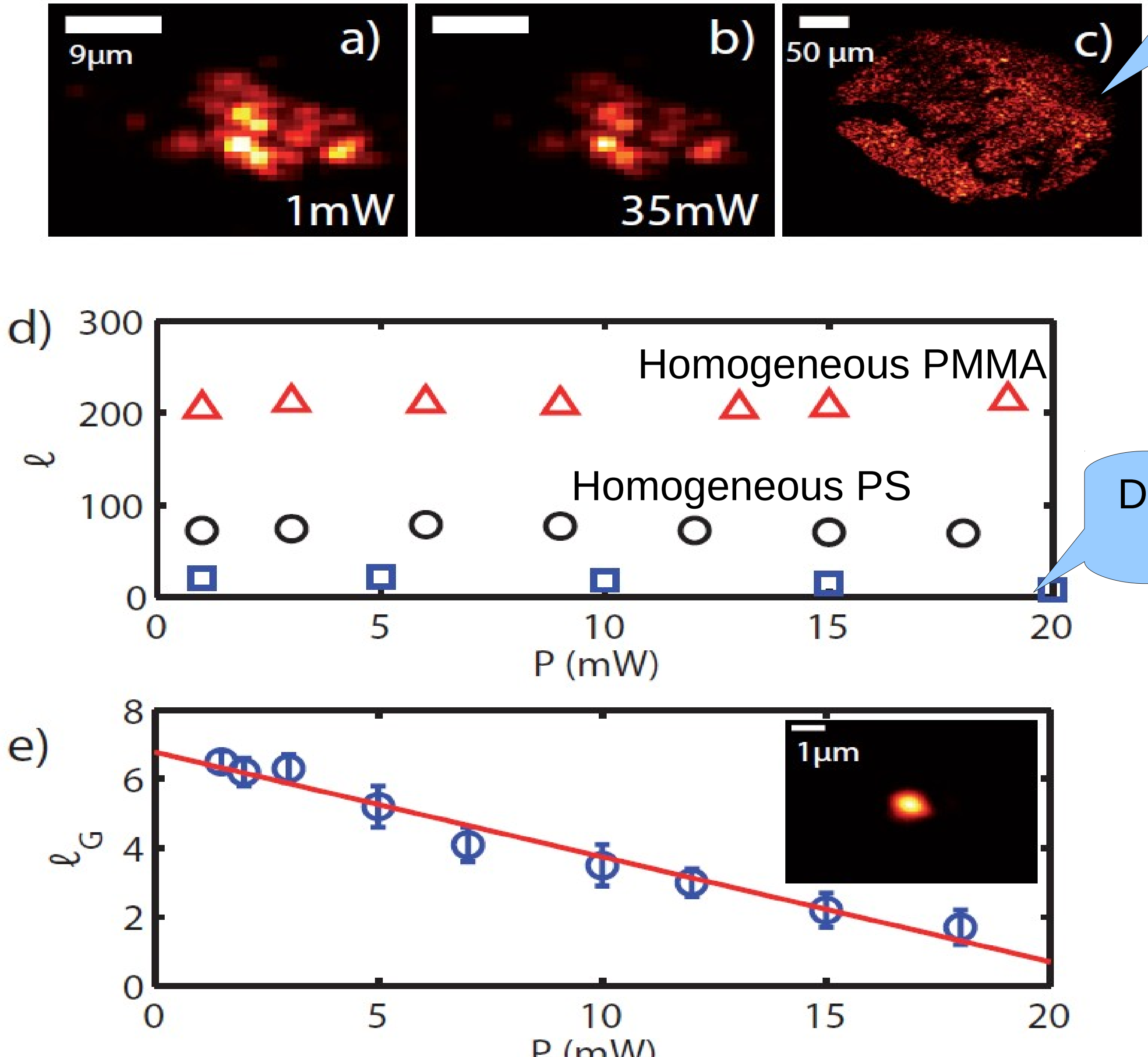
CCD1
CCD2
Scanning
fiber
4 µm
OBJ1
OBJ2
S
Fiber


Output Spectrum
Input spectrum
Intensity (a.u.)
2
1.5
1
0.5
0
795
800
805
810
815
820
825
830
835
840
λ (nm)
7 mW
5μm
At any spatial location there are several localized modes at different frequencies

