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Tactical Production and Lot Size Planning with Lifetime Constraints: A Comparison of Model Formulations

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Abstract

In this work we face a variant of the capacitated lot sizing problem, a classical problem addressing the issue of aggregating lot sizes for a finite number of discrete periodic demands that need to be satisfied, thus setting up production resources and eventually creating inventories, while minimizing the overall cost. In the proposed variant we take into account lifetime constraints, which model products with maximum fixed shelflives due to several possible reasons, including regulations or technical obsolescence. We propose four formulations, derived from the literature on the classical version of the problem and adapted to the proposed variant. An extensive experimental phase on two datasets from the literature is used to test and compare the performance of the proposed formulations.

Keywords: Tactical Production Planning, Lot Sizing, Lifetime Constraints, Perishability, Mathematical Models

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1. Introduction

The tactical production and lot sizing problem aims at satisfying demands related to one or more products, which are assumed to be forecasted or known in advance, and are distributed over a time horizon of interest. To this end, it is required to make choices related to when items should be produced, and the related setup operations for the production resources performed. Furthermore, since resources may have limited availability and multiple types of items may compete for it, pre-production and thus inventory holding might be a favorable or necessary strategy. Thus the objective is to fulfill all product demands in a timely manner while minimizing total costs, i.e., setup costs and inventory holding ones.

Depending on the specific trade-off between setup costs, inventory costs, and available resources, holding items in inventory for even large portions of the planning horizon may appear as an attractive strategy. However, in many real-world cases this might cause products to pass their useful lifetime and therefore impose high costs due to inventory loss not only in terms of lost value, but also in terms of utilization of resources employed in vain, e.g., machine time, raw materials, metals, and energy increasing CO_2-levels. This is notably disquieting, e.g., in the food industry, where significant losses occur during handling, processing, and distribution of products. Results of a study conducted for the International SAVE FOOD Congress (Gustavsson et al., 2011) state that 1.3 billion tons of food per year are wasted on a global scale, which is around one third of food produced worldwide for human consumption. This is particularly valid for developing countries, where more than 40% of food losses are due to processing, whereas total losses in industrialized countries are as high, but with more than 40% mainly occurring at the retailer and consumer level. In industrialized countries, unsatisfied customers waiting for their products or being concerned about quality represent a critical issue as well.

Deterioration has a great influence not only on inventory management, but on every area of the production and supply chain planning processes where items are stocked or forced to wait due to uncertain demand, technical matters, variability, or disruptions; see Pahl and Voß (2014) and the references therein. In general, we distinguish deterioration and perishability, where the first refers to items with an ongoing process of decay while being stored, that continuously loose their utility (see for example Dave (1986); Wee (1993); Darlington and Rahimifard (2006)). In contrast, perishable items can be regarded as items constrained by fixed, maximum lifetimes. This holds for items that become obsolete or unusable at some point in time, because of various reasons, e.g., laws and regulations that predetermine their shelflives (Boukas and Liu, 2001; Ketzenberg and Ferguson, 2005). The definitions of deterioration and perishability or lifetime restrictions clearly depend on the specification of item quality. This relates to the physical state of items, their “behavior” over time, customers’ opinions expressed in demand functions defining the remaining value of items as well as their functional characteristics; see Pahl and Voß (2014). Functional deterioration and depreciation due to value loss have the same problem source, but require different actions. For instance, deteriorating food that might develop toxins should be separated from fresh food inventory while items that loose their perceived utility can be kept in the same place with items at different lifetime states.

Apart from engineering issues to enhance the production process, related coordinated planning throughout the supply chain is an important starting point to this type of problems. Despite a long tradition in the Operations Research area to integrate deterioration effects in models for planning, there are only a few approaches that integrate lifetime constraints to develop production and supply chain plans accordingly, so that wastage and disposal are avoided, minimizing resource utilization and increasing environmental consciousness and sustainability; see also Pahl and Voß (2014).

In this paper, we address this important issue by integrating lifetime constraints into formulations for a classical variant of the lot sizing problem, the capacitated lot sizing problem (CLSP), in which different items have to be produced by a single production unit (machine) with a limited capacity per time period. We test their performance on a wide set of test instances derived from the literature, taking into account several different settings for lifetime durations. The aim is to determine which of them are better suited to represent this class of problems. To the best of our knowledge, such a comparison has not been undertaken in the literature, and represents an important preliminary step towards the development of efficient exact approaches.

The rest of the work is organized as follows. Section 2 contains an overview of the existing literature. In Section 3 we introduce four classical formulations for the capacitated lot sizing problem, and show how they can be adapted to incorporate lifetime constraints. In Section 4 we analyze and compare the performance of the different formulations on some datasets coming from the literature. Finally in Section 5 we present some final remarks.

2. Literature Overview

The CLSP was shown to be NP-Hard in earlier works such as Florian et al. (1980) and Bitran and Yanasse (1982). Due to the complexity of the problem, heuristic approaches for the CLSP and its variants have been proposed in many works (Dixon and Silver, 1981; Haase, 1996; Meyr, 2000; Alfieri et al., 2002; Gupta and Magnusson, 2005; de Araujo et al., 2007; Caserta and Voß, 2013). A survey on meta-heuristics for the CLSP was presented in Jans and Degraeve (2007).

Exact approaches for the problem have also been faced in several works (Barany et al., 1984; Eppen and Martin, 1987; Pochet and Wolsey, 1988; Stadtler, 1996; Belvaux and Wolsey, 2000, 2001), which focused on either valid equalities for the natural formulation of the problem (which we describe in Section 3.1) or in proposing reformulations. In particular, some works proposed network flow-based reformulations deriving from the shortest path problem (Eppen and Martin, 1987; van Hoesel and Kolen, 1994; Alfieri et al., 2002) or the multi-commodity flow problem (Pochet and Wolsey, 1988) while reformulations based on plant location problems can be found in Stadtler (1996) and Alfieri et al. (2002). In Karimi et al. (2003) the authors present a survey on models and solution approaches for several CLSP variants. Recently, in de Araujo et al. (2015) the author studied the CLSP with setup times, modeled as capacity constraints, presenting a hybrid approach combining Lagrangean relaxation and column generation.

An important distinction made in the literature with respect to discrete-time period models is the classification between *big bucket* and *small bucket* models, where the big bucket ones model rather long time periods, so that the setup and production for several products can be carried on in a single bucket. Given the assumptions that will be mentioned in next section, the CLSP belongs to big bucket models, also known in the literature as *macro period* models, a name deriving from material requirements planning (MRP) terminology (see Eppen and Martin (1987), Billington et al. (1983), Voß and Woodruff (2006) for more details). Small bucket models, on the other hand, generally include scheduling information and are used to model factors such as individual product setups and switch-offs, thus considering more detailed information about the shop floor (see also Pahl et al. (2011)).

