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A GENERAL APPROACH FOR GEOMETRICAL CALIBRATION OF MACHINERY AND ITS APPLICATION TO A PARALLEL KINEMATIC MANIPULATOR

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Abstract: Geometric machine calibration is a widespread technique used to increase machine accuracy thanks to a set of measures and adequate algorithms that permit identification and, generally, compensation of all relevant repetitive errors in the structure. Traditional strategies are challenged by the vast kinematic variety of Parallel Kinematic Machines, today under development for industrial applications. An original approach is proposed and implemented to calibrate machine with generic architectures and varying geometrical errors. The analysis time is very short because the kinematic model is numerically obtained from a customized multi-body package and then automatically elaborated with numerical routines. As an example, the methodology is applied, in simulation, to a PKM with 3 translational degrees of freedom.

Keywords: Calibration, Parallel Manipulator

1. INTRODUCTION

A huge family of machines, used in very different industrial sectors, from machine tools, to robot and manipulators, are built to position a tool, or end-effector, into space, following the requirements of the desired task. Their positioning accuracy is affected by several error sources, like servo error, thermal structure expansion, static and dynamic structure flexibility and manufacturing accuracy of the machine itself. A possible approach to reduce the effect of geometrical inaccuracies is based on "software compensation": the basic idea is to identify, by specific tests, the real machine geometry and use it to drive the machine instead of the nominal one. The precision of the calibrated machine depends on several factors, like the number of available controlled axes, the error sources stability in time and the accuracy of the calibration procedure and instrumentation. Geometrical calibration has been widely used in the machine tool sector, where machine kinematics is often trivial (e.g. a machine with three orthogonal axes) and in the robotic sector (where common accuracy requirements are not so stringent). A new challenge is emerging in the growing sector of machines based on parallel kinematics structures ("PKM", as the machine used hereafter as test case, shown in Figure 1): many of their applications are a compromise of typical machine tool and robotic tasks, producing a tricky combination of kinematic complexity and high accuracy requirements.

Geometrical accuracy is still a critical factor for PKM industrial deployment: accurate and easy SW calibration would permit to speed up machine assembly, based only on rough alignment, and to obtain more accurate machines.

Several works have been published in this field, coping with the large kinematics variety typical of PKMs (Boer *et alii*, 2000), but they are often affected by two lacks: they are related to a specific machine architecture and/or they consider only a subset of possible error sources, because they exploit analytical simplifications valid only for the nominal kinematics (e.g. the axes of the revolute joints that constitute an universal joint must be coplanar and orthogonal).

To overcame these limitations an original approach is herein proposed and implemented in a specific software environment, in order to permit calibration of machines with various architecture (parallel or serial) and any kind of geometrical errors.

After a short presentation of the design environment, the proposed approach is presented and applied to the model of a 3 dof PKM developed by ITIA-CNR.

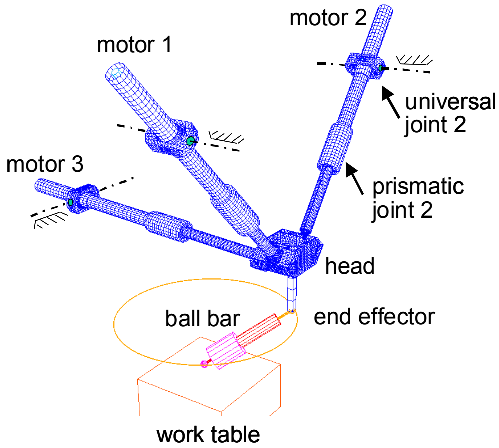

Fig. 1. The test case PKM (without fixed frame), with a Ball Bar device used for calibration (here presented by its Finite Element Model).