Nonlinear regime

- at any wavelength we study the localization profile Vs power

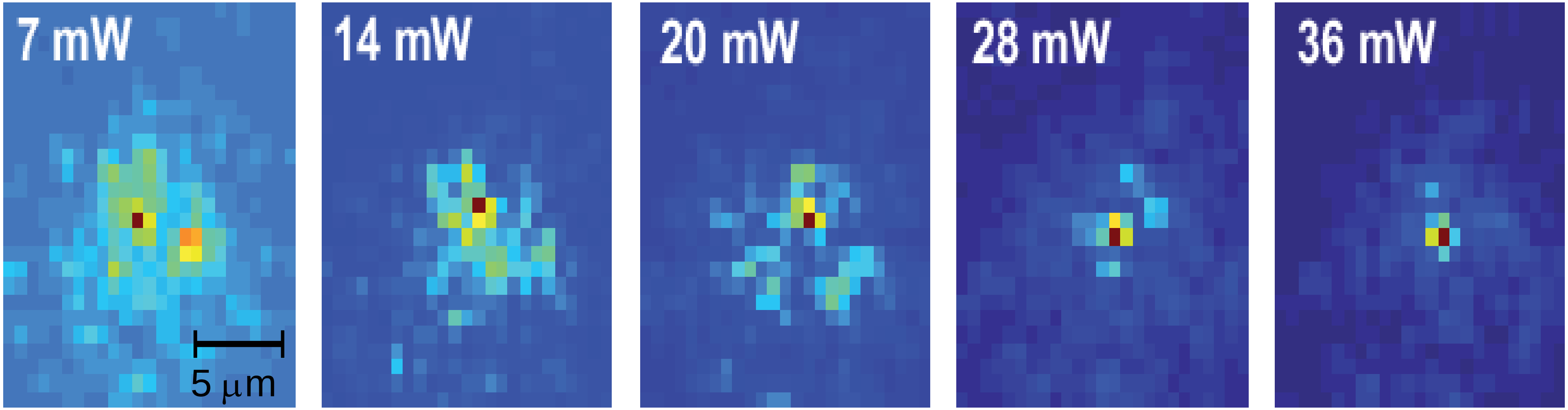
Measurement of critical power



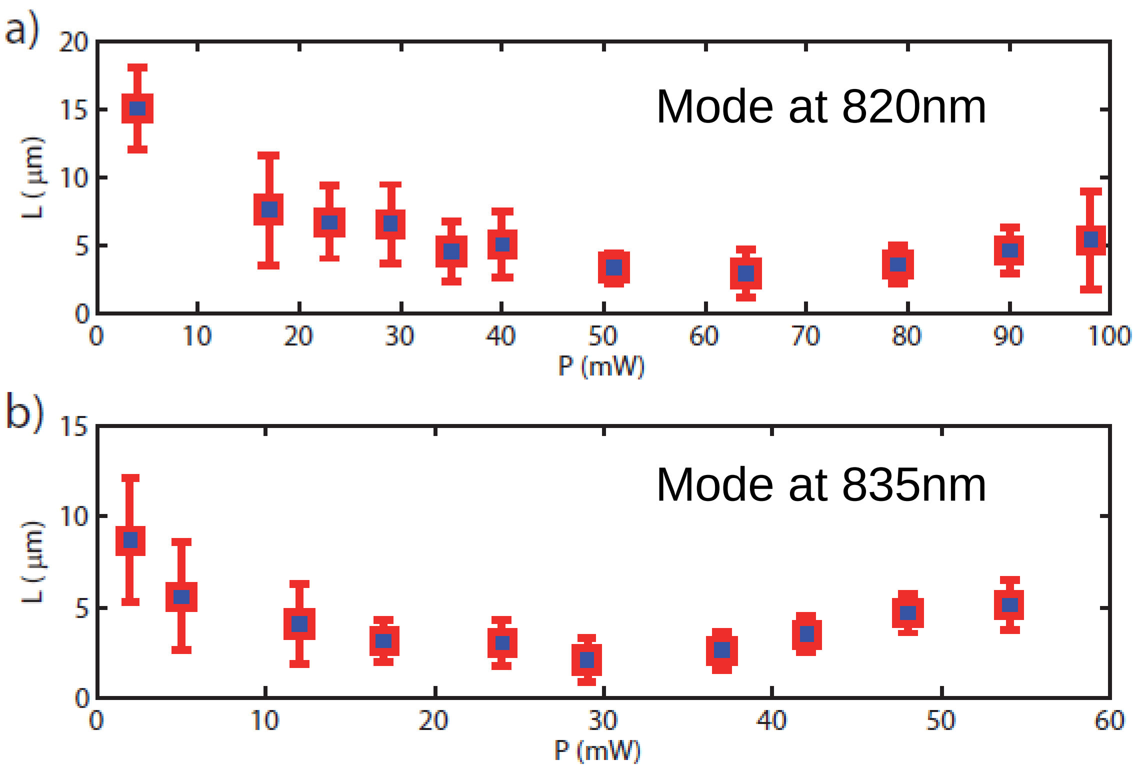
Homogeneous fiber

Disordered fiber fiber

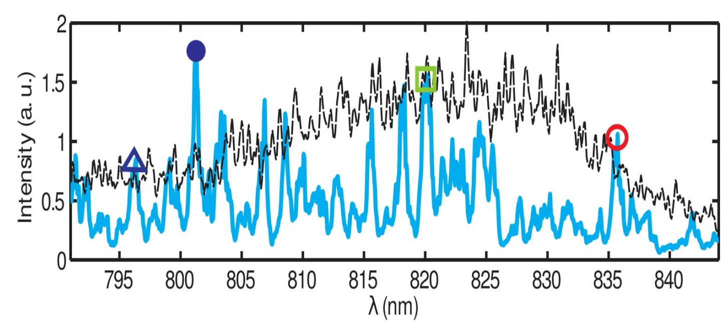
Pc=14mW



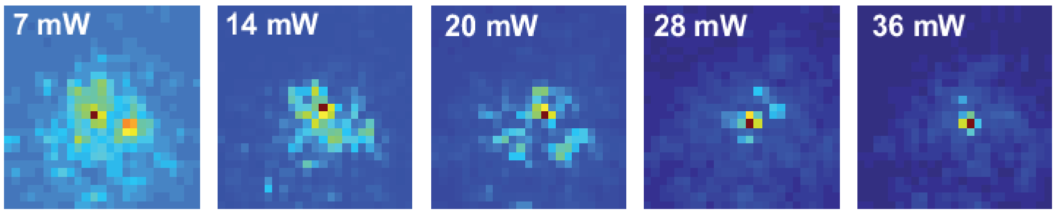
Mode profile at 820nm



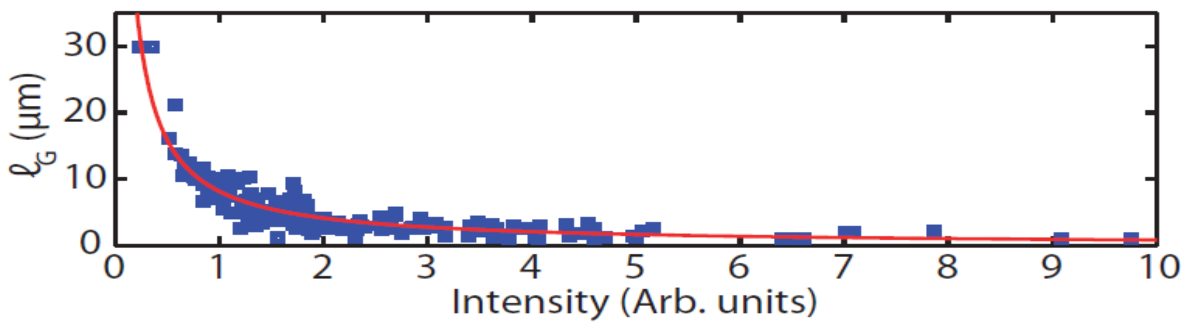
We observe focusing of any of the localized mode when incresing power



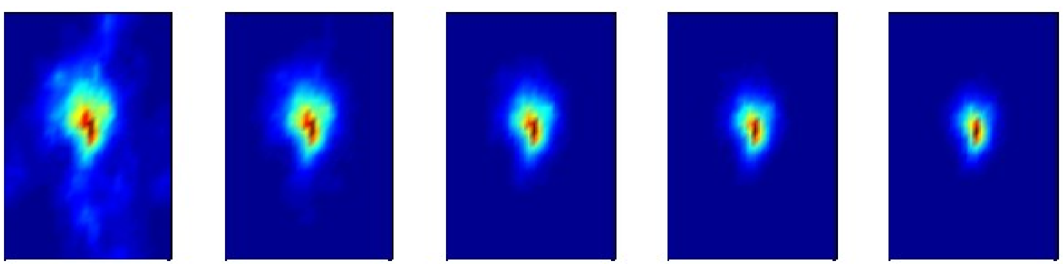
2D SELF-FOCUSING of Anderson localizations



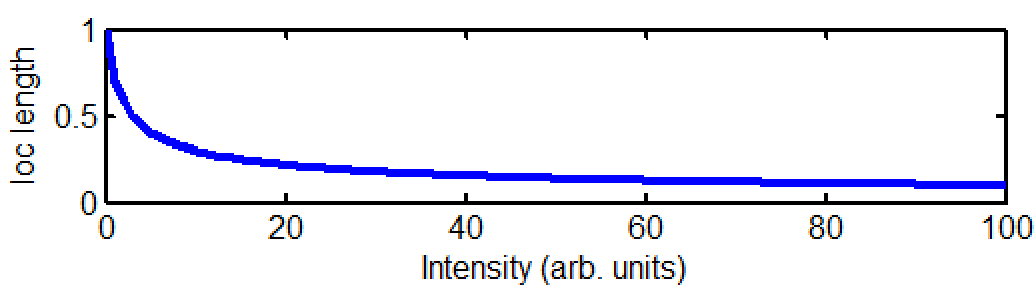
Experiments



Localization length
Versus
Intensity
(50 modes)



