

About scaling properties of relative velocity between heavy particles in turbulence

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Abstract. We present results obtained from high-resolution direct numerical simulations (DNS) of incompressible, statistically homogeneous and isotropic turbulence, up to a Taylor scale based Reynolds number $Re_\lambda \simeq 200$ and with millions of heavy particles with different inertia. In our set-up, particles are assumed to be spherical and rigid, they simply move by viscous forces, such as the Stokes drag. The velocity statistics is found to be extremely intermittent, with an almost bi-fractal behavior. Here, we consider also a new data analysis for the stationary distribution of *rescaled longitudinal velocity difference* and further assess the intermittent character of the heavy particles velocities, characterized by the presence of quasi-algebraic tails.

1. Introduction

The acceleration of a dilute suspension of spherical heavy particles in a turbulent fluid can be described as:

$$\dot{\mathbf{X}} = \mathbf{V}, \quad \dot{\mathbf{V}} = -\frac{1}{\tau_s} [\mathbf{V} - \mathbf{u}(\mathbf{X}, \mathbf{t})], \quad (1)$$

where the dots indicate time derivatives. The particle position and velocity are $(\mathbf{X}(t), \mathbf{V}(t))$, respectively, while $\mathbf{u}(\mathbf{X}(t), t)$ is the incompressible Eulerian fluid velocity evaluated at the particle position. Particle inertia is quantified by means of *Stokes number*, St . The Stokes number is defined as $St = \tau_s/\tau_\eta$, i.e. the ratio between the particle response time τ_s and the flow Kolmogorov timescale $\tau_\eta = (\nu/\varepsilon)^{1/2}$, where ν is the flow kinematic viscosity and ε the average rate of energy injection. The response time $\tau_s = 2a^2\rho_f/(9\nu\rho_p)$ depends on the particle radius a , and the ratio between the fluid ρ_f and particle density ρ_p . In this set up, particle radius is much smaller than the Kolmogorov scale of the flow $a \ll \eta$, and the Reynolds numbers at the particle size is much smaller than one. Also, gravity acceleration, as well as hydrodynamical interactions and particle-particle integrations are neglected.

Point particle approximation is working pretty well for particles sizes smaller or of the order of

the viscous scale, as direct comparison between numerics and experiments has shown Cencini et al. (2006); Ayyalasomayajula et al. (2006). Finite size effects becomes important only for large particles see e.g. Qureshi et al. (2007); Xu & Bodenschatz (2008); Qureshi et al. (2007); Calzavarini et al. (2009); Homann & Bec (2010).

Thanks of the simple form of the above dynamics, a large amount of theoretical and numerical work has been done, particularly on two features: heavy particles spatial distribution and velocity statistics, Because of inertia, non trivial correlations between particle positions and structures of the underlying flow appear. Heavy particles are expelled from vortical structures Bec et al. (2006b), and *preferentially concentrate* in specific regions of the flow. Hence, strong inhomogeneities in the particle spatial distribution develop, depending on the relative importance between inertial forces and turbulent advection.

Inertia is responsible also for the formation of *fold caustics* (also called the *sling effect*), which results in large probabilities that close particles have important velocity differences Bec et al. (2005); Wilkinson & Mehlig (2005); Wilkinson, Mehlig & Bezuglyy (2006); Falkovich & Pumir (2007). In other words, in a small limited region of the turbulent flow, heavy particles can be found that move with very different velocities. Hence, particularly in the limit of moderate to large inertia, the motion of heavy particles one can not use a continuous hydrodynamic description for the motion of heavy particles.

Preferential concentration and caustics can modify the rate of particles collisions in turbulent flows: in particular, for atmospheric physics, astrophysics, and engineering applications it is crucial to assess how these effects depend on the Reynolds number Re_λ of the flow (see e.g. Derevyanko, Falkovich, & Turitsyn (2008); Xue, Wang & Grabowski (2008)). In addition, both effects have an impact on the way pairs of inertial particles separate in turbulent flows, Bec et al. (2010a).

In this paper, we present a brief summary of previous results concerning scaling of velocity differences between inertial particles, Bec et al. (2010b, 2011), and a new data analysis concerning the probability density function (PDF) of the rescaled velocity differences of heavy particles, namely

$$\sigma = \dot{\mathbf{R}}/R, \quad \mathbf{R}(t) = \mathbf{X}_1(t) - \mathbf{X}_2(t).$$

This can be viewed as a longitudinal velocity gradient of an *effective particle velocity field*. The behavior of this quantity has been previously discussed in a series of works (see Wilkinson & Mehlig (2005); Wilkinson, Mehlig & Bezuglyy (2006); Piterbarg (2002); Derevyanko et al. (2007); Bec, Cencini & Hillerbrand (2007)), where the advecting flow is random and smooth. When the advecting flow has no time persistent structures, the dynamics of σ becomes independent of the separation R at very small scales. It is not obvious to observe the same behavior in real flows, where time correlations and structures play important roles. Moreover, the PDF is found to have power-law tails, and a phenomenological argument can be formulated relating the algebraic behavior with events where the particles approach each other almost ballistically Bec, Cencini & Hillerbrand (2007). Here, we present results from Direct Numerical Simulations of an incompressible, three-dimensional fully developed turbulent flow at moderate Taylor scale based Reynolds number, $Re_\lambda \simeq 200$, and seeded with heavy particles.