2. THE VIRTUAL PROTOTYPING ENVIRONMENT

The described approach has been implemented in the Virtual Prototyping Environment for Parallel Kinematic Machine development, built by ITIA-CNR in the last years. Leaving a more complete description to previous papers (Leonesio *et alii*, 2001), we recall here that the VPE-PKM has been conceived to support design of generic machinery characterized by a complex kinematics, producing in a short time all key indicators, like singularity analysis, workspace, actuators required force/torque, internal effort in the structure, effect of lumped compliances and of manufacturing tolerances.

These capabilities have been obtained joining the general modelling power of a commercial multi-body ("MB") software (ADAMSTM, MDInc) with the efficiency of PKM-specific analysis routines (implemented in MatlabTM, The MathWorks, Inc.), as depicted in Figure 2. The MB package has been customised, for example adding routines for automatic workspace exploration, while kinematic analyses are performed in the mathematical environment using linear mechanism models produced by the MB package in each point of the workspace selected by the user. Geometrical errors are considered like "fictitious machine actuators", able to move the End-Effector in a uncontrolled way. This information is extracted from the machine linearized model, computing a so called "Extended Inverse Jacobian" "EIJ" (similarly to what is usually done to analyse the relationship between End-Effector motion and main machine actuators motion):

$$\delta x_{\text{end effector}} = EIJ \cdot \delta q_{\substack{\text{fictitious}\\ \text{joints}}}, \qquad (1)$$

that relates End-Effector motion (6 dof in space) with motions at different fictitious joints. The Extended Inverse Jacobian is the base of all calibration analyses, but can be used also to establish manufacturing tolerances and assembly procedures, given a desired accuracy at the EE.

3. THE TEST MACHINE

The machine used to simulate the calibration procedure is a PKM with 3 translational dofs (plus 2 rotational dofs at the wrist, not considered here) built by ITIA-CNR for light deburring operations in a shoe manufacturing plant. The machine is a Tsai platform, where the moving platform is connected to the fixed base by three similar variable length struts, with universal joints at both ends (it is therefore classified as a 3-RRPRR mechanism). Under specific conditions on the orientation of revolute joints axis, the machine posses pure translational degrees of freedom.

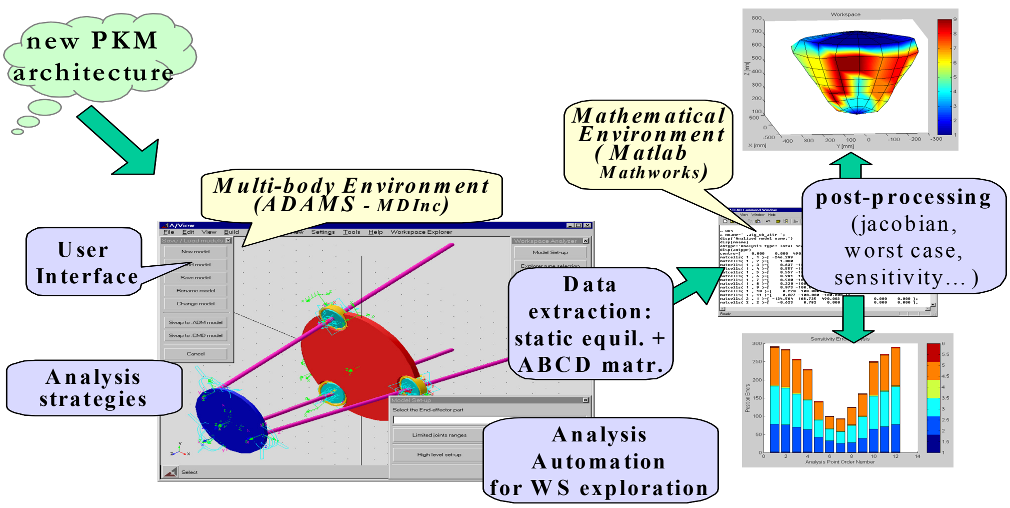

Fig. 2. Architecture of the Virtual Prototyping Environment

The nominal machine kinematics can be easily obtained writing vector equations representing the closure of three loops connecting the fixed and moving platform (O-O_1 vector) through the three struts, as represented in Figure 3:

$$\mathbf{x} + [\mathbf{R}]\mathbf{b}_i = \mathbf{A}_i + q_i\mathbf{w}_i\,, \qquad (2)$$

adopting the following definitions:

- **x** position of O1UVW with respect to OXYZ
- **bi** position of the ideal joint Bi center referred to O1UVW
- **Ai** position of the ideal joint Ai center referred to OXYZ
- q_i length of the ith leg,
- $\mathbf{w}_i$ unit vector defining the ith leg with respect to OXYZ

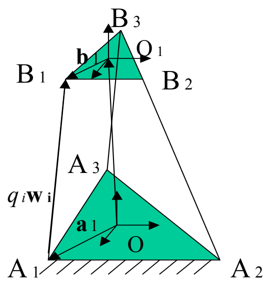

Fig. 3. Vector kinematics, used for machine control

These equations have been solved to get the inverse kinematics and coded in the Numerical Control (NC), to drive the machine. It is important to analyse the parameters involved in these formulas, because the calibration procedure has to produce corrected values for them, to minimize the residual error: it is not possible to compensate other geometrical errors (for example a universal joint composed by two non-orthogonal revolute joints).

A first step in calibration is the selection of the "Error Sources" (ES) in the machine: as a general rule, each joint can introduce Error Sources corresponding to its constrained degrees of freedom. While the calibration algorithm is able to identify and eliminate redundant ESs (i.e. correlated ESs), anyhow, to reduce the analysis time, is useful to adopt a methodology that suggest independent ESs, like the Denavit-Hartemberg scheme. For the presented test case, the ESs illustrated in Figure 4 have been defined. They are all translational: location of the three universal joints ("UJ") connected to ground (3 dofs x 3), referred to the work table origin, plus relative location of the UJs located at both ends of each strut (5 x 3 ES) plus one ES for each strut, to model the error of the controlled axis that modulates strut length. Similarly, 3 dofs x 3 joints are needed to describe the location of the three UJs on the mobile platform, referred to the tool attachment point. In total there are 9+15+3+9 = 36 ESs.

Some ESs can interest geometrical features that are not taken into account by the vector kinematics: for example, strut bending or torsion. The VPE can evaluate also their effect, in order to know how a simplified kinematic model implemented in the Numerical Control) limits the possibility to perform an accurate calibration.

The "importance" of each ES depends on three independent aspects, taken into account by the VPE:

1. **How machine precision is measured**. Depending on the specific production task, the error to minimize has to be defined as a particular mixture of translational and rotational EE errors; e.g. axial error are usually not critical in drilling operations.
2. **How each ES produces EE motion, in all workspace**. This relationship, linearized for each pose, is represented by the EIJ.
3. **How much error is expected for each ES**. This is a function of the manufacturing process: typically location errors in small components are small, while joint location error on large structure is much bigger.