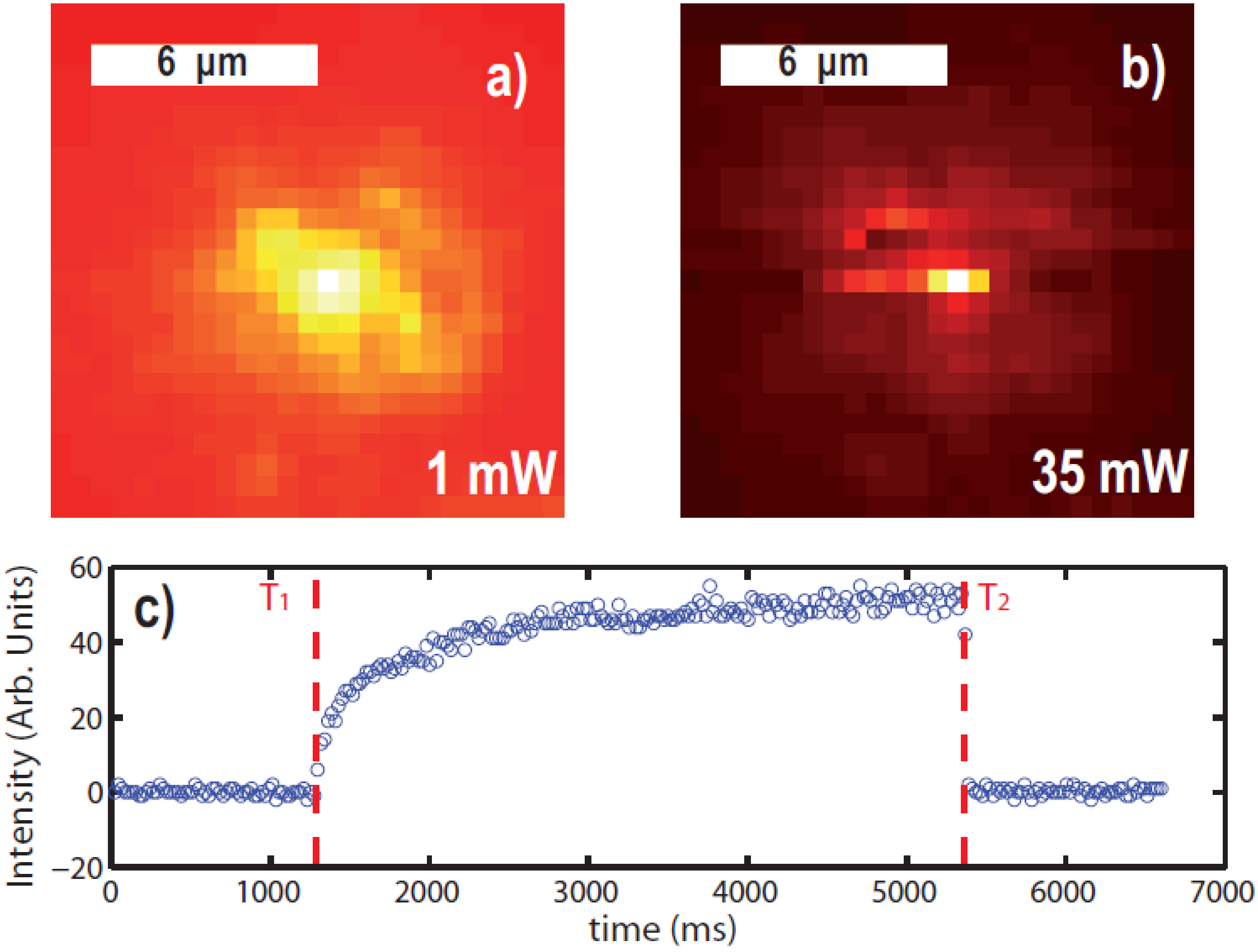
Numerically calculated
bound states of the 2D-NLS
with Gaussian disorder



Theory from
the variational approach
Folli, Conti, OL 2011
Conti, PRA, 2012

Which the origin of the observed nonlinear focusing ?

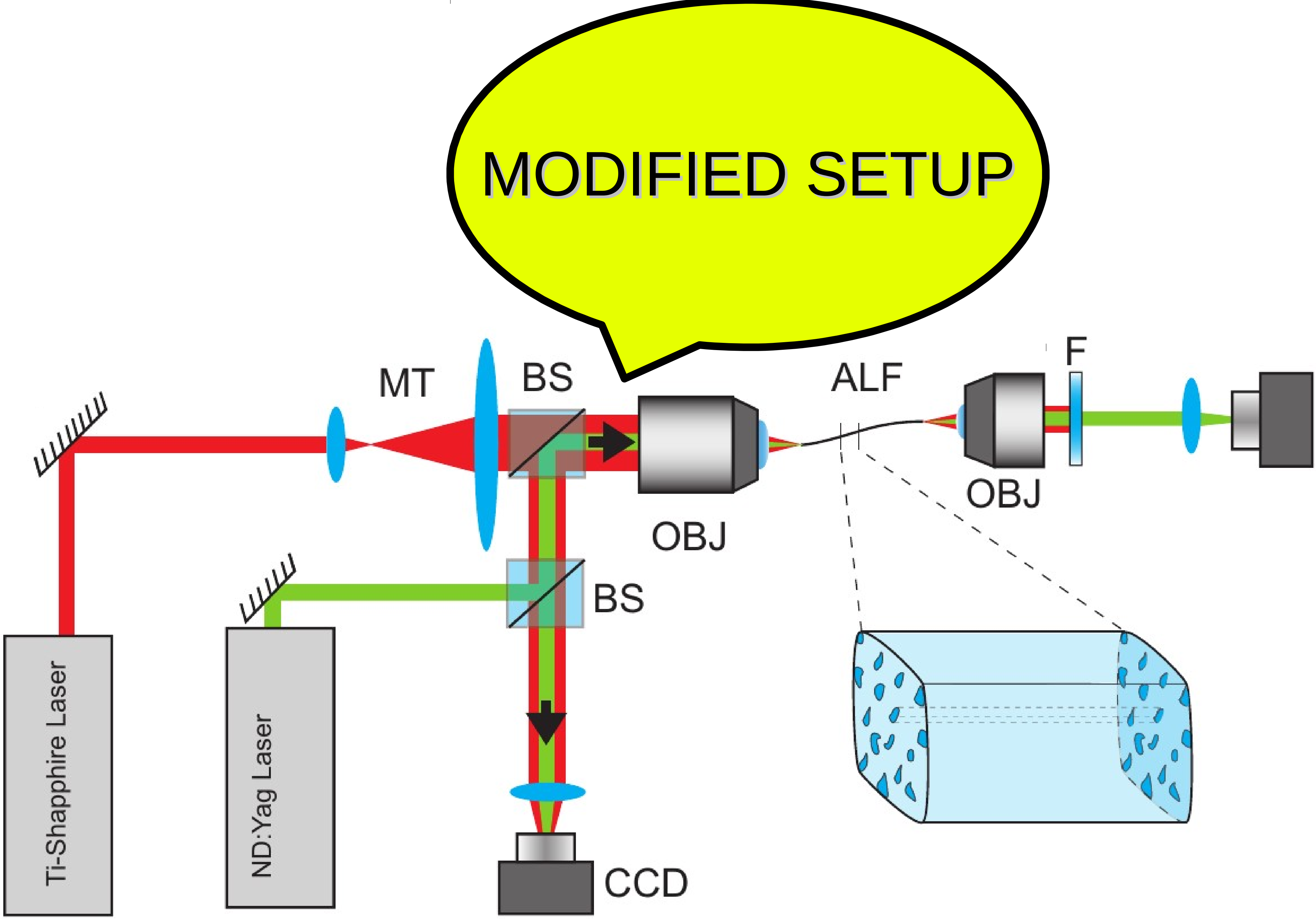
- it's thermal !