A recent survey on deterioration and lifetime constraints in models for production and supply chain planning shows that not many formulations exist to integrate such issues in optimization models; see Pahl and Voß (2014). Moreover, discrete deterministic dynamic optimization formulations for production and inventory planning including lifetime restrictions are rare. Mixed-integer programming (MIP) models including lifetime restrictions have been proposed, e.g., by Hsu (2000); Förster et al. (2006); Entrup et al. (2005); Amorim et al. (2011); Pahl and Voß (2010); Pahl et al. (2011).

We note that there are mainly two types of formulations to integrate lifetime constraints in optimization models in the literature that are relevant to this research. One is the restriction regarding the number of periods that items can be held in inventory. It modifies the time index and restricts it by the item lifetimes, so that it is named *index transformation* by Pahl and Voß (2014); see also Inequality (9) on Page 9. This formulation can be used in inventory and lot-size (I&L) as well as in transportation formulations; see also Stadtler (1996).

MIPs including the index transformation formulation have been proposed by Förster et al. (2006); Entrup et al. (2005); Amorim et al. (2011). The other formulation has been developed by Pahl and Voß (2010) and used in I&L models such as the ones presented in the work of Pahl et al. (2011); Pahl (2012). It calculates the amount of perishable items by taking into account the sum of items produced in a period restricted by their lifetime less the sum of those items used for demand satisfaction until the current regarded time period as well as the sum of those that have already been disposed in previous periods. Different to the index transformation formulation, it allows for perishability and related disposal whereas the index transformation prohibits it. Moreover, in case of minimal lot size constraints, the index transformation formulation might be violated and thus cannot be applied.

3. Model Formulations

In this work, we take into account classical assumptions for the CLSP. Namely, production is planned for a time horizon divided into a discrete number of time periods (or *time buckets*), demands are known in advance, and there are no constraints related to minimal lot sizes or precedence constraints among different items. Demands for each time bucket must be satisfied with no delays, and can be satisfied by items produced in the same time bucket. With respect to setup costs, they are independent from the produced amount, and they incur in each time period in which a given item is produced. Regarding inventory, costs are linear, initial inventory is assumed to be zero, and final inventory levels are not considered.

For each of the four formulations presented in this section, we start by reviewing the formulation related to the basic CLSP, and then show how the formulation can be modified to incorporate lifetime constraints. We start by introducing some notation for the parameters of the problem, that will be used throughout the paper.

Let:

- $\mathcal{N} = \{1, \ldots, N\}$ be the set of the products;
- $\mathcal{T} = \{1, \ldots, T\}$ be set of discrete time buckets;
- $d_{nt} \geq 0$, $\forall\ n \in \mathcal{N}$ and $t \in \mathcal{T}$, be the amount of demand for product n at time t;
- $r_n \geq 0$, $\forall\ n \in \mathcal{N}$, be the amount of resources requested to produce one unit of product n;
- $R_t \geq 0$, $\forall\ t \in \mathcal{T}$, be the total amount of resources available for production at time t;
- $set_n \geq 0$, $\forall\ n \in \mathcal{N}$, be the setup cost to be paid to produce any amount of product n in a given time period;
- $inv_n \geq 0$, $\forall\ n \in \mathcal{N}$, be the inventory cost to store one unit of product n for a time period;
- $ft_n \in \mathcal{T}$, $\forall\ n \in \mathcal{N}$, be the first time period with non-zero demand of item n;
- $D_{ntq} \geq 0$, $\forall\ n \in \mathcal{N}$ and $\{t, q\} \in \mathcal{T}$ with $t \leq q$, be the total demand for item n from time t to q (that is, $d_{nt} + \ldots + d_{nq}$).
- $c_{ntq} \geq 0$, $\forall\ n \in \mathcal{N}$ and $\{t, q\} \in \mathcal{T}$ with $t \leq q$, be the total inventory cost that should be paid if D_{ntq} is produced at time t (*i.e.* $inv_n(d_{n(t+1)} + 2d_{n(t+2)} + \ldots (q-t)d_{nq}))$
- $\theta_n \geq 0$, $\forall\ n \in \mathcal{N}$, be the lifetime associated with product n; that is, any unit of item n produced at time $t \in \mathcal{T}$ can be used to satisfy demands up to time period $\max\{t + \theta_n, T\}$.

The presented formulations include a natural formulation of the problem, also referred to in the literature as I&L (Stadtler, 1996; Alfieri et al., 2002), a multi-commodity flow formulation (Pochet and Wolsey, 1988), a plant location-based formulation (Stadtler, 1996; Alfieri et al., 2002), and a shortest path-based formulation (see Eppen and Martin (1987); Stadtler (1996); Alfieri et al. (2002)).

3.1. I&L Formulation

The I&L formulation makes use of the following variables:

- $S_{nt} \in \{0,1\}$, $\forall\, n \in \mathcal{N}$ and $t \in \mathcal{T}$, representing the decision on producing some units of product n at time t, and hence pay the related setup cost;
- $I_{nt} \geq 0$, $\forall\, n \in \mathcal{N}$ and $t \in \mathcal{T}$, representing the number of units of product n stored in inventory at the end of time t, defining the related inventory cost;
- $X_{nt} \geq 0$, $\forall\, n \in \mathcal{N}$ and $t \in \mathcal{T}$, representing the number of units of product n produced at time t.

The classical I&L formulation is given as follows:

$$\textbf{[I\&L]} \quad \min \sum_{n=1}^{N} \sum_{t=1}^{T} (set_n S_{nt} + inv_n I_{nt}) \tag{1}$$

s.t.

$$I_{n1} = X_{n1} - d_{n1} \qquad n = 1,\ldots,N \tag{2}$$

$$I_{nt} = I_{n(t-1)} + X_{nt} - d_{nt} \qquad n = 1,\ldots,N, t = 2,\ldots,T \tag{3}$$

$$\sum_{n=1}^{N} r_n X_{nt} \leq R_t \qquad t = 1,\ldots,T \tag{4}$$

$$X_{nt} \leq D_{ntT} S_{nt} \qquad n = 1,\ldots,N, t = 1,\ldots,T \tag{5}$$

$$S_{nt} \in \{0,1\} \qquad n = 1,\ldots,N, t = 1,\ldots,T \tag{6}$$

$$I_{nt} \geq 0 \qquad n = 1,\ldots,N, t = 1,\ldots,T \tag{7}$$

$$X_{nt} \geq 0 \qquad n = 1,\ldots,N, t = 1,\ldots,T \tag{8}$$

Objective function (1) minimizes the sum of setup and inventory costs. For each product n the related setup cost is paid once for each time period t where some production of n is planned, while inventory costs are linear and depend on the stocked quantities. Constraints (2)-(3) impose inventory balance constraints, i.e. consistency among production variables, inventory variables and demands, for the first time period and the subsequent ones, respectively (recall that by definition inventory is empty at the beginning and at the end of the planning horizon). Constraints (4) impose the limitations on production resources availability, while Constraints (5) bind the binary setup variables with production variables.