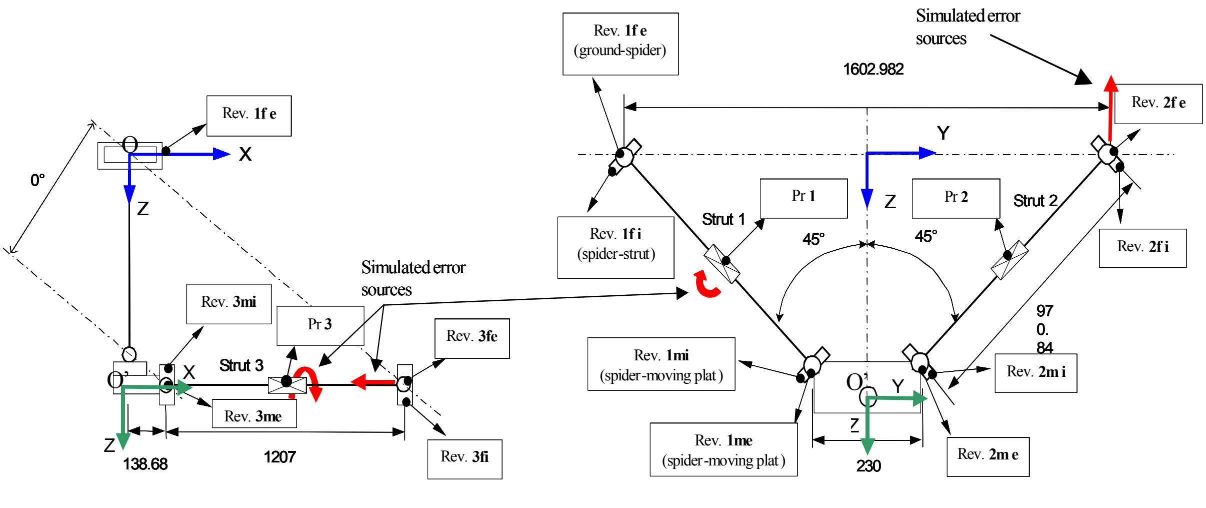

Fig. 4. Architecture of the Virtual Prototyping Environment

This kind of analysis is performed by the VPE, producing for example graphs like that one in Figure 5, showing how each ES contribute to EE error.

Aside single "Error Sources", the VPE can also identify "Error Functions", when a specific Error Source cannot be considered as a constant displacement for all machine poses, for example the positioning error of a controlled axis that varies the strut length, that generally depends on strut extension, e.g. because of an error in the ball screw lead (maybe due to thermal expansion). In this case the user can specify that an ES is an "Error Function EF(x)", indicating the independent "x" dimension in the model (in this case strut extension). The VPE sw computes the Extended Jacobian as usual, saving also the corresponding "xi" values. In the post-processing phase (in Matlab) the EF can assume different values for each pose (in contrast to the unique value considered for normal Error Sources), but this errors are constrained by the shape selected for the Error Function. At the moment two possibilities are considered: a polynomial function (for linear axes) and sine/cosine shapes (for rotational axes).

The linear system to be solved for calibration contains the corresponding constrain equations (here for the linear case):

$$m_i = EJ_i \begin{Bmatrix} ES_1 \\ \dots \\ EF(x_i) \\ \dots \\ ES_N \end{Bmatrix}; EF_j(x_i) = a_j x_i + b_j, \quad (3)$$

where a_j and b_j are coefficients describing, in our example, respectively the lead error and a constant offset error in the analysed strut.

Any measurement strategy and device (Ball Bar, laser interferometer, comparator) introduce new Error Sources in the calibration procedure, due to two different causes:

1. the device measurement error.
2. the device location in respect to the examined machine.

These measurement ESs have to be taken into account in the calibration procedures: the user must define in the mathematical environment the equations relating these ES and the calibration measurements. The analysis of their influence is not the goal of this paper, nevertheless in the Test Case an ES is defined for the radial measurement error and 6 others for the locations of the Ball Bar ends, referred one on the fixed table, and the other to the machine head. Summing these to the 36 ESs considered in the machine kinematics, a total of 43 ESs is got.

The calibration algorithm requires the EIJ on each point where calibration measures are performed. The EIJ are not obtained directly by the linearized model computed in the multi-body package, for two basic reasons:

1. the VPE must support the user in selecting the most efficient set of calibration measurements, therefore calibration set-ups are still undefined in the multi-body environment.
2. the EIJ is typically a smooth function in the workspace (i.e. far from singularities), thereafter an exact computation of the EIJ in each pose would require unnecessarily heavy computations in the multi-body environment.

The VPE is thereafter based on EIJ interpolation on a N-dimensional grid (where N is the number of degrees of freedom of the machine). Each single EIJ coefficient is interpolated by linear, cubic (used in this case) or spline functions, under user control, to estimate the EIJ in all points where calibration measures are performed.

Interpolation accuracy depends on the variability of the jacobian, on the number of nodes on which the interpolation is built and can be investigated for a specific machine comparing interpolated EIJs with EIJs directly computed by the VPE on a set of control poses.