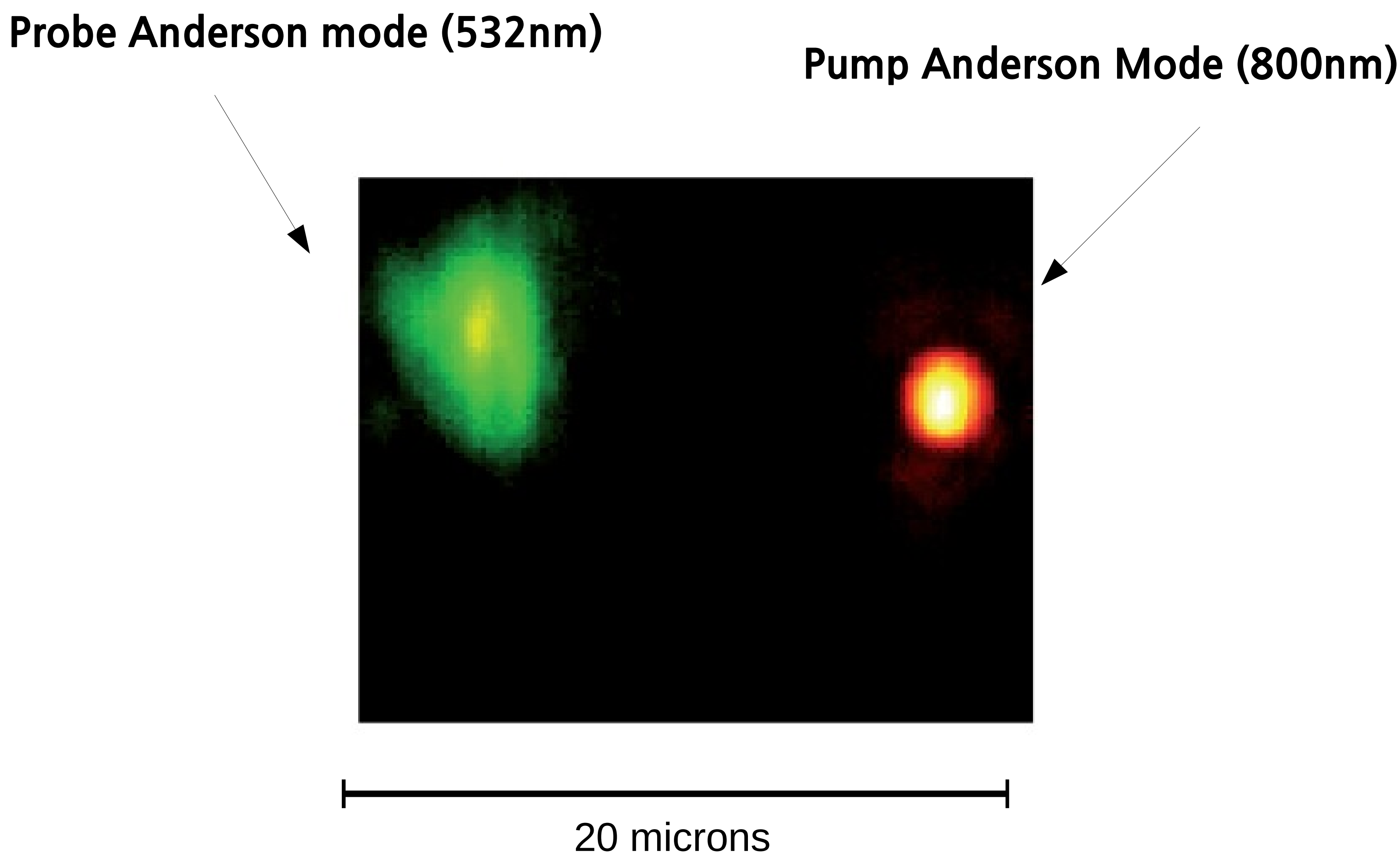


Timescale is compatible with thermal effects
(PMMA and PS absorb the infrared light)

Action at a distance between Anderson localizations in nonlinear nonlocal media

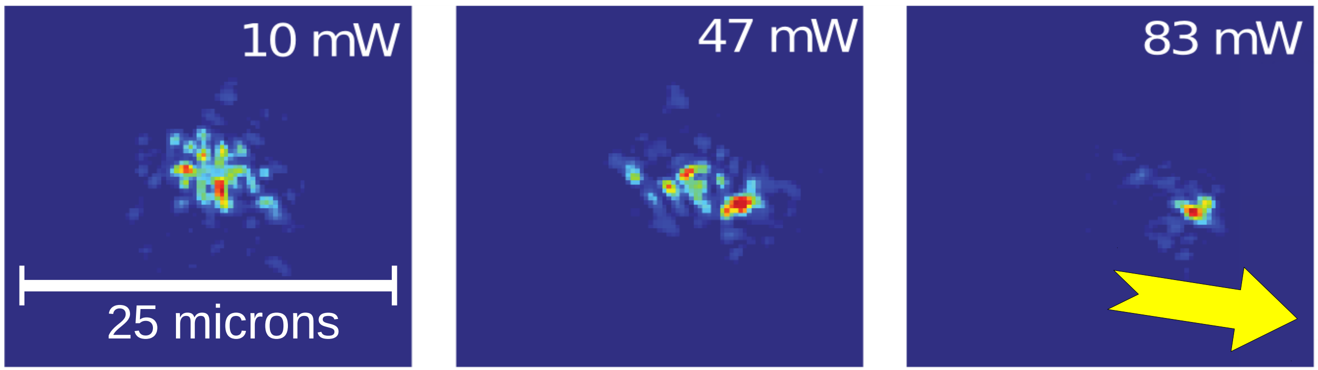
- thermal nonlinearity is nonlocal!