It may be noted that the formulation does not impose the inventory to be empty at the end of the time horizon, although given an optimal solution with non-empty final inventory, an equivalent optimal solution without the production excess surely exists.

Now, let us consider perishability constraints. In all our formulations, we assume that items are used on a first-in-first-out basis, meaning that the demand of each product n at time t is satisfied by using first the units produced in the oldest previous time period. Recall that, for a given product n, all units produced at time 1 can be used up to time $1 + \theta_n$. More generally, for any $t = \theta_n + 1, \ldots, T$, the earliest time of production for units of n used to satisfy the demand d_{nt} is $t - \theta_n$. Therefore, similarly to other works (see (Pahl and Voß, 2010)), we impose that for any such value of t, the total production of n from time 1 to $t - \theta_n$ does not exceed the cumulative demand D_{n1t}, as follows:

$$\sum_{q=1}^{t-\theta_n} X_{nq} \leq D_{n1t} \qquad n = 1, \ldots, N, t = (\theta_n + 1), \ldots, T \tag{9}$$

For instance, for $t = \theta_n + 1$, and therefore $t - \theta_n = 1$, these constraints impose that the production of n in the first time instant (i.e. X_{n1}) is not greater than the overall demand of the item up to time $\theta_n + 1$, since this production cannot be used to satisfy demand of n at time $\theta_n + 2$ or later. For $t = \theta_n + 2$, it is imposed that the production of n in the first two time instants is not greater than $D_{n1(\theta_n+2)}$, and so on.

Furthermore, by a similar reasoning, the right-hand side coefficient in Constraints (5) can be reduced by substituting the constraints with

$$X_{nt} \leq D_{nt(t+\theta_n)} S_{nt} \qquad n = 1, \ldots, N, t = 1, \ldots, T - \theta_n \tag{10}$$

$$X_{nt} \leq D_{ntT} S_{nt} \qquad n = 1, \ldots, N, t = (T - \theta_n) + 1, \ldots, T \tag{11}$$

where (11) is related to time periods t such that $t + \theta_n > T$.

3.2. Multi-Commodity Flow Formulation

The multi-commodity flow formulation makes use of disaggregated production and inventory variables. That is, in addition to variables S_{nt} described in Section 3.1, it uses the following ones:

- $i_{ntq} \geq 0$, $\forall\ n \in \mathcal{N}$ and $\{t, q\} \in \mathcal{T}$ with $t < q$, representing the number of units of product n stored at the end of time t to satisfy demand at time q;
- $x_{ntq} \geq 0$, $\forall\ n \in \mathcal{N}$ and $\{t, q\} \in \mathcal{T}$ with $t \leq q$, representing the number of units of product n produced at time t to satisfy demand at time q.

The formulation is the following:

$$[\mathbf{MC}] \quad \min \sum_{n=1}^{N} \Big(\sum_{t=1}^{T} set_n S_{nt} + \sum_{t=1}^{T-1} inv_n \sum_{q=t+1}^{T} i_{ntq} \Big) \tag{12}$$

s.t.

$$x_{n11} = d_{n1} \qquad n = 1, \ldots, N \tag{13}$$

$$x_{n1t} = i_{n1t} \qquad n = 1, \ldots, N, t = 2, \ldots, T \tag{14}$$

$$i_{n(t-1)t} + x_{ntt} = d_{nt} \qquad n = 1, \ldots, N, t = 2, \ldots, T \tag{15}$$

$$i_{n(t-1)q} + x_{ntq} = i_{ntq}$$

$$n = 1, \ldots, N, t = 2, \ldots, (T-1), q = (t+1), \ldots, T \tag{16}$$

$$\sum_{n=1}^{N} r_n \sum_{q=t}^{T} x_{ntq} \leq R_t \qquad t = 1, \ldots, T \quad (17)$$

$$x_{ntq} \leq d_{nq} S_{nt} \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = t, \ldots, T \quad (18)$$

$$S_{nt} \in \{0, 1\} \qquad n = 1, \ldots, N, t = 1, \ldots, T \quad (19)$$

$$i_{ntq} \geq 0 \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = 1, \ldots, T, t < q \quad (20)$$

$$x_{ntq} \geq 0 \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = 1, \ldots, T, t \leq q \quad (21)$$

Objective function (12) assumes the same meaning of (1). Constraints (13)-(16) impose consistency among the disaggregated production and inventory variables, and ensure that demands are fulfilled. Constraints (17) model resource availability restrictions, and correspond to Constraints (4) in the previous formulation, while Constraints (18) are the disaggregated version of the coupling constraints (5). Indeed, if any production of item n takes place at time t for some time instant $q \geq t$, and therefore $x_{ntq} > 0$, then S_{nt} must be equal to 1. The value of x_{ntq} is trivially bounded by d_{nq}; therefore, the disaggregated constraints allow to use such small coefficients.

Since the additional variables directly express the production period t and the consumption period q of product units, modeling lifetime constraints for a given product n simply requires to impose disaggregated production variables to 0 if $q > t + \theta_n$. The related inventory variables can be imposed to 0 as well. Such constraints are therefore expressed as follows:

$$x_{ntq} = 0$$
$$n = 1, \ldots, N, \quad t = 1, \ldots, T - (\theta_n + 1), \quad q = (t + \theta_n + 1), \ldots, T \quad (22)$$

$$i_{ntq} = 0$$
$$n = 1, \ldots, N, \quad t = 1, \ldots, T - (\theta_n + 1), \quad q = (t + \theta_n + 1), \ldots, T \quad (23)$$

3.3. Plant Location Formulation

The plant location-based model is basically a reformulation of the multicommodity flow one, which does not use inventory constraints. Instead, production variables x_{ntq} with $q > t$ can be implicitly used to express and evaluate inventory levels. As noted in (Alfieri et al., 2002), the model defines a plant location formulation where plants are located temporally instead of physically.

