More Observations on the Lower-Lying Hadron Spectrum

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Abstract. The equation $m = m_0(1 - q)^{-1/2}$ is shown to produce two distinct types of numerical coincidences involving the mass values of the lower-lying mesons and baryons.

In the quark model, the large differences of mass between hadron states belonging to a same multiplet of a fixed spin and parity are related to their strange quark and antiquark content. It could therefore be asked whether an empirical rule concerning the mass values of, say, the lower-lying hadrons might be found which depends on the number of their strange constituents. We have considered the following three hadron multiplets: the baryon octet of spin-parity $J^P = 1/2^+$, the baryon decuplet of $J^P = 3/2^+$, and the meson nonet of $J^P = 1^-$. For each of these multiplets we have calculated, separately, the relative differences of the linear or square mass values, as follows

(1) $$g_i(\wp) = (m^i - m_0{}^i)/m^i$$

with $i = 1,2$; where m is the mass of a given hadron $\wp$ and m_0 is the mass of the lowest-lying hadron, $\wp_0$, in the multiplet considered (in all cases, $\wp_0$ has strangeness number $S = 0$). We have found that the values of these ratios for the various hadron states of a same multiplet fit the following simple empirical relationship

(2) $$g_i(\hat{\sigma}) \simeq g_i(\sigma_1)\, C_i^{\,n(\hat{\sigma})}$$

where i) σ_1 is the hadron with $|S| = 1$ of lowest mass in the multiplet; ii) $\hat{\sigma}$ is any hadron, other than σ_1, which contains at least one strange quark s or antiquark $\bar{s}$; iii) C_i is the same constant for all three multiplets, which is given by

(3) $$C_i = \left[1 + g_i(\Lambda)\right]^{2/i}$$

with

(4) $$g_i(\Lambda) = (m_\Lambda^{\ i} - m_p^{\ i})/m_\Lambda^{\ i} \quad ,$$

m_p and m_Λ being the mass of the proton and $\Lambda(1116)$ hyperon, respectively, i.e. $g_i(\Lambda) \equiv g_i(\sigma_1)$ for the $1/2^+$ baryon octet (we have $C_1 = 1.3432$; $C_2 = 1.2926$); and iv) the exponent of C_i , $n(\hat{\sigma})$, is an integer equal to the sum of the s-quarks and $\bar{s}$-antiquarks for the hadron $\hat{\sigma}$. Equation (2) can be made explicit for mass m as follows

(5) $$m^i \simeq m_0^{\ i}/\left[1 - \exp(\alpha_i + \nu\,\gamma_i)\right]$$

where $\gamma_i = \ln C_i$ is the same constant for all three multiplets; $\alpha_i = \ln\left[g_i(\sigma_1)\right]$ depends on the multiplet considered; and ν is an integer defined as: $\nu(\sigma_1) = 0$, $\nu(\hat{\sigma}) = n(\hat{\sigma})$. In Table 1, a set of mass values satisfying eqs. (2) or (5) are reported for our three hadron multiplets. For the 1^- meson nonet, the $\omega(783)$ meson has not been mentioned because, as the $\rho(770)$ meson, it has no strange quark content, and its mass is a few MeV greater than the mass of ρ. The 0^- meson nonet has not been taken into account because some of its states contain a mixing of an $s\bar{s}$ pair with other differently flavored pairs.

Equation (1) for $i = 2$ can be shown to produce further, independent numerical coincidences involving the mass values of the lowest-lying hadrons with strangeness $S = 0$ and $|S| = 1$. Let us rewrite eq.(1) in the following form

$$(6) \qquad m = m_0(1 - q)^{-1/2} = \\ = m_0\left[1 + (1/2)q + (3/8)q^2 + (5/16)q^3 + \ldots\right] \quad ,$$

where m_0 is now the mass of the pion or the nucleon, according to whether m is the mass of a meson or a baryon. Then, for each of the following $|S| = 1$ hadrons, K(496), Λ(1116), Σ(1193), and Σ(1385), consider the decay which is reported in Table 2. If we impose that the mass value of the pion produced in the decay of both the K meson and the Λ hyperon, or the value of the difference of mass with regard to m_0 of the immediately lighter $S = -1$ hyperon produced in the decay of both Σ(1193) and Σ(1385), are yielded by the sum of the minimum number of terms $\mathcal{N}$ - starting from the first-order term onwards - of the series development in eq.(6), then we find that the corresponding value for mass m matches the value of the experimental mass for all the hadrons considered. For each of the decays $K \rightarrow \pi\pi$ and $\Lambda \rightarrow N\pi$, we have reported the variant which gives the highest value for mass m. The value of m for Σ(1385) has been calculated from the average mass value of the Σ(1193) isospin multiplet. In Table 2, a typical decay of the charmed baryon of lowest mass Λ_c^+ is also reported; if we calculate m from a mass value equal to the sum of the masses of the two mesons produced in this decay, we again have a numerical coincidence involving the mass of the decaying hadron.

Captions

Table 1. Mass values satisfying eqs.(2) or (5) for three hadron multiplets.

Table 2. Hadron decays from which an empirical rule based on eq.(6) produces numerical coincidences with the hadron experimental mass values. The value marked "*" is the average over the isospin multiplet concerned.

J^P		hadron	$n(\hat{\gamma})$	mass (MeV) for i=1	mass (MeV) for i=2
$1/2^+$	γ_0	N (939)	–	938	938
$1/2^+$	γ_1	Λ(1116)	–	1116	1116
$1/2^+$	$\hat{\gamma}$	Σ(1193)	1	1194	1191
$1/2^+$	$\hat{\gamma}$	Ξ(1317)	2	1317	1314
$3/2^+$	γ_0	Δ(1232)	–	1232	1232
$3/2^+$	γ_1	Σ(1385)	–	1382	1387
$3/2^+$	$\hat{\gamma}$	Ξ(1530)	2	1532	1531
$3/2^+$	$\hat{\gamma}$	Ω^-(1672)	3	1672	1670
1^-	γ_0	ρ (770)	–	769	769
1^-	γ_1	K^*(892)	–	891	893
1^-	$\hat{\gamma}$	ϕ(1020)	2	1021	1020

Table 1

decay	$\mathcal{N}$	m (MeV)	exp mass (MeV)
$K^+ \rightarrow \pi^0 \pi^+$	3	499	494
$\Lambda \rightarrow p \pi^-$	1	1119	1116
$\Sigma^0 \rightarrow \Lambda \gamma$	1	1190	1192
$\Sigma(1385) \rightarrow \Sigma \pi$	1	1388	1384*
$\Lambda_c^+ \rightarrow p K^- \pi^+$	2	2285	2282

Table 2