2. DNS formulation

The flow phase is described by the Navier-Stokes equations for the velocity field $\mathbf{u}(\mathbf{x}, t)$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_f} \nabla p + \nu \Delta \mathbf{u} + \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0. \quad (2)$$

N	Re_λ	η	δx	ε	ν	τ_η	t_{dump}	δt	T_L
512	185	0.01	0.012	0.9	0.002	0.047	0.004	0.0004	2.2

Table 1. Eulerian parameters for the data analyzed. N is the number of grid points in each spatial direction; Re_λ is the Taylor-scale Reynolds number; η is the Kolmogorov dissipative scale; $\delta x = \mathcal{L}/N$ is the grid spacing, with $\mathcal{L} = 2\pi$ denoting the physical size of the numerical domain; $\tau_\eta = (\nu/\varepsilon)^{1/2}$ is the Kolmogorov dissipative time scale; ε is the average rate of energy injection; ν is the kinematic viscosity; τ_{dump} is the time interval between two successive dumps along particle trajectories; δt is the time step; $T_L = L/U_0$ is the eddy turnover time at the integral scale $L = \pi$, and U_0 is the typical large-scale velocity. As for the dispersed phase, $N_{\text{tot}} = 12 \times 10^7$ is the total number of advected particles; among these, $N_t = 5 \times 10^5$ is the number of trajectories of heavy particles for each Stokes number, saved at frequency $\tau_\eta/10$, and $N_p = 7.5 \times 10^6$ is the number of particles per Stokes stored at frequency $10\tau_\eta$. The error bars on all statistically fluctuating quantities are of the order of 10%. The set of Stokes number is: $St \in [0.0; 0.16; 0.27; 0.37; 0.48; 0.59; 0.69; 0.80; 0.91; 1.01; 1.12; 1.34; 1.60; 2.03; 2.67; 3.31]$.

The statistically homogeneous and isotropic external forcing \mathbf{f} injects energy in the first low wave number shells, by keeping constant their spectral content. Here we give a reference table 1, summarizing the main parameters of the DNS. Further details can be found in Bec et al. (2006); Cencini et al. (2006).

Kinematic viscosity is chosen such that the Kolmogorov length scale $\eta \approx \delta x$, where δx is the grid spacing. The numerical domain is cubic and 2π -periodic in the three directions of space. We use a fully dealiased pseudo-spectral algorithm with 2nd order Adam-Bashforth time-stepping. In the statistically stationary turbulent flow, we injected the inertial particles with 16 Stokes numbers (see table). Once the particles have relaxed onto their stationary state, we start the productive run which lasted about 5 large-scale eddy turn over times.

In Bec et al. (2010b), it was discussed the stationary statistics of moments of velocity differences,

$$S_p(r, St) = \left\langle |\dot{R}|^p \mid R=r \right\rangle,$$

between heavy particles in the same DNS at Re_λ and also at a higher Reynolds number $Re_\lambda \simeq 400$.

3. Scaling properties of velocity increments

At very small separations, $r \ll \eta$, in the viscous range of the Eulerian velocity, it is found that velocity increments between pairs of particles depend on the relative importance of two types of statistical realizations: *smooth events*, where particle velocity is approximately the same of the fluid velocity, and *caustic contributions*, when two particles at very close positions exhibit very different velocities. Inertia tunes the statistical weight of them, varying between the tracers limit (at $St = 0$) where a smooth differentiable behavior is observed, to the ballistic limit at very large Stokes, where velocity differences are independent of the separation. Moreover, scaling exponents ξ_p of velocity difference structure functions $S_p(r, St) \propto r^{\xi_p}$ show a quasi bi-fractal behavior, see Fig. (1), which is the fingerprint of the quasi-singular velocity realizations or caustics. In particular, for small p the exponents ξ_p hardly deviate from the smooth value $\xi_p = p$, while at large orders, they saturate to an asymptotic value $\xi_\infty(St)$, monotonically decreasing with increasing inertia, Bec et al. (2010b).