The formulation is the following:

$$\textbf{[PL]} \quad \min \sum_{n=1}^{N} (\sum_{t=1}^{T} set_n S_{nt} + \sum_{t=1}^{T-1} inv_n \sum_{q=t+1}^{T} (q-t) x_{ntq}) \tag{24}$$

s.t.

$$\sum_{t=1}^{q} x_{ntq} = d_{nq} \qquad n = 1, N, q = 1, \ldots, T \tag{25}$$

$$\sum_{n=1}^{N} r_n \sum_{q=t}^{T} x_{ntq} \leq R_t \qquad t = 1, \ldots, T \tag{26}$$

$$x_{ntq} \leq d_{nq} S_{nt} \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = t, \ldots, T \tag{27}$$

$$S_{nt} \in \{0, 1\} \qquad n = 1, \ldots, N, t = 1, \ldots, T \tag{28}$$

$$x_{ntq} \geq 0 \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = 1, \ldots, T, t \leq q \tag{29}$$

Objective function (24) minimizes the sum of setup costs and inventory costs as for the previous models. For each production variable x_{ntq} expressing a positive inventory level (that is, such that $q > t$), the related inventory cost is obtained by multiplying the unitary storage cost inv_n for the number of time periods during which the inventory holding phase takes place (which is $(q - t)$). Constraints (25) make sure that all the production of a given item n targeted to a given time period q is used to satisfy the related demand d_{nq}. Constraints (26) and (27) are resource and coupling constraints, and are the same as (17) and (18), respectively.

Lifetime constraints can be imposed by adding Constraints (22), as for the multi-commodity flow formulation.

3.4. Shortest Path Formulation

The shortest path-based formulation makes use of the binary variables S_{nt} introduced in Section 3.1. Furthermore, it uses an additional set of variables

$z_{ntq} \in [0,1]$, $\forall\, n \in \mathcal{N}$ and $\{t,q\} \in \mathcal{T}$ with $t \leq q$, representing the fraction of the cumulative demand D_{ntq} produced at time t. The formulation is the following:

$$\textbf{[SP]} \quad \min \sum_{n=1}^{N} \sum_{t=1}^{T} (set_n S_{nt} + \sum_{q=t}^{T} c_{ntq} z_{ntq}) \tag{30}$$

s.t.

$$\sum_{t=1}^{T} z_{n1t} = 1 \qquad n = 1, \ldots, N \tag{31}$$

$$\sum_{q=t}^{T} z_{ntq} = \sum_{k=1}^{t-1} z_{nk(t-1)} \qquad n = 1, \ldots, N, t = 2, \ldots T \tag{32}$$

$$\sum_{n=1}^{N} r_n \sum_{q=t}^{T} D_{ntq} z_{ntq} \leq R_t \qquad t = 1, \ldots, T \tag{33}$$

$$\sum_{q=ft_n}^{T} z_{ntq} \leq S_{nt} \qquad n = 1, \ldots, N, t = 1, \ldots, T \tag{34}$$

$$S_{nt} \in \{0,1\} \qquad n = 1, \ldots, N, t = 1, \ldots, T \tag{35}$$

$$z_{ntq} \in [0,1] \qquad n = 1, \ldots, N, t = 1, \ldots, T, q = 1, \ldots, T, t \leq q \tag{36}$$

As introduced in Section 3, c_{ntq} values represent the full inventory cost to produce D_{ntq} at time t. Therefore, objective function (30) represents the sum of setup and inventory costs for this formulation as well.

The formulation is based on the structure of the single-item lot sizing problem without capacity constraints. Indeed, in this case, it can be demonstrated that the optimal solution is composed of full cumulative productions for future time periods. Therefore, the z_{ntq} variables are binary and the solution is represented by a shortest path. In this sense, Constraints (31)-(32) can be interpreted as flow conservation constraints. Even when z_{ntq} variables are relaxed, these constraints impose the overall amount of produced items to correspond to the total demand. Let $z_{ntq} > 0$; this means that $z_{ntq} D_{ntq}$ units of item n will be produced at time t. The effect of Constraints (31)-(32) is to impose that $\sum_{t=1}^{T} z_{n1t} D_{n1T} = D_{n1T}$ units are produced for each item n. Consider, for instance, the case in which a single item (with index 1) has to be produced over a time horizon of length 8, and suppose that the following non-zero values (that satisfy Constraints (31)-(32)) are assigned to z_{1tq} variables: $z_{112} = 0.8$, $z_{135} = 0.8$, $z_{115} = 0.2$, $z_{168} = 1$. It follows that the overall number of produced units is $0.8(D_{112} + D_{135} + D_{168}) + 0.2(D_{115} + D_{168}) = D_{118}$.

Note that in absence of capacity constraints, production of item n in an optimal solution would start at time ft_n and, therefore, Constraints (31) would be written as $\sum_{t=ft_n}^{T} z_{nft_nt} = 1 \ \forall n \in 1,\ldots,N$, as in Alfieri et al. (2002). Conversely, imposing the flow to start at time instant 1 for each product n allows production before ft_n, which might be necessary due to capacity constraints. However, it should be noted that in this case $z_{ntq} > 0$ does not correspond to actual production if $q < ft_n$, and therefore the related setup cost S_{nt} should not be paid. Therefore, Constraints (34) force S_{nt} to value 1 only if $z_{ntq} > 0$ for some values of $q \geq ft_n$.

Finally, Constraints (33) are the capacity constraints and therefore are equivalent to (26) for the plant location-based formulation.

To consider lifetime constraints, it is not possible to directly impose $z_{ntq} = 0$ for each $q > t + \theta_n$. Indeed z_{ntq} values are used to determine the time buckets in which items should be produced, as well as production levels; in particular, if $z_{ntq} > 0$ and q is not the final time bucket, then item n will be produced at time t and again at time $q + 1$. Hence, sometimes positive z_{ntq} values for $q > t + \theta_n$ can be required in an optimal solution. Therefore, similarly to Constraints (9) for the I&L Formulation, we model lifetime constraints by imposing that for any product n and for any $t = \theta_n + 1,\ldots,T$ the amount of production from time period 1 to $t - \theta_n$ is not greater than D_{n1t}. Since for any given $\tau = 1,\ldots,(t-\theta_n)$ the amount of units of n produced in τ is given by $\sum_{q=\tau}^{T} D_{n\tau q} z_{n\tau q}$, such constraints are expressed as follows:

$$\sum_{\tau=1}^{t-\theta_n} \sum_{q=\tau}^{T} D_{n\tau q} z_{n\tau q} \leq D_{n1t} \qquad n = 1,\ldots,N, \quad t = (\theta_n + 1),\ldots,T \tag{37}$$

4. Computational Study

For the comparison, we use two different sets of test instances that can be found in the literature, that is,

- **Dataset 1:** instances proposed in Alfieri et al. (2002), provided by the authors;
- **Dataset 2:** instances proposed in Trigeiro et al. (1989), provided by C. Sürie[2].