The following table shows the tolerance of each ES, deduced from mechanical drawings and components catalogues, inserted in the VPE.

Error source type	**Maximum expected value**
Joint location on the fixed structure	70 μm
Location error in joint spiders	20 μm
Joint location on each strut	30 μm
Actuator accuracy (ball screw)	40 μm
Joint location on the mobile platform	15 μm
Strut bending due to static and dynamic loads	0.02°
Strut torsion due to static and dynamic loads	0.06°

Table 1. Expected manufacturing accuracies

Figure 5 shows the EE location error (in the Y direction) produced by such ESs (named as in Figure 4): for our machine all universal joints produce similar effects on the EE, but the most influencing are the ES related to the fixed structure, because of their larger tolerances.

4. CALIBRATION TEST CASE

The machine multi-body model has been modified introducing 4 geometrical errors: the first 2 errors (see Tab.2) are present in the machine model used by the Numerical Control; the second ones, instead, are non compensable errors, since they are not included in vector kinematics (described in section III). The considered ESs have been classified among the most relevant ones in the previous analysis.

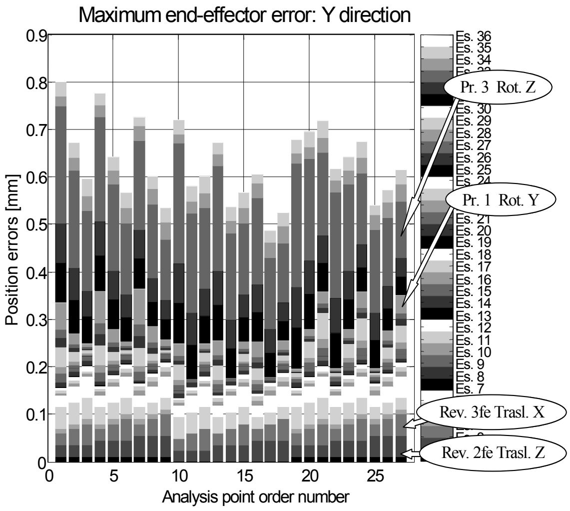

Fig. 5. Analysis of ES effect on End Effector location error

Error Source	description	value
ES 3: Rev. 2fe	Revolute between ground & spider, Left strut, axial dir.	50 μm
ES 7: Rev. 3fe	Revolute between ground & spider, Back Strut, axial dir.	60 μm
ES 22: Pr1	Prismatic coupling in the middle of right strut, bending.	0.01°
ES 33: Pr3	Prismatic coupling in the middle of horizontal strut	0.05°

Table 2. Introduced geometrical errors

The next step is to define the measurements needed to calibrate the machine: in the present study, it was decided to measure EE location error on circular trajectories, using a “Ball Bar” (“BB”) device because they are very practical instruments with fast set-up and accurate measurements (e.g. measuring range: ± 5 mm, system accuracy: ± 1 μm, measuring resolution down to 0,01 μm). Similar procedures can be developed in the VPE for other instruments, like laser interferometers or laser trackers.

As the Ball Bar does not provide information on rotations of the whole machine around the adopted fixed point, to fully identify machine location in space is necessary to select three different set-ups, connecting three different points on the Fixed Platform (Af, Bf, Cf) to three points on the Moving Head (Am, Bm, Cm respectively), as shown in Figure 6. For the experiment herein presented, a single circular trajectory with a radius of 150 mm has been simulated for each set-up.

During motion the Ball Bar registers all distance variations between its attachment points (producing measures similar to Figure 7): 51 uniformly distributed samples of the radial error have been collected for each circle , giving a total of 153 samples, used for machine calibration.

