CONGRESSO

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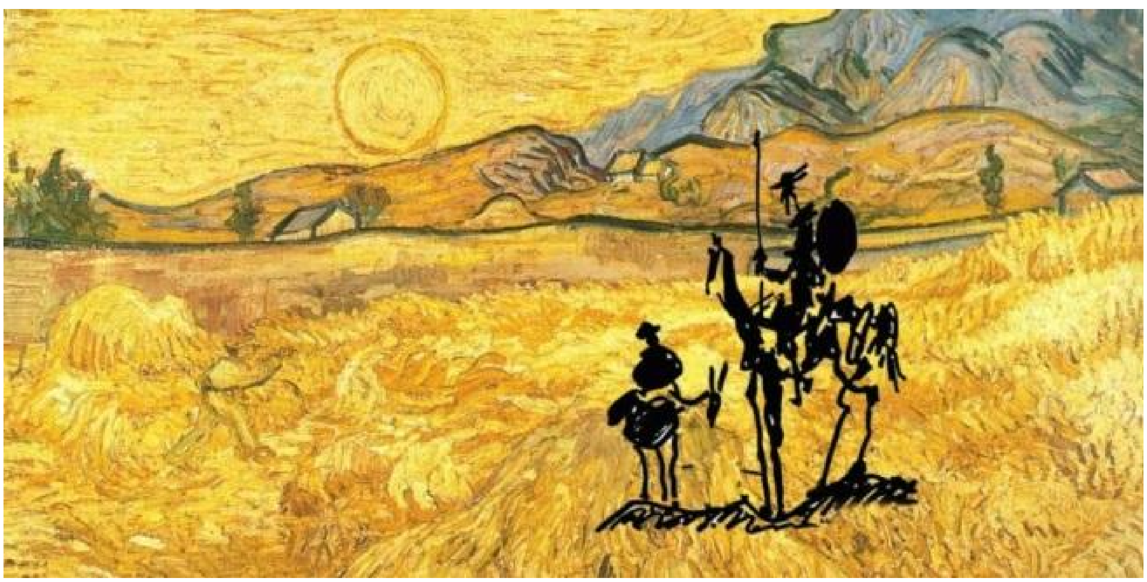

Singular asymmetric plasma structures

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Asymmetry is often regarded as a perturbation of an otherwise symmetric state of affairs. Strongly asymmetric structures are nevertheless observed in several states of matter. Plasmas, in particular, offer a fertile setting for the observation and laboratory reproduction of nonlinear, strongly asymmetric, long lived structures, which cannot be conceived as being small perturbations of any symmetric counterpart. Since a few years, at CNR-IPCF, we have been carrying out research on asymmetric plasma structures, ranging from raw data analysis to the development of new mathematical tools and numerical algorithms. Our research is based on the statistical description of plasmas conceived as a set of multi species particles, electric and magnetic fields. One major result is that, beside nonlinearity, the emergence of asymmetry is intimately connected to the onset of singularities, i.e. infinities, in the model distributions of certain physical quantities. Owing to their recently proved stability properties, the novel asymmetric singular inhomogeneous structures thus found are likely to play an important role in plasma science. Our work recently received special mention at the 100th congress of the Italian Physical Society.

Singular asymmetric plasma structures

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ABSTRACT. Asymmetry is often regarded as a perturbation of an otherwise symmetric state of affairs. Asymmetric structures are nonetheless observed in several states of matter. Plasmas, in particular, offer a fertile setting for the observation of nonlinear, strongly asymmetric, long lived structures, which cannot be interpreted as small perturbations of any symmetric counterpart. At CNR-IPCF, we carry out research on asymmetric plasma structures, ranging from raw data analysis to the development of new mathematical tools and numerical algorithms. Our research is based on the statistical description of plasmas conceived as a set of multi-species particles, electric and magnetic fields. One major result is that, beside nonlinearity, the emergence of asymmetry is intimately tied to the onset of singularities, i.e. infinities, in the model distributions of certain physical quantities. Owing to their superior stability properties, the novel asymmetric singular inhomogeneous structures thus found are likely to play an important role in plasma science. Our work recently received special mention at the annual congress of the Italian Physical Society.