Dataset 1 includes 20 instances with 500 products and 15 time periods. Instances are generated according to three parameters, $d-prob$, $t-cap$ and tbo. The $d-prob$ parameter represents the probability of having non-zero demand during each time period; whenever there is demand, its value was set to a random value. The $t-cap$ parameter represents the tightness of capacity constraints with respect to the total demand; $t-cap = 0$ would correspond to the uncapacitated case, while with $t-cap = 1$ the available capacity would be equal to the required one. Finally, tbo represents the time between orders; it is used to calculate setup costs as a function of tbo and inventory costs, using relationships deriving from classical Economic Order Quantity analysis. The authors considered values $d-prob = \{0.9, 0.5\}$, $t-cap = 0.9$, $tbo = 2, 4$, and generated 5 instances for each combination of such parameters, leading to the final set of 20 instances.

The dataset proposed by Trigeiro et al. (1989) is composed of 751 instances. The authors generated instances containing 6 to 30 items and 15 to 30 time periods. Average product demands are equal to 100 per period, with several different coefficients of variation considered, while the capacity used per unit of production is always set to 1. The ratio of setup to inventory costs was set to either 0.267 or 1.333. Since most of these instances can be solved within few seconds, a subset of 35 among the nontrivial ones has been selected to be

[2]Instances available at the following address: `http://www.suerie.de/testsets.htm#CLSPL_Testset`

part of Dataset 2. More particularly, it is composed of 20 instances with 10 products and 20 time periods (using the naming convention provided by C. Sürie, instances 0441-0445, 0468, 0470-0473, 0502, 0531, 0561-565 and 0578-0580), 10 instances with 20 products and 20 time periods (instances 0711-0715 and 0735-0739), and 5 instances with 30 products and 20 time periods (instances 0936-0940).

Since the instances belonging to both datasets did not originally take into account lifetime constraints, the lifetime of the products had to be defined in order to run our computational experiments. For both datasets, we considered five different scenarios. In more detail, for each of the two datasets, we considered two scenarios in which all products have the same lifetime; in one of the two cases (called *short fixed* from now on) the lifetime value was chosen to be relatively small, while in the other one (*long fixed*) it was chosen to be at least half the length of the planning horizon. Moreover, we considered two scenarios (*short variable* and *long variable*) in which each item has a lifetime value chosen between two different alternatives. Finally, we also considered the case in which perishability constraints are not included. Such a comparison does not constitute the main focus of this work; however, it is reported for the sake of completeness.

For Dataset 1, in the short fixed scenario each of the 500 products has a lifetime equal to 2 time periods, while in the long fixed one it is equal to 8 time periods. In the short variable scenario, given the set of products $\{s_1, \ldots, s_{500}\}$, the ones with an assigned odd index have a lifetime equal to 3, while the others have lifetime 2; in the long variable one, these lifetime values are equal to 8 and 4, respectively.

For the instances belonging to Dataset 2, that have much fewer products and a longer planning horizon, the lifetime is equal to 3 time periods for the short fixed scenario and equal to 10 time periods for the long fixed one. In the short variable scenario, items with an assigned odd index have lifetime 5, while the others have lifetime 3. In the long variable one, these lifetime values are equal to 10 and 5, respectively.

Overall, taking into account the five different scenarios, a total number of 100 instances for Dataset 1 and 175 instances for Dataset 2 were considered.

For all formulations, both datasets and all the above described scenarios, we compared the quality of the solutions found within a time limit using the CPLEX 12.5.1 solver. Furthermore, as will be shown, we also compared the strength of their linear relaxations. Finally, in order to further investigate the effect of lifetime constraints in terms of solution value and required computational time, for both datasets and a single formulation (namely the Plant Location one) we also considered the case where the lifetime of each product is equal to 0,1,2,3,4,5 or 10.

All the formulations were implemented using the AMPL mathematical programming language, and tests were performed on an Intel Xeon 2 GHz workstation with 8 GB of RAM. A time limit equal to 1 hour was considered for each run. For all the tests, the relative MIP gap tolerance value was set to 0, as the default value of the solver resulted sometimes in slightly different optimal objective function values when the same instances were solved using different models.

Tables 1-7 and Figure 1 summarize the main findings of our computational study. In these tables, headings *PL*, *MC*, *I&L* and *SP* stand for the Plant Location, Multi-Commodity, I&L and Shortest Path formulation, respectively.

Table 1 reports a comparison on the quality of the solutions returned by the different formulations for Dataset 1, in the five different scearios. Each table entry, whose row and column headings refer to formulations, reports how many times (out of the 20 instances) the formulation indicated by the row heading found a better solution than the other. Each entry related to a *#opt* column heading reports instead how many times a certified optimal solution within the time limit was found using the formulation indicated by the row heading. Finally, for each formulation the two entries under the *overall* column heading report the sum of the number of times in which the related formulation found a better solution and an optimal solution over the five scenarios, respectively.

It can be noticed that PL and MC outperform significantly the other two

Table 1: Dataset 1: Solutions quality comparison

	no perishability					$\theta_n = \{4, 8\}$					$\theta_n = 8$				
	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP
PL	11	—	3	15	4	12	—	1	15	6	11	—	1	15	5
MC	11	5	—	15	3	12	6	—	15	6	10	7	—	15	8
I&L	5	0	0	—	0	5	0	0	—	0	5	0	0	—	0
SP	9	5	6	15	—	9	5	5	15	—	9	5	2	15	—

	$\theta_n = \{2, 3\}$					$\theta_n = 2$					overall	
	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#wins	#opt
PL	15	—	2	14	9	15	—	4	10	10	114	64
MC	14	3	—	14	9	15	1	—	10	10	127	62
I&L	6	0	0	—	3	8	0	0	—	8	11	29
SP	8	0	0	11	—	8	0	0	4	—	88	43

formulations, by finding overall 64 and 62 optimal solutions out of 100, respectively. Overall, PL and MC find a better solution than some other formulation 114 and 127 times, respectively. Conversely, SP finds better solutions 88 times, individuating 43 optimal solutions, while I&L finds better solutions in 11 cases and optimal solutions in 29 cases.