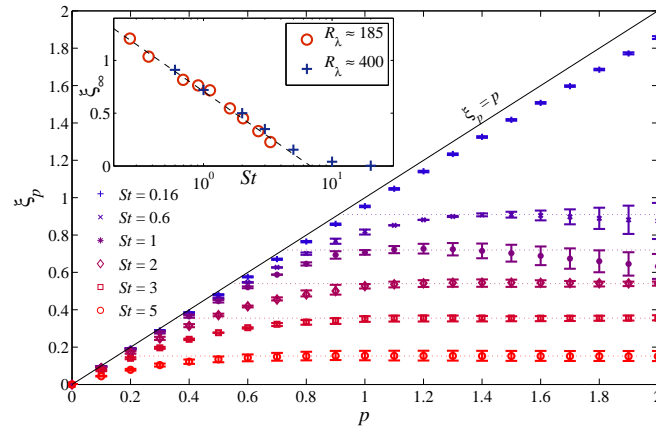


Figure 1. Leading scaling exponents of velocity increments at changing St and for different orders. Notice the behavior very close to a bifractal distribution with a superposition of smooth and singular –discontinuous– contributions. Such behaviors as been recently rigorously proven for a class of stochastic flows in 1d systems by Gustavsson & Mehlig (2010). Inset value of the saturation exponent ξ_p for $p \rightarrow \infty$ as a function of Stokes. Here there are also data from a simulation at larger Reynolds, Bec et al. (2010a)

The signature of such extreme intermittency has to show up also in the probability density function of the rescaled longitudinal velocity differences, σ_{St} . The interest for this quantity is because it is clearly better suited to show the appearance of quasi-singularities in the velocity statistics, i.e., large velocity differences at nearby positions. Moreover, as an effective particle velocity gradient, it can be also useful for modeling purposes.

In Figure 2, we show the probability density function of the rescaled longitudinal velocity difference $P(\sigma)$ for a moderate Stokes number $St = 3.3$, and at different scale separations smaller or of the order of the viscous scale η (the behavior is similar for other Stokes numbers $St \geq 0.6$, i.e. as soon as saturation of velocity structure functions scaling exponents is observed). It appears that the behavior in 3D fully developed turbulence resembles that observed in random, structures-less flows. For not too large values of σ , the distributions for different separations $R = r$ collapse onto a single one with a fat, almost algebraic behavior. Such algebraic behavior can be explained using recent rigorous results obtained for 1d systems by Gustavsson & Mehlig (2010). In that paper, the probability to observe a caustic with a velocity increment Δv conditioned to a small separation R is predicted to go as $P(\Delta v) \propto (\Delta v)^{D_2 - d - 1}$. Where D_2 is the correlation dimension of the fractal set where particles are sitting. Moreover, D_2 is also connected to the saturation value of the scaling exponents by the relation: $D_2 = d - \xi_\infty$ (see Bec et al. (2011); Gustavsson & Mehlig (2010)). The slope for the pdf of σ at fixed R must then go as $-1 - \xi_\infty$. This is consistent with the bi-fractal scenario for velocity difference structure functions $S_p(r; St)$. In conclusion, we have shown that velocity increments between two inertial particles show very singular statistical properties, reflected in both the saturation of large order moments and in the quasi algebraic tail of PDF of *coarse-scale* velocity gradients, i.e. longitudinal velocity increments normalized with the separation between two particles. Both issues may have a strong impact in the modelization of extreme events of collisional kernel.

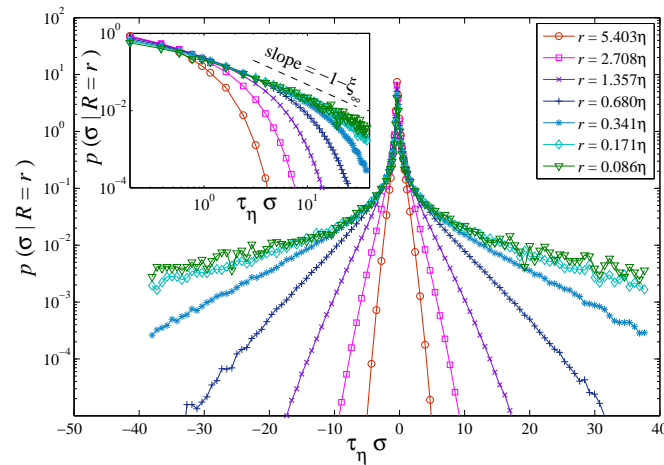


Figure 2. Probability density function of the (rescaled) longitudinal velocity difference σ for various values of particles separation R and for $St = 3.3$. Inset: same for the right tail in log-log coordinates. The algebraic tails possess a slope $-1 - \xi_\infty$, where ξ_∞ is the saturation scaling exponent of velocity increment structure functions.

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