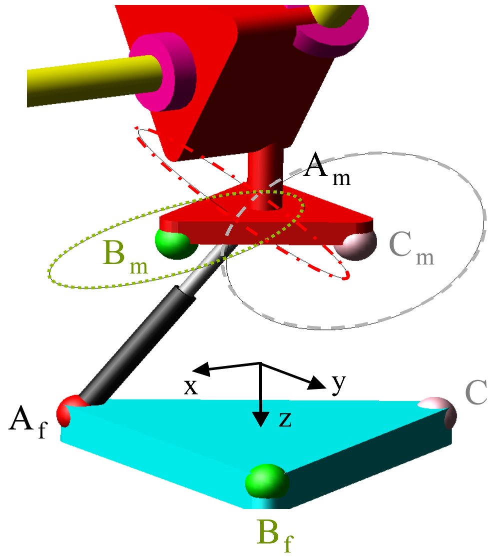

Fig. 6. Circular trajectories used for calibration in the test case

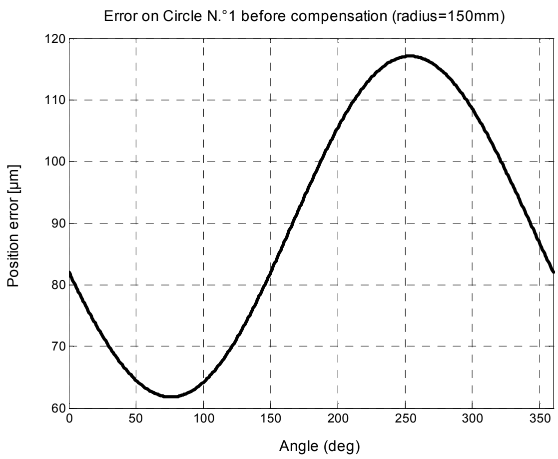

Fig. 7. Radial error before calibration (circle in the YZ plane)

In order to obtain the Extended Inverse Jacobian matrix corresponding to all measurement poses, the EIJ has been interpolated over a grid of 252 points, with the following dimensions: X axis 380 mm, Y axis 380 mm, Z axis 760 mm, centred in the machine workspace.

Starting from the EIJ interpolated the EE motion due to each ES is projected on the measurement direction, given by the Ball Bar axis. In this way a matrix M is built, that correlates ESs and calibration measures:

$$\{measure_i\} = [M] \cdot \{ES_j\}. \tag{4}$$

The M matrix is then examined to identify, if any, "not-observable" ESs, corresponding to columns of M characterized by very small values. If these ESs are important, the user must define new measurements, more influenced by those ESs. In the test case the following ESs have been discarded because their effect was smaller than one hundredth of the maximum coefficient in the M matrix: 1, 4, 8, 9, 13, 16, 17, 18, 19, 20, 24, 25, 29, 30, 36, 37, 38, 39, 40, 41, 42.

Then the kept part of the M matrix has been examined to locate correlated ESs. ES can be correlated for two reasons:

1. their effect on EE motion is equivalent in the whole workspace (e.g. two axial ESs that modify the strut length at the two ends of it). The user is free to select a dependent ES to eliminate.
2. their effect on the chosen calibration measures is equivalent: the user has to decide if tests have to be extended with other set-ups able to distinguish between considered ESs (e.g. changing the Ball Bar connection point on the moving and/or fixed platform).

In the VPE, when a cluster of correlated ESs is identified (basing of the correlation thresholds defined), one of them is automatically removed, trying to preserve a list of "preferred" ESs specified by the user (e.g. because they correspond to physical regulation points of the machine). After this analysis 10 ESs have been kept in the test case (2, 3, 5, 6, 7, 22, 26, 33, 35, 43).