Looking at the five lifetime scenarios, it can be noticed that the performance of the PL and MC formulations improve when shorter lifetimes are considered, while the opposite appears to hold when considering the SP one. Indeed, in the scenario without perishability, SP finds 9 optimal solutions, and the same holds for both the long fixed and the long variable scenarios. MC and PL find 11 optimal solutions in the case without perishability, 12 optimal solutions in the long variable one and either 11 or 10 in the long fixed one. When the short lifetime constraints are considered, the number of optima found decreases to 8 for SP, and further increases to either 14 or 15 for both MC and PL. This behavior can be understood by considering that lifetime constraints involve a reduction of the number of variables on which a choice has to be made for PL and MC (see Constraints (22)-(23)). Clearly, the smaller the θ_n value is, the higher is the number of variables which are set to 0. On the contrary, adding Constraints (37) make the SP formulation harder to solve. The I&L natural formulation shows overall the worst performance, testifying the need for stronger formulations. This is an expected result, consistent with the literature (see Stadtler (1996); Alfieri et al. (2002)). In particular, it is worth noting that it never finds better solutions than PL or MC.

Table 2 further analyzes these results. For each scenario, each couple of formulations appearing in our tables, and each instance x, let $val(FC, x)$ and $val(FR, x)$ be the objective function values found by using the formulation indicated by the column heading and the row heading, respectively; the percentage gap among these values is calculated as $(100 - \frac{100val(FC,x)}{val(FR,x)})$. We do not consider the cases in which $val(FC, x) = val(FR, x)$, that is, we focus on the cases in which the two formulations report different solution values; on this subset of instances, each entry in the upper part of Table 2 reports the average of the percentage gaps multiplied by 100, while entries in the lower part of the table contain the standard deviation for such values. This choice was taken in order to improve the legibility of the tables, since the gap values are usually very small with respect to objective function values.

Table 2: Dataset 1: Gaps comparison on solutions with different values

Average values

	no perishability				$\theta_n = \{4, 8\}$				$\theta_n = 8$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	0.12	-30.83	0.26	—	0.32	-32.17	0.27	—	0.89	-33.11	0.59
MC	-0.12	—	-30.90	0.15	-0.32	—	-32.32	0.07	-0.89	—	-33.59	-0.12
I&L	30.55	30.61	—	30.70	31.86	32.01	—	32.06	32.78	33.26	—	33.18
SP	-0.26	-0.15	-30.99	—	-0.27	-0.07	-32.37	—	-0.59	0.12	-33.51	—

	$\theta_n = \{2, 3\}$				$\theta_n = 2$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	-0.03	-11.57	-6.05	—	-0.03	-5.39	-14.8
MC	0.03	—	-11.56	-6.04	0.03	—	-5.38	-14.79
I&L	11.52	11.51	—	7.64	5.38	5.37	—	-7.83
SP	6.04	6.03	-7.67	—	14.75	14.74	7.81	—

Standard deviation values

	no perishability				$\theta_n = \{4, 8\}$				$\theta_n = 8$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	0.45	43.58	0.47	—	0.32	45.62	0.53	—	1.98	46.36	1.41
MC	0.45	—	43.68	0.23	0.32	—	45.81	0.38	1.98	—	46.98	0.53
I&L	43.17	43.26	—	43.43	45.16	45.35	—	45.50	45.90	46.51	—	46.48
SP	0.47	0.23	43.85	—	0.53	0.38	45.96	—	1.41	0.53	46.95	—

	$\theta_n = \{2, 3\}$				$\theta_n = 2$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	0.09	18.37	7.75	—	0.14	6.02	16.95
MC	0.09	—	18.34	7.75	0.14	—	5.98	16.91
I&L	18.28	18.25	—	16.88	6.01	5.97	—	11.45
SP	7.74	7.73	16.94	—	16.88	16.84	11.41	—

From our description of the gap evaluation, it follows that entries with negative values in Table 2 identify the cases in which the formulation related to the row heading finds on average better solutions than the other, while the opposite holds for the entries with positive values.

It can be noted that PL and MC report very similar results, since in the cases without perishability the average gap on instances with different solution values is equal to 0.0012%. When perishability is considered, it is equal to 0.0003% in the worst case for short lifetime scenarios and 0.0089% in the worst case for the long ones. The SP formulation confirms its good performance in absence of perishability or when long lifetime values are considered. Indeed, in two out of three cases, it finds on average better solutions than all the others, and in the remaining one ($\theta_n = 8$) only MC finds on average better solutions. However, in the other two cases, it finds on average worse solutions than the others in all cases except one (where it outperforms I&L by 0.07767% on average).

The I&L formulation is always outperformed by PL and MC in terms of average gap, with the highest gaps being related to the scenarios without perishability or with long lifetime values. This could be expected given the high number of suboptimal solutions returned by I&L in this scenario. Unsurprisingly, the standard deviation values show a low level of dispersion when the best-performing formulations are compared, and larger dispersion values in the other cases.

Table 3 compares the formulations in terms of average computational time ratios. For each scenario, each couple of formulations $F1$, $F2$ and each instance x which can be solved optimally using both formulations within the time limit, let $time(F1, x)$, $time(F2, x)$ be the related computational times. The ratio among the two values is evaluated as $\frac{time(F1,x)}{time(F2,x)}$. Each entry in Table 3 with a *ratio* row heading reports the average ratio between the computational times of the two formulations indicated by the related column heading, evaluated on the subset of instances for which both are solved within the time limit. Each entry with a # *ins* row heading and the same column heading reports the size of such subset.

Table 3: Dataset 1: Time comparisons among instances with found optimal solutions

	MC/PL	I&L/PL	SP/PL	I&L/MC	SP/MC	I&L/SP
			no perishability			
# ins.	11	5	9	5	9	5
ratio	0.97	17.76	2.97	15.98	2.87	4.43
			$\theta_n = \{4, 8\}$			
# ins.	12	5	9	5	9	5
ratio	1.13	12.67	4.34	10.98	3.89	2.56
			$\theta_n = 8$			
# ins.	10	5	9	5	9	5
ratio	1.15	13.35	3.23	12.90	2.94	3.65
			$\theta_n = \{2, 3\}$			
# ins.	14	6	8	6	8	5
ratio	1.15	12.69	13.07	14.10	14.62	1.23
			$\theta_n = 2$			
# ins.	15	8	8	8	8	5
ratio	1.17	14.97	16.15	12.63	13.51	0.99

From the above mentioned average ratio definition, it follows that for the entries in a given column with heading $F1/F2$ the $F1$ formulation is on average faster if the *ratio* value is strictly lower than 1, and slower if it is strictly higher. It can be noted that MP and PL show a very similar performance in terms of computational time as well, since the ratios are close to 1 for each scenario. Conversely, solving the problem using the other two formulations appears to be significantly more time intensive. In all cases in which I&L is compared to MC or PL, it is at least 10.98 times (and up to 17.76 times) slower on average than them. The SP formulation also always performs worse than MC and PL; however, a noticeable deterioration of the performances can again be noticed in the scenarios with more restrictive lifetime constraints. Indeed, in the cases without perishability the ratio value is up to 2.97, in the scenarios with long lifetimes it is up to 4.34, and in those with short lifetimes SP and I&L perform very similarly on average (it may be noticed that the $I\&L/SP$ ratio is close to 1 for $\theta_n = \{2, 3\}$ and $\theta_n = 2$).