Background

Observations show that stable strongly asymmetric nonlinear structures are as likely to emerge in Nature and in artifacts as their equally nonlinear symmetric counterparts (Fig 1).

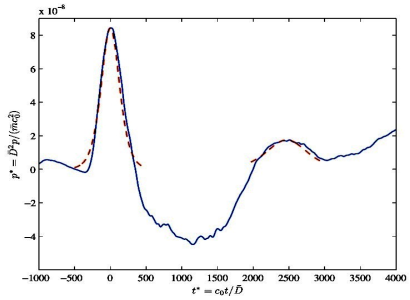

Fig 1: Asymmetric waveform in a granular material imperfectly fitted by a symmetric Korteweg de Vries soliton [1]. Reproduced under permission.

Other effects are thus needed, beside nonlinearity, to explain those structures. Causes due to large imperfections in their hosting medium [1] make them unstable and are thus inadequate.

Context

To analyze the emergence of asymmetry at a fundamental level we conceive plasmas as a set of multi-species particles in electromagnetic fields.

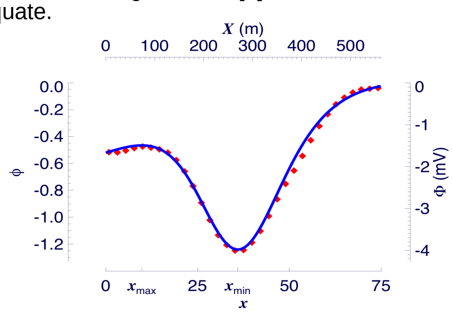

Fig 2: Asymmetric electric potential waveform in a plasma fitted by our model profile (continuous line).

The large ratio of the particle mean free path to the size of the structures (the Knudsen number, typically upwards of one hundred, in the applications so far envisaged) affords the adoption of the collisionless Vlasov model, as derived from the equations of Statistical Mechanics.

Singular distributions

A first result of our work is the determination of the electric potential in the structures by the sole knowledge of its morphology (Fig 2).

This potential is then used to find the particle distributions by inverting Poisson integral equation, much in the same way as done in image retrieval from tomographic data.

A second highlight is that those distributions are singular [2]. Singular solutions to the Vlasov model seem to have gone unnoticed in the over seventy years since it was introduced.

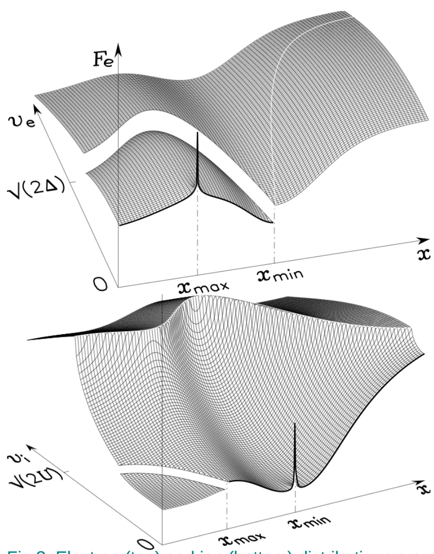

Fig 3: Electron (top) and ion (bottom) distributions are jump discontinuous on their separatrix and log singular at the bottom of their potential wells.

Related as it is to the collisionless hypothesis, singularity is checked by collisional processes. These however wear plasma structures away and besides their treatment faces severe difficulties.

Inversion lemmas

In a third contribution we offer a better alternative to collisional effects by turning the singular solutions of Vlasov equation into the boundary values of certain well behaved sectionally analytic functions of complex velocity.

To do so, we proved two inversion lemmas (a fourth valuable product of our research) which connect the Riemann-Liouville and Weyl fractional transforms to the finite Hilbert and Stieltjes transforms (Fig 4).

A fifth advance captures the relevant properties of singular particle distributions by simple mathematical formulas which allow to actually see them (Fig 3) and to model realistic potentials in a manageable way and in remarkable agreement with observations (Fig 2).