The linear system corresponding to the kept section of M is solved in a Least Mean Square sense, producing the ES values listed in the following table, in very good agreement with the corresponding theoretical values:

ES n°: id	**identified value**	**real value**
ES 3: Rev. 2fe	50.001 μm	50 μm
ES 7: Rev. 3fe	59.99 μm	60 μm
ES 22: Pr1	0.01°	0.01°
ES 33: Pr3	0.0499°	0.05°
Max. of all other ESs	1.2E-3	0

Table 3. Identified Error Sources

While identification of many ESs can be very useful to better qualify the machine and optimise the manufacturing accuracy, only few ESs are present in the NC kinematic routines (e.g. the vector kinematic exposed in section III), permitting thereafter a software error compensation ("Rev. 1f" and "Rev. 3fe" in the test case); others (like "Pr. 1" and "Pr. 3") are not directly compensable. The VPE can investigate how much error, compared to the expected global machine error, is possible to compensate with the Numerical Control.
Figure 10 shows the residual radial error on the circle in the YZ plane after this full identification and partial compensation (solid line A): the maximum error is now around ±28 mm (due to the not compensated ESs).

For compensation purposes is generally more effective to identify only ESs present in NC kinematics, because the Least Square approach will try to use them also to compensate as much as possible the ESs not represented (so the optimal value is not the real value of each ES!). A second identification has been performed, limited to the NC ESs: the result is curve B in Figure 10, with a residual error of ±12 mm.

A different approach can be adopted: enrich the NC inverse kinematics in order to model some of the previously disregarded ESs, to permit their compensation. As an example, the effect of ES "Pr3" (torsion of the horizontal strut) on the end-effector position has been added. This is usually not done because the inverse kinematics became rapidly very complex when many ESs are considered: to avoid this drawback, linearizations are performed exploiting the fact that typically ESs assume very small values, compared to machine dimensions.

Since the posterior strut is connected to moving platform via an universal joint, a torsion (φ_1) of this strut produces a rotation of the moving platform (φ_2) in accordance with the following well-known relationship (see Figure 8):

$$\tan\varphi_1 = \tan\varphi_2 \cdot \cos\alpha. \tag{5}$$

Approximating the tangent with its argument, we obtain:

$$\varphi_1 = \varphi_2 \cdot \cos\alpha. \tag{6}$$

Angle α can be easily deduced from geometry as a function of Y, Z coordinates of the universal joint that connects the moving platform to the horizontal struts, itself depending from end-effector position (see Figure 8):

$$\cos\alpha = \frac{\sqrt{\left(b_{3y} - A_{3y}\right)^2 + \left(b_{3z} - A_{3z}\right)^2}}{q_3} \qquad (7)$$

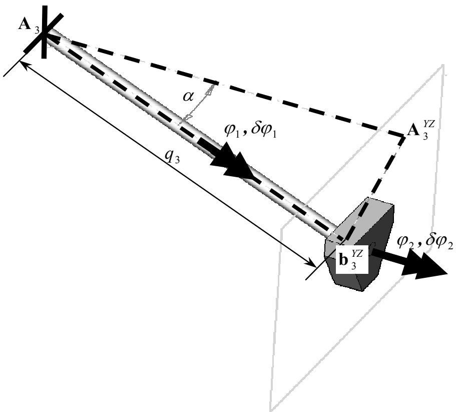

Fig. 8. Rotation transmission through universal joint

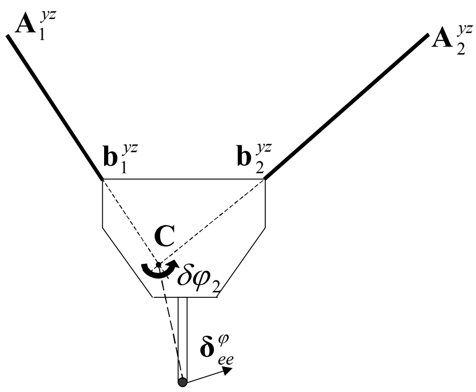

Fig. 9 End-effector displacement due to a virtual rotational displacement of moving platform.