Looking at results for Dataset 2 (Tables 4-6), similar conclusions can be drawn. It may be observed that PL, MC and SP can individuate roughly the

Table 4: Dataset 2: Solutions quality comparison

	no perishability					$\theta_n = \{5, 10\}$					$\theta_n = 10$				
	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP
PL	20	—	2	26	5	24	—	2	23	6	21	—	2	25	5
MC	20	4	—	26	3	24	1	—	22	6	21	2	—	25	5
I&L	2	0	0	—	0	3	0	0	—	1	3	0	0	—	0
SP	18	4	3	25	—	19	1	1	21	—	19	2	1	25	—
	$\theta_n = \{3, 5\}$					$\theta_n = 3$					overall				
	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#opt	vs. PL	vs. MC	vs. I&L	vs. SP	#wins	#opt			
PL	23	—	2	21	6	26	—	2	15	9	151	114			
MC	23	1	—	21	5	25	1	—	16	8	146	113			
I&L	2	0	0	—	0	5	1	0	—	4	6	15			
SP	16	1	1	20	—	14	0	1	12	—	118	86			

same percentage of optimal solutions on both Dataset 1 and Dataset 2. In more detail, PL and MC solve with success 64% and 62% of the instances for Dataset 1, respectively, and around 65% of them for Dataset 2. The SP formulation individuates 43% of the optima for Dataset 1 and around 49% of them for Dataset 2. On the contrary, by using the I&L formulation, 29% of the Dataset 1 instances are solved with success, while the same holds for less than 9% of the Dataset 2 instances, which suggests that the classical formulation is even more unfit to solve the problem on instances with longer planning horizons.

Looking at the average gap and standard deviation values in Figure 5, it can be seen that SP is always outperformed by PL and MC, while I&L is always outperformed by the other three formulations. Analyzing the average time ratio values, it is clear that the ratio keeps being very close to 1 for all scenarios between PL and MC, with the other two formulations being significantly slower (up to 13 times slower in the case of MC, and over 300 times slower in the case of I&L).

Table 7 summarizes the comparison between linear relaxation values, for all the formulations and all the considered scenarios. The MC and PL formulation always have identical linear relaxation values, which are also the strongest among all formulations. For each scenario, each formulation $F1$ and each instance x, let $valR(F1, x)$ be the optimal objective function value of the linear programming relaxation of $F1$ for x. The percentage gap between the value provided by the linear relaxation of $F1$ and the one of MC (or PL) for instance

Table 5: Dataset 2: Gaps comparison on solutions with different values

Average values

	no perishability				$\theta_n = \{5, 10\}$				$\theta_n = 10$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	-3.96	-27.75	-4.66	—	4.08	-27.83	-9.84	—	-2.77	-18.29	-8.95
MC	3.94	—	-26.83	-3.04	-4.12	—	-29.74	-11.61	2.76	—	-17.84	-8.59
I&L	27.57	26.66	—	27.00	27.64	29.54	—	25.70	18.20	17.76	—	15.71
SP	4.65	3.03	-27.18	—	9.82	11.58	-25.86	—	8.93	8.57	-15.78	—

	$\theta_n = \{3, 5\}$				$\theta_n = 3$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	-2.29	-13.38	-1.89	—	-1.13	-6.64	-9.63
MC	2.24	—	-13.06	-1.06	1.12	—	-6.43	-9.26
I&L	13.34	13.02	—	13.35	6.63	6.42	—	1.22
SP	1.87	1.06	-13.39	—	9.59	9.21	-1.24	—

Standard deviation values

	no perishability				$\theta_n = \{5, 10\}$				$\theta_n = 10$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	13.18	32.37	11.08	—	18.38	33.29	11.06	—	12.20	23.50	10.54
MC	13.16	—	31.51	4.67	18.41	—	33.65	12.64	12.20	—	22.23	10.94
I&L	32.02	31.15	—	31.51	32.94	33.31	—	31.22	23.32	22.08	—	20.70
SP	11.06	4.67	31.87	—	11.03	12.60	31.50	—	10.51	10.92	20.86	—

	$\theta_n = \{3, 5\}$				$\theta_n = 3$			
	vs. PL	vs. MC	vs. I&L	vs. SP	vs. PL	vs. MC	vs. I&L	vs. SP
PL	—	22.48	13.85	15.20	—	8.39	7.14	19.14
MC	22.50	—	15.42	1.77	8.39	—	5.81	19.98
I&L	13.79	15.33	—	15.74	7.13	5.80	—	11.96
SP	15.20	1.77	15.83	—	19.02	19.86	11.93	—

x is evaluated as $(100 - \frac{100valR(F1,x)}{valR(MC,x)})$. Each entry in Table 7 under a *F1/MC,PL* heading is an average over all percentage gaps between the relaxations of $F1$ and MC (or PL) over all the instances of the considered dataset and scenario type.

It can be seen that the relaxed SP formulation generally returns solutions that are close to the ones of MC and PL. Indeed it returns identical results for the scenarios without perishability and in the long fixed ones. Overall, in the remaining scenarios, the percentage gap grows up to 1.17% for Dataset 1 and up to 2.41% for Dataset 2. Conversely, the relaxed I&L formulation always returns solutions that are very far from those of MC and PL, with percentage gaps between 32.02% and 61.45% for Dataset 1 and between 36.73% and 59.28% for Dataset 2. Our results agree with the comparison carried out in Alfieri et al. (2002), who, for Dataset 1 and the case without perishability, reported identical

Table 6: Dataset 2: Time comparisons among instances with found optimal solutions