Inversion of Abel-Weyl equation

$$\int_a^w \mathrm{d}u \frac{f(u)}{\sqrt{(w-u)}} = \int_w^b \mathrm{d}u \frac{g(u)}{\sqrt{(u-w)}}$$

$$f(w) = \frac{1}{\pi} P \int_a^b \mathrm{d}t \frac{\sqrt{(t-a)}}{\sqrt{(w-a)}} \frac{g(t)}{t-w}$$

Fig 4: One of the two inversion lemmas we proved to solve Poisson integral equation [3].

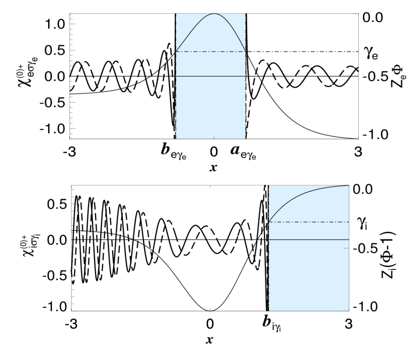

Fig 5: Trapped electron (top) and ion (bottom) modes of the free-streaming part of the Vlasov operator in the Fourier transformed velocity space.

Spectroscopy

The plasma structures thus found are able to emit, absorb and scatter waves much in the same way as an atom. Determining the continuous and discrete spectra, the subtle degeneracy and unfolding the distributional nature of their oscillation free and bound eigenstates is a sixth result of our analysis allowing for their unprecedented view in suitable transform spaces (Fig 5) and for the proof of their completeness (Fig 6).

Expansion of an arbitrary function ⇒ COMPLETENESS

$$h_\alpha(s,q_x,q_y) = \sum_{s_\alpha=\pm}\int_{-\infty}^{\infty}\mathrm{d}c_\alpha \int_{-V_\alpha(x)}^{\infty}\mathrm{d}\gamma_\alpha \left(A_\alpha^{s_\alpha}+B_\alpha^{-s_\alpha}\right)\langle h|\chi^{s_\alpha}_{\alpha k_y\sigma c_\alpha\gamma_\alpha}\rangle_{x,q_x,q_y}\,\chi^{s_\alpha}_{\alpha k_y\sigma c_\alpha\gamma_\alpha}(x,q_x,q_y). \quad (21)$$

arbitrary function; superposition; arbitrary coefficients; scalar product; eigenfunction

Fig 6: Expansion of an arbitrary function in terms of the eigenfunctions of the Vlasov operator.

Stability

Thermdynamic free energy is a valuable tool to determine the stability to external perturbations of plasma structures having singular particle distributions: a seventh feat of our work is that such structures are the most stable (Fig 7).

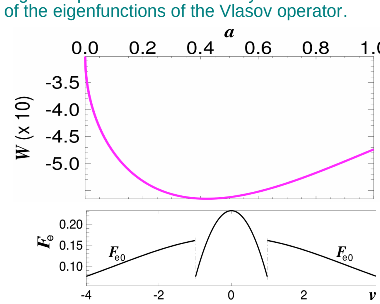

Fig 7: The plasma free energy W (top) has a minimum when the ratio of the plasma boundary potentials is a=0.4. The corresponding most stable structure has a singular electron distribution (bottom) and an asymmetric potential (Fig 2).

Numerical schemes

Due to the singularity of their particle distributions, kinetic simulations of asymmetric plasma structures are computational challenges. An objective of our current work is to reduce their memory foot-print by using high order compact difference schemes in transform spaces: order 9 schemes are very promising (Fig 8).

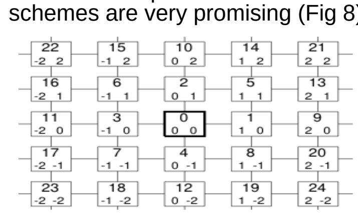

Fig 8: The ninth order stencil for the Fourier transformed Vlasov equation.

References

[1] S R Hostler *et al*, *Phys Rev E* **72**, 31303, 2005.

[2] L Nocera, *Ann Phys*, 323, 2482, 2008.

[3] L Nocera, L J Palumbo, *Phys Plasmas*, **20**, 12107, 2013.