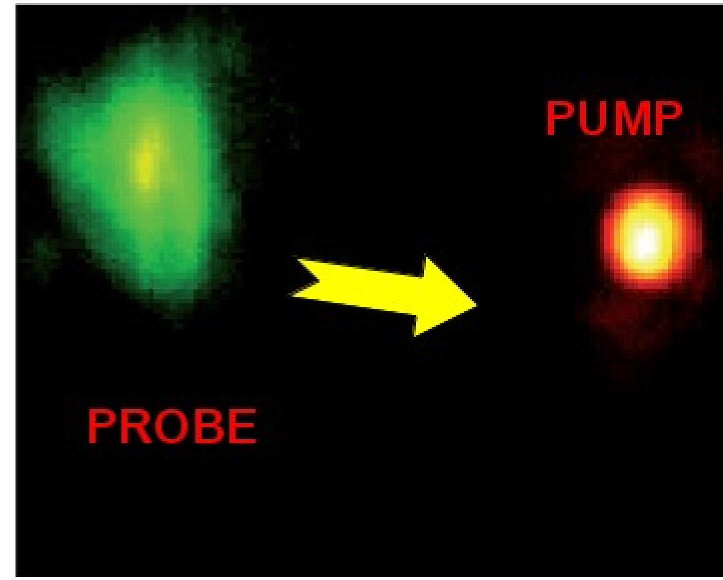

MODIFIED SETUP
MT
BS
OBJ
ALF
F
OBJ
BS
Ti-Shapphire Laser
ND:Yag Laser
CCD

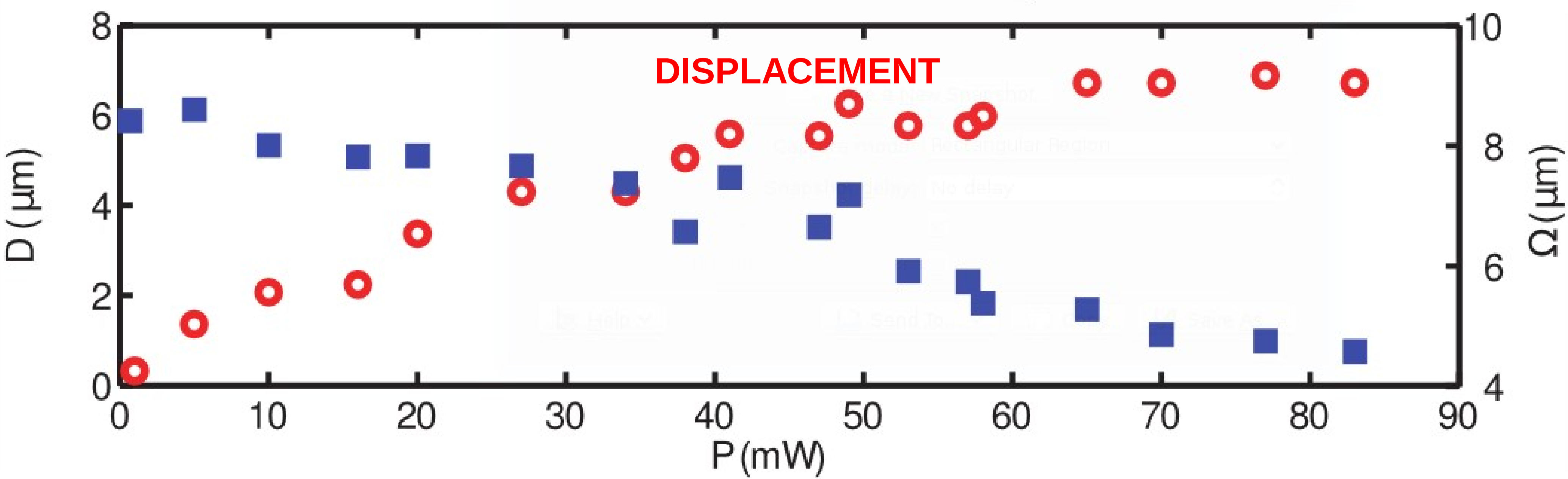

Probe Anderson mode (532nm)
Pump Anderson Mode (800nm)
20 microns

The size of the probe changes with the pump power !

Probe Anderson mode (532nm)