Now, centre **C** of rotation $\boldsymbol{\varphi}_2$ has to be determined to calculate the corresponding end-effector displacement. It can be noticed that such point **C** has to belong to YZ projections of both strut 1 and 2, just because in this plane these struts behave like two rods (see Figure 9); therefore, point **C** is determined by their intersection, solution $(\overline{\eta_1, \eta_2})$ of the following vector equation:

$$\begin{aligned} \mathbf{C} &= \mathbf{A}_1^{yz} + \left(\mathbf{b}_1^{yz} - \mathbf{A}_1^{yz}\right) \cdot \eta_1 = \\ &= \mathbf{A}_2^{yz} + \left(\mathbf{b}_2^{yz} - \mathbf{A}_2^{yz}\right) \cdot \eta_2 . \end{aligned} \qquad (8)$$

End-effector displacement can be easily calculated linearizing kinematics and expressing it as a cross product between rotation arm and rotation vector:

$$\boldsymbol{\delta}_{ee}^{\varphi} = \delta\varphi \cdot \mathbf{k} \wedge \left(\mathbf{EE} - \mathbf{C}\right). \qquad (9)$$

Without kinematics linearization, positions of universal joints $\mathbf{b}_i$ can not be considered as constant parameters in error displacement computation, since they depend from the machine position itself: iterative methods would have to be used to solve inverse kinematics, implicating an harmful, useless waste of time.

Using this "extended inverse kinematics" it is possible to compute corrected references positions compensating for the modelled error, as depicted in Figure 10 by the dotted line C: maximum trajectory error (always on a circle in ZY plane) is now about 4 mm, mostly due to the effect of the remaining uncompensated ES ("Pr.1") .

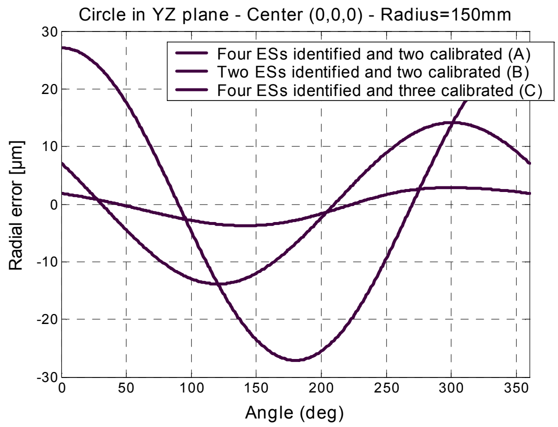

Fig. 10. Radial error after different SW compensations strategies.

5. CONCLUSION

The developed methodology and software permit to quickly generate equations needed to perform a geometric calibration of a generic Parallel Kinematics Machine, using various calibration devices and considering any Error Source, even not represented in the machine nominal kinematic model. To permit full compensation of the most significant ESs, it has been shown how an extended inverse machine kinematics can be developed (exploiting linearization when possible) to take into account ESs not included in the nominal kinematic model.

Further developments are focused on the analysis of measurement errors on the identification accuracy, before performing the calibration of the real machine.

6. AKNOWLEDGMENT

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REFERENCES

Amit J. Patel, Kornel F. Ehmann, "Calibration of a hexapod machine tool using a redundant leg", Int J of Machine Tools and Manufacture, 2000, Elsevier

Benjamin W. Mooring "Fundamentals of manipulator calibration, 1991 " John Wiley & Sons

Boër, C.R., Molinari-Tosatti, L., Smith K.S., Eds., 1999, PKM: Theoretical Aspects and Industrial Requirements, Springer Verlag, London.

Leonesio Marco, Irene Fassi, Giacomo Bianchi, Lorenzo Molinari-Tosatti, Parallel Mechanisms Design in a Virtual Prototyping Environment, Proceedings of the 32nd ISR, Seul, 19-21 April 2001

Parenti-Castelli V. et Di Gregorio R. A translationnal 3-dof parallel manipulator. In ARK, pages 49-58, Strobl, 29 June-4 July, 1998

Parenti-Castelli, Di Gregorio "Influence of manifacturing errors on the kinematic performances of the 3 UPU-parallel mechanism"

Taddei Mario, Meccanica Applicata alle machine, vol.12, p. 18, Liguori Editore, Napoli, IT