Kinetic quasimodes in a plasma double layer

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Outline

Structure of the work: three stages

1. Strong electrostatic double layers in a multispecies plasma facing a boundary in space
2. Closure of the linearized Vlasov-Poisson eigenvalue problem (including particle trapping and sizeable pressure gradients), its eigenfunctions and their completeness.
3. The dispersion function of the eigenvalue problem and its roots in the physical and unphysical sheets

Four results of this work to remember

1. High DC voltage jumps in a plasma may be simply associated with beam-less, thermal electron and logarithmic trapped ion distributions.
2. Modes of electrostatic warm plasma oscillations in a double layer have a purely real continuous spectrum and a complete set of singular eigenfunctions.
3. There is also a new quasimode corresponding to new unphysical roots of the plasma dielectric function.
4. The frequency shift and continuum damping rate of the quasimode are proportional to its wavenumber, to the layer's width and voltage jump and to the electron's temperature.

Model double layers: known and open issues

Focus		*Main drawbacks*
One or more very hot species [1]	➡	Only weak double layers
Drifting or beam ions and/or electrons [2]	➡	Unsuitable boundary conditions
Numerical integration or ab initio numerical simulations [3,4]	➡	No analytical potential distribution $\Rightarrow$ no progress . . .

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Our double layer: what we need and what we aim at

A different kind of simple analytical trapped particle distributions ➡ Simple analytical potential distribution

Inhomogeneous plasma closures: known and open issues

Focus		*Main drawbacks*
Cold fluid plasma [5]	➡	No DC field, no kinetics
Density discontinuity [6]	➡	Sub Debye-length issues, no continuum damping
Numerical kinetic closure or ab initio simulations [7]	➡	No scaling laws for frequency and damping

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Our closure: what we need and what we deal with

- Narrow double layer
- Strong double layer
- Small temperatures

➡

- Sizeable electric field
- Significant particle trapping
- Simple scaling laws

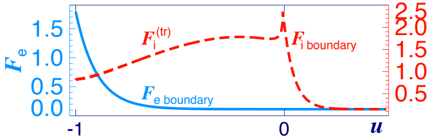
The steady state particle distributions

$$F_{\mathrm{e}}(u) = F_{\mathrm{e}}(u)|_{\mathrm{boundary}} = F_{\mathrm{eM}}(u) = \sqrt{\beta_{\mathrm{e}}}\, e^{-\beta_{\mathrm{e}} u}/\sqrt{\pi},\ \forall u,$$

$$F_{\mathrm{i}}(u) = F_{\mathrm{i}}(u)|_{\mathrm{boundary}} = F_{\mathrm{iM}}(u) = \sqrt{\beta_{\mathrm{i}}}\, e^{-\beta_{\mathrm{i}} u}/\sqrt{\pi},\ u \geq 0,$$

$$F_{\mathrm{i}}^{(\mathrm{tr})}(u) = +F_{\mathrm{iM}}(u)\,\mathrm{erfc}(\sqrt{\beta_{\mathrm{i}}}|u|) - F_{\mathrm{eM}}(u)\,\mathrm{erfi}(\sqrt{\beta_{\mathrm{e}}}|u|) + \frac{c}{\pi}K^2[3d(2b-5+5z)\tanh^{-1}\sqrt{z} + 2(e-8z)\sqrt{z}],\ u < 0,$$

$$z = Bu,\quad B, b, \beta_{\mathrm{e}}, \beta_{\mathrm{i}}, c, d, e, K, \mathit{constants}$$



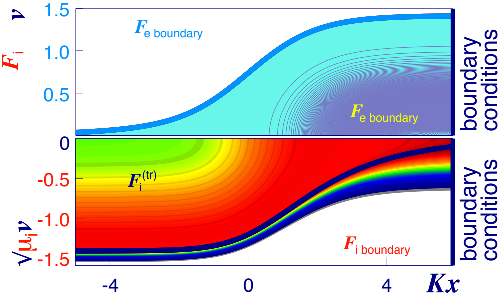


Figure 1 : The distributions of electrons and ions in the double layer vs. paricle energy u (right) and in phase space $x - v$ (left)

The steady state potential distribution

$$\Phi(x) = [A + tanh(Kx)]^2, \quad n_e = e^{\beta_e(\Phi - 1)}, \quad n_i = n_e - \Phi''$$

$$A > 1$$

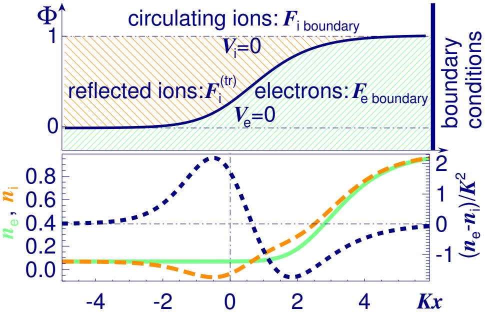


Figure 2 : The potential (top) and density (bottom) distributions in the double layer