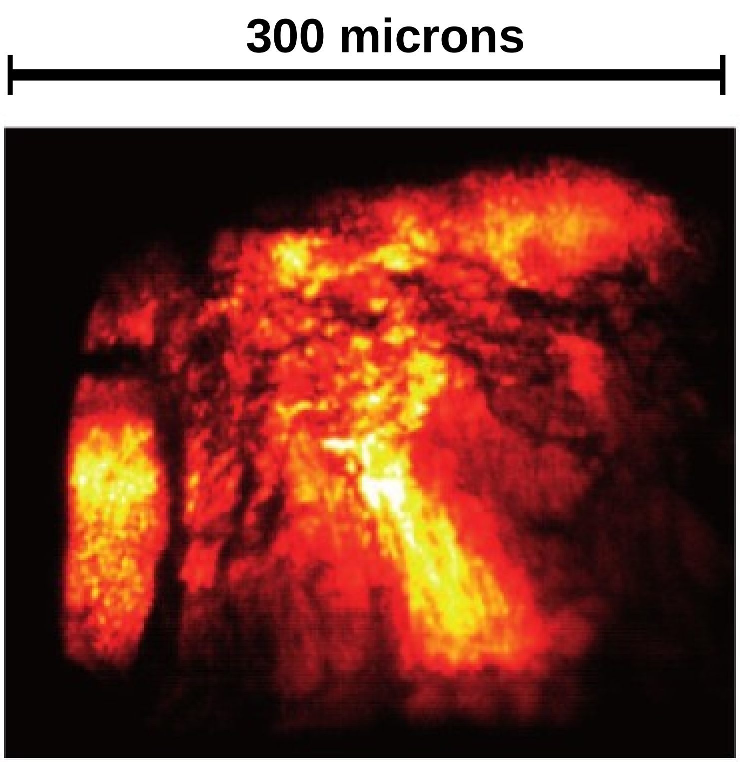


The migration of the multicolor Anderson localization

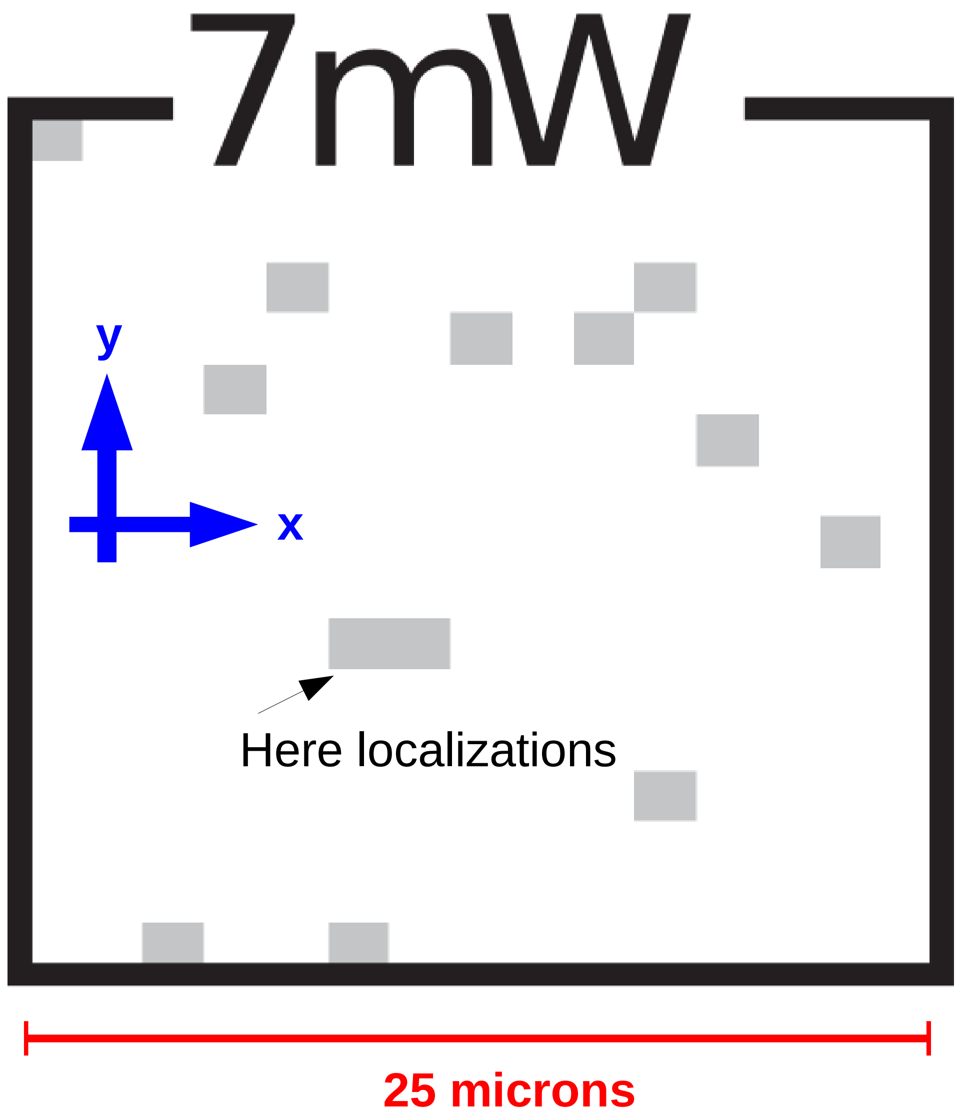
A form of transport in the Anderson regime

Density map of localizations

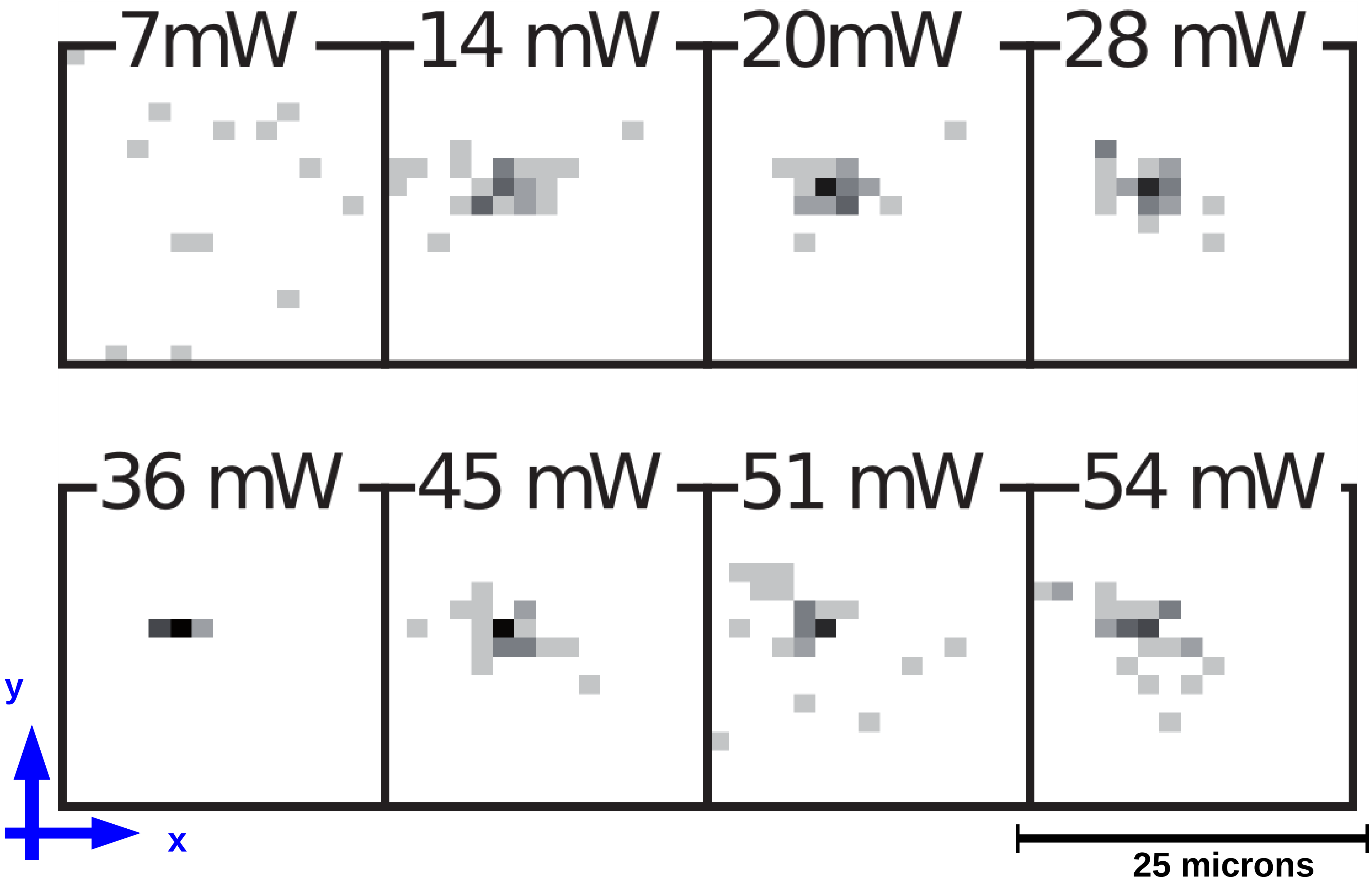
- We count the states in any spatial location