	MC/PL	I&L/PL	SP/PL	I&L/MC	SP/MC	I&L/MC
			no perishability			
# ins.	19	2	18	2	18	2
ratio	1.10	11.10	1.28	11.09	1.21	7.11
			$\theta_n = \{5, 10\}$			
# ins.	23	3	19	3	19	3
ratio	1.00	7.41	1.95	7.85	2.03	3.89
			$\theta_n = 10$			
# ins.	20	3	18	3	19	3
ratio	0.98	8.07	1.56	9.06	1.63	5.83
			$\theta_n = \{3, 5\}$			
# ins.	22	2	16	2	16	2
ratio	1.14	12.50	9.35	8.55	9.07	3.26
			$\theta_n = 3$			
# ins.	24	5	14	5	14	5
ratio	1.11	320.69	13.31	317.19	12.30	9.63

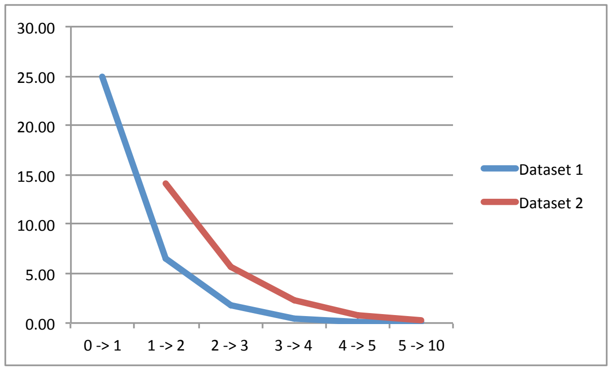
Table 7: Average percentage gaps for linear relaxations

scenario type	Dataset 1		Dataset 2	
	SP/MC,PL	I&L/MC,PL	SP/MC,PL	I&L/MC,PL
no perishability	0.00	61.45	0.00	59.28
$\theta_n = \{4, 8\}$	0.04	53.58	0.12	52.66
$\theta_n = 8$	0.00	58.92	0.00	57.30
$\theta_n = \{2, 3\}$	0.72	36.81	1.24	42.30
$\theta_n = 2$	1.17	32.02	2.41	36.73

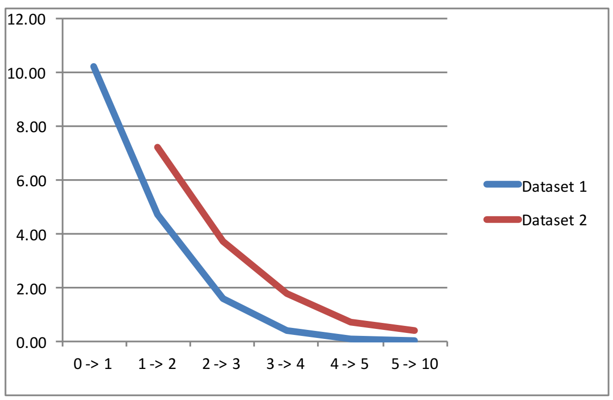
linear relaxation results for PL and SP, and a significant gap between them and I&L.

Finally, in Figure 1, the results of our sensitivity analysis on the value of product lifetimes are summarized. As previously introduced, we solved all instances using a single formulation, considering a lifetime θ_n for each product equal to 0,1,2,3,4,5 or 10. From the previous tests, it is clear that MC and PL proved to be the best-performing formulations, with very similar performances among the two. For this sensitivity analysis, we chose the Plant Location formulation since it was able to find a slightly larger number of optimal solutions.

Figure 1(a) reports the average percentage improvement that can be obtained by switching from each considered value of $\theta_n < 10$ to the following one. In more detail, for each dataset, an "$a \to b$" value on the x-axis corresponding



(a) Average percentage gaps



(b) Standard deviation

Figure 1: Solution gaps obtained for different θ_n values

to a "*num*" value on the y-axis means that considering $\theta_n = b$ brings on average a *num*% improvement with respect to $\theta_n = a$. Figure 1(b) reports the standard deviation for the same percentage improvement values.

The scenario $\theta_n = 0$, in which items have to be used in the same time period in which they are produced, is not reported for Dataset 2 since a single instance (that is, instance 0735) is feasible for this scenario. Also, three instances for Dataset 1, $\theta_n = 0$ and two instances for Dataset 2, $\theta_n = 1$ are not feasible and therefore averages and standard deviation are evaluated on the remaining instances when these scenarios are considered. For both datasets, the improvements that can be obtained by considering larger product lifetimes get smaller and smaller. For Dataset 1, solutions for $\theta_n = 1$ are on average 24.88% smaller than the ones for $\theta_n = 0$; solutions for $\theta_n = 2$ are on average 6.47% smaller than the ones for $\theta_n = 1$; solutions for $\theta_n = 3$ are on average 1.78% smaller than the ones for $\theta_n = 2$, and percentage gaps are below 1% in the other cases. For Dataset 2, solutions for $\theta_n = 2$ are on average 14.08% smaller than the ones for $\theta_n = 1$, solutions for $\theta_n = 3$ are on average 5.7% smaller than the one for $\theta_n = 2$, solutions for $\theta_n = 4$ are on average 2.26% smaller than the ones for $\theta_n = 3$, and gaps get below 1% in the other cases. Standard deviation values follow a similar pattern, going from 10.19 to 0.04 for Dataset 1, and from 7.19 to 0.38 for Dataset 2.

5. Conclusions

In this work, we addressed the CLSP with Lifetime Constraints which considers the case of products becoming unusable after a maximum lifetime, as opposed to the case of deteriorable items, which gradually lose their utility over time. One may consider, for instance, the rapid obsolescence of technology products, due to continuous innovation. Four formulations derived from the study of the literature on capacitated lot sizing models have been adapted to face this variant. The four formulations have been tested on a wide set of test instances, belonging to two well-known and used datasets available in the literature. The natural I&L formulation resulted to be the worst performing, in agreement with the literature on the basic version of the problem. The main finding of our work is that, while their performances appeared to be comparable to the shortest path formulation for the classical problem, the plant location-based and the multicommodity fomulations resulted to be consistently the best performing ones when the perishability constraints are taken into account. A reason for this refers to the fact that those formulations allow a “natural preprocessing” regarding the used variables (that is, related variables are set to zero). Moreover, a sensitivity analysis on the product lifetimes revealed that this factor can greatly influence the solution value, and that therefore it is important to take it into account to avoid economic and environmental waste.

Further research will be focused on proposing meta-heuristic algorithms to find good solutions for complex instances in reasonable time, as well as proposing extensions of the problem taking into account other significant parameters coming from real-world applications, such as setup times and setup carry-overs where the latter are employed in aggregate models to already take into account sequencing of multiple items. A further interesting aspect is the inclusion of rework options for items that passed their useful lifetime. There already exists considerable research in this direction (see, e.g., Görler and Voß (2016); Pahl and Voß (2016)). Nevertheless, formulations and models have not been extensively tested regarding the solution process performance.

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