Eigenfunctions of the Liouville operator

$$\chi^{s_\alpha}_{\alpha \mathbf{k}_\perp \sigma \gamma_\alpha \mathbf{c}_\alpha} = \frac{C_\alpha}{u_{\alpha\gamma_\alpha}} e^{-\mathrm{i} s_\alpha (\sigma - \mathbf{k}_\perp \cdot \mathbf{c}_\alpha)\xi_{\alpha\gamma_\alpha} + \mathrm{i} s_\alpha q_x u_{\alpha\gamma_\alpha} + \mathrm{i}\mathbf{q}_\perp \cdot \mathbf{c}_\alpha}$$

$$u_{\alpha\gamma_\alpha}(x) = \sqrt{[2(\gamma_\alpha + V_\alpha(x))/\mu_\alpha]}, \; \xi_{\alpha\gamma_\alpha}(x) = \int dx/u_{\alpha\gamma_\alpha}(x)$$

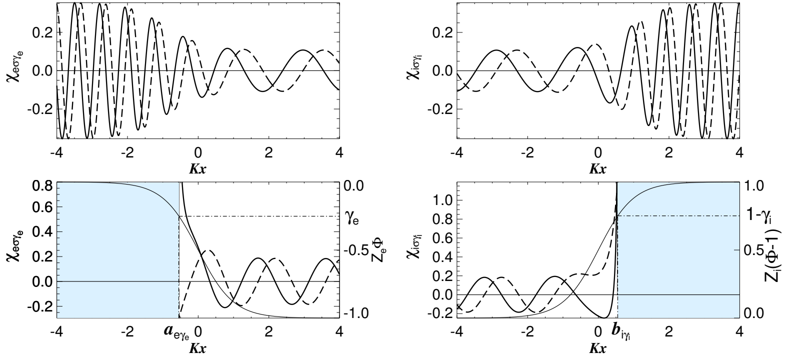


Figure 3 : Real (continuous line) and imaginary (dashed line) parts of the free electron (top left), free ion (top right), trapped electron (bottom left) and trapped ion (bottom right) eigenfunctions of the Liouville operator in the double layer.

The wave equation

$$\underbrace{\nabla^2(\omega_\mathrm{p}^2-\omega^2)e_{x\mathbf{k}\omega}=\omega_\mathrm{p}^{2\prime}D_x^{-1}e_{x\mathbf{k}\omega}}_{cold\ \ plasma\ \ terms}+D_x[\omega_\mathrm{p}^{2\prime}e_{x\mathbf{k}\omega}+(\omega_\mathrm{p}^2-\omega^2)n_{\mathbf{k}\omega}]$$

$$\mathbf{k}\perp\hat{\mathbf{x}},\ \ D_x=\frac{\partial}{\partial x},\qquad \overbrace{n_{\mathbf{k}\omega}(x)=\int_{-\infty}^{\infty}dsN(x,s)e_{x\mathbf{k}\omega}(s)}^{Vlasov}$$

Singular integral eigenvalue problem

$$(\omega_\mathrm{p}^2-\omega^2)e_{x\mathbf{k}\omega}(x)=\int_{-\infty}^{\infty}ds\,Q(x,s)e_{x\mathbf{k}\omega}(s)$$

Q : continuous integral kernel

Adjoint operator

$$\int_{-\infty}^{\infty}ds\,H(s,x)=-1$$

Fredholm theorem

Continuum eigenfunctions

$$e_{x\mathbf{k}\omega}(x)=D_{\mathbf{k}\omega}\delta(x-x_\omega)+P\frac{\psi(x)}{\omega^2-\omega_\mathrm{p}^2}$$

$$\psi(x)=-\int_{-\infty}^{\infty}ds\,H(x,s)\psi(s)$$

Dispersion function

$$D_{\mathbf{k}\omega}=\int_{-\infty}^{\infty}ds\frac{\psi(s)}{\omega^2-\omega_\mathrm{p}^2}$$

Narrow layer

$ka \ll 1$

$a = \textit{layer width}$

Strong layer

$[(e\Delta\Phi/a)/m_{\mathrm{e,i}}]/a \lesssim \omega^2$

$\Delta\Phi = \textit{voltage jump}$

Cold plasma

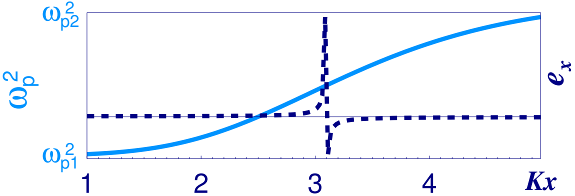
$v_{The,i} \ll \omega/k$

$v_{The,i} = \textit{thermal speed}$

Degenerate solution

$H(x,s) = \mathcal{H}(x)\mathcal{K}(s)$

$\psi(x) = \mathcal{H}(x)$



Nonphysical roots of the dispersion function (quasimodes)

$$\Re\omega = \underbrace{\frac{\sqrt{[\omega_{p1}^2+\omega_{p2}^2]}}{\sqrt{2}}}_{\textit{cold fluid limit}}\left[1+\frac{1}{8}ka\left(1+\frac{e\Delta\Phi/(m_e a^2)}{\omega_{p1}^2+\omega_{p2}^2}-4\frac{k_B T_e}{e\Delta\Phi}\right)\right]$$

$$\Im\omega = -\frac{\pi}{8}(ka)\frac{\sqrt{[\omega_{p1}^2+\omega_{p2}^2]}}{\sqrt{2}}\left(\frac{e\Delta\Phi/(m_e a^2)}{\omega_{p1}^2+\omega_{p2}^2}+4\frac{k_B T_e}{e\Delta\Phi}\right)$$

References

[1] H. Schamel (1972) *Plasma Physics* **14**, 915.

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[6] R. Guernsey (1969) *Phys. Fluids* **12** 1852.

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