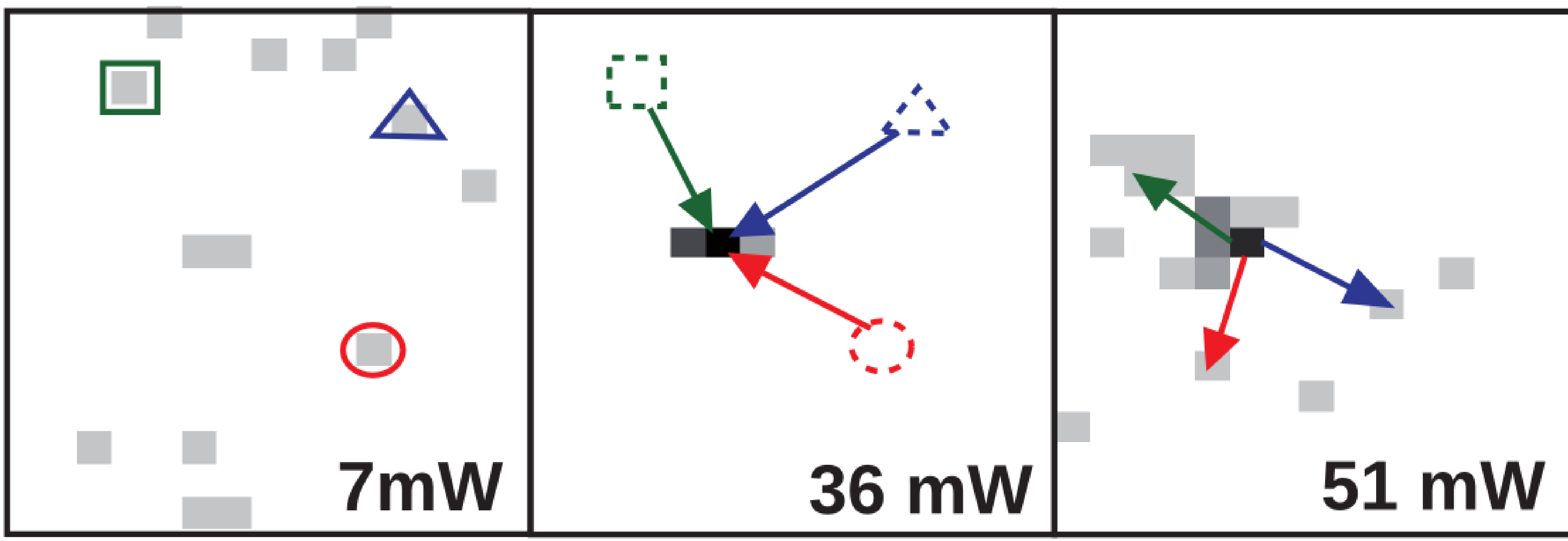


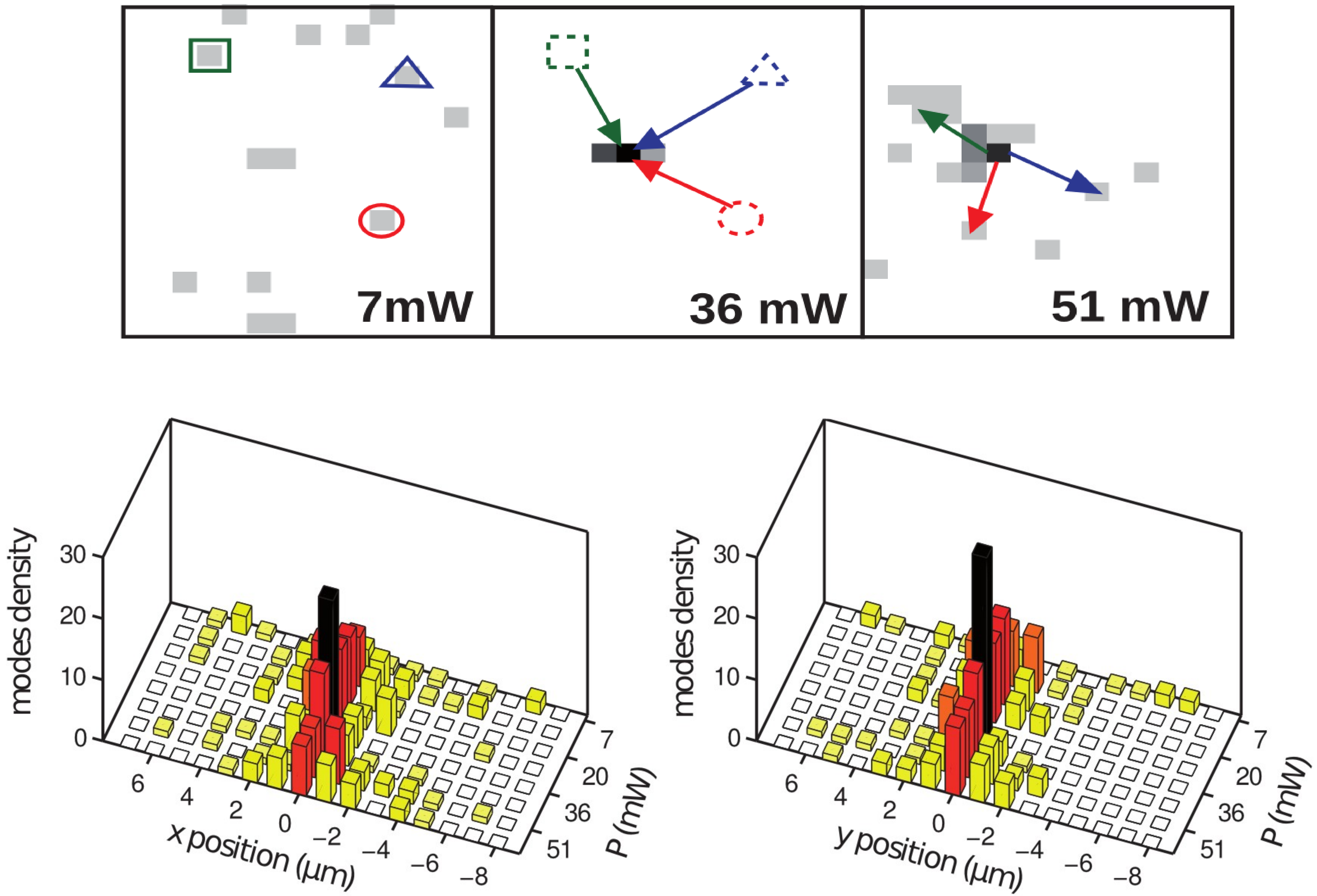
FIBER OUTPUT



Density map of locs Vs power




7mW
36 mW
51 mW


7mW
36 mW
51 mW
modes density
30
20
10
0
6
4
2
0
−2
−4
−6
−8
x position (μm)
7
20
36
51
P (mW)
modes density
30
20
10
0
6
4
2
0
−2
−4
−6
−8
y position (μm)
7
20
36
51
P (mW)

Model with nonlocal nonlinearity

$$2ik\frac{\partial A}{\partial z} + \nabla_{x,y}A + 2k^2\frac{\Delta n}{n_0}A = 0,$$

$$\Delta n = n_{\mathrm{PS}} - n_{\mathrm{PMMA}} = \Delta n_R + \Delta n_{\mathrm{NL}}$$

$\Delta n_R(x, y)$ due to the disorder

$$\Delta n_{\mathrm{NL}} = \int K(x - x', y - y')|A|^2(x', y')dx'dy'.$$

$$\Delta n_{\mathrm{NL}} \cong K(x, y)\int |A|^2 d\mathbf{r} \cong P\left(\Delta n_1 + \frac{r^2}{2}\Delta n_2\right).$$

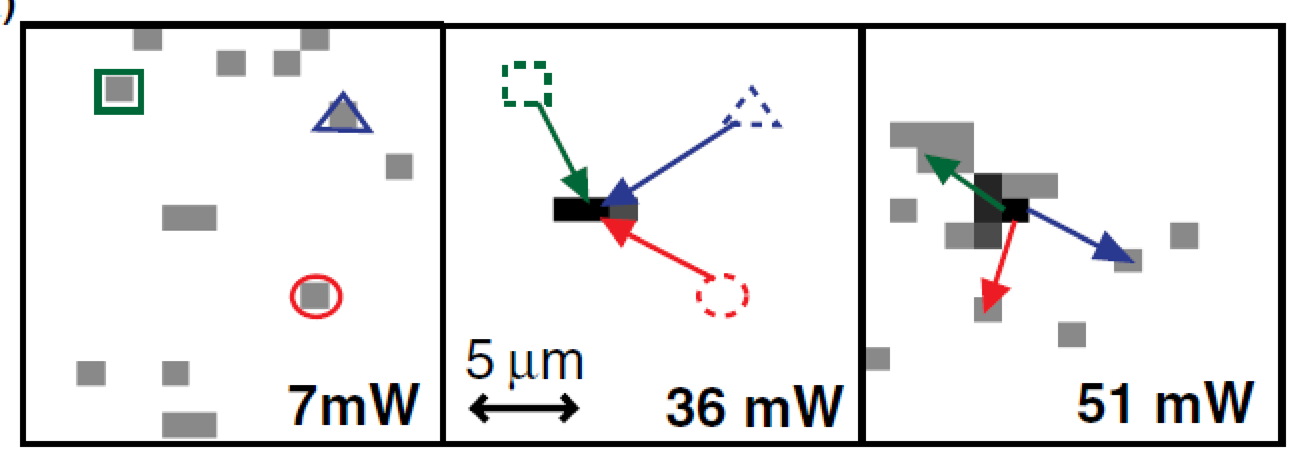
Collective coordinates

$$P_p \frac{d^2\mathbf{r}_p}{dz^2} = \int I_p(\mathbf{r} - \mathbf{r}_p) \nabla_{x,y} \frac{\Delta n_{NL}}{n} d\mathbf{r},$$

$$\Delta n_{\mathrm{NL}} = \sum_{q=1}^{N} \Delta n_{\mathrm{NL},q} \cong \sum_{q=1}^{N} \frac{P_q \Delta n_2}{2} (\mathbf{r} - \mathbf{r}_q)^2.$$

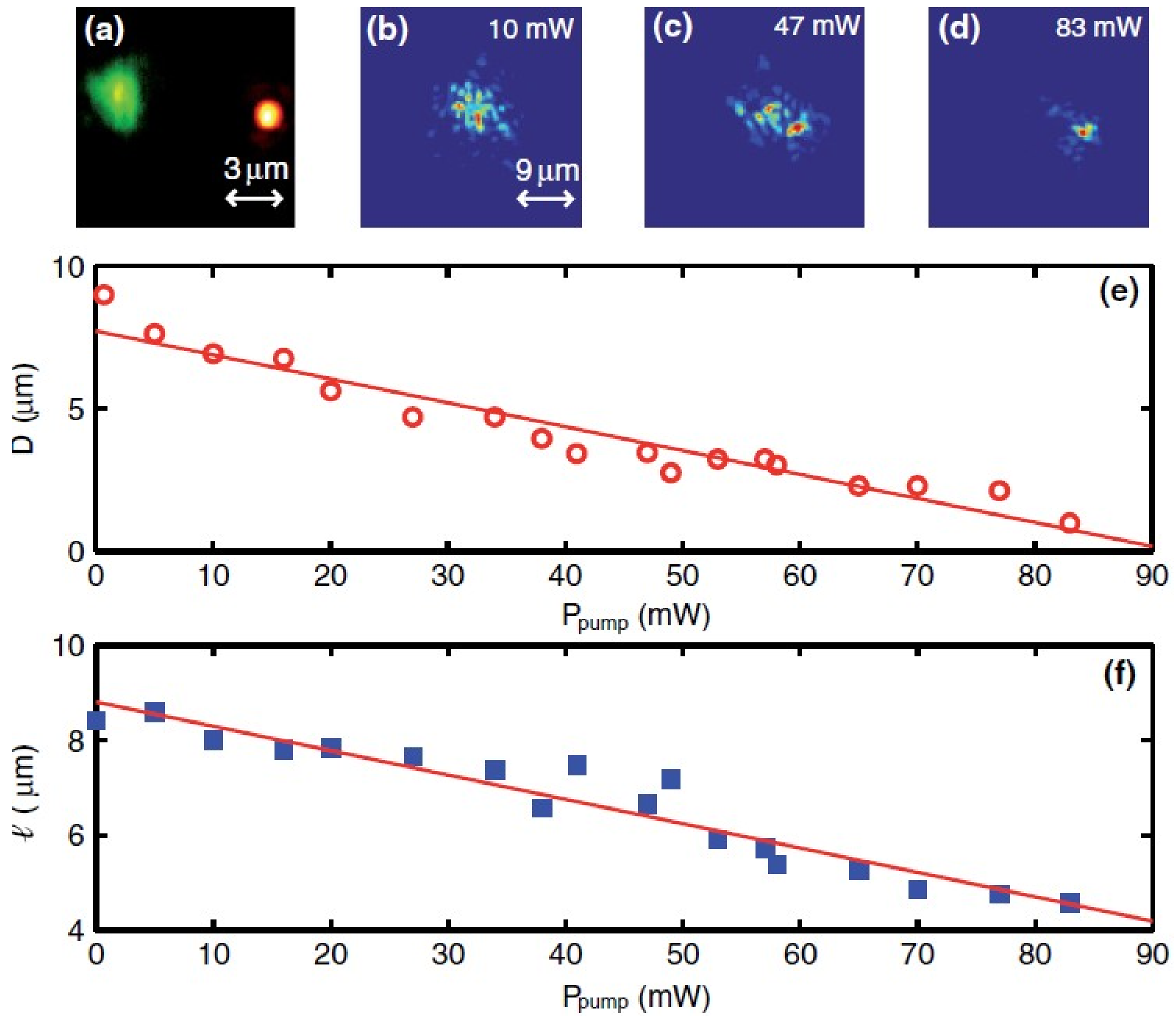
$$P_p \frac{d^2\mathbf{r}_p}{dz^2} = -\nabla_{x_p,y_p} \sum_{q=1}^{N} \frac{|\Delta n_2| P_q P_p}{2n_0} |\mathbf{r}_p - \mathbf{r}_q|^2.$$

(a)



Action at a distance for two states

$$D(z) = D(0)\left(1 - \frac{|\Delta n_2| z^2}{2n_0} P_{\text{pump}}\right).$$

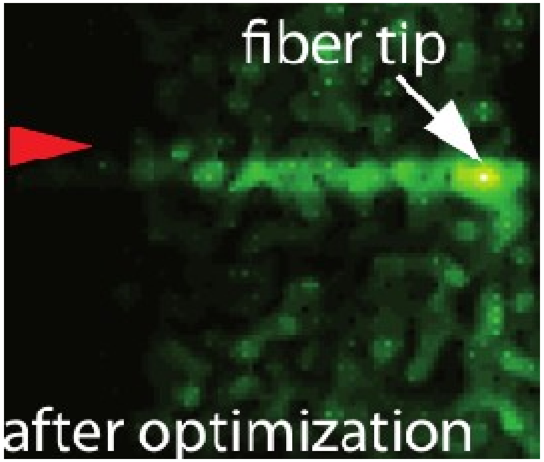


Leonetti, Karbasi, Mafi, CC, PRL 112, 193902 (2012)

Conclusions

- Nonlinearity and nonlocality in 2D disorder fibers
- Action at a distance
- Transport in the Anderson regime
- Incoherent Anderson states

  and interative focusing (see poster)
- Variational theoretical approaches



THANKS !

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