TECHNIQUE AND ACCURACY ESTIMATION OF SIRIO ATTITUDE DETERMINATION

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ABSTRACT

This paper describes the general technique used to determine the SIRIO attitude and certain systematic errors, the biases, which affect the onboard sensor measurements.

This technique is implemented in the Attitude Determination Program (ADP) used at CNUCE for determining SIRIO attitude.

The ADP produces very accurate attitude and bias determinations. During the various mission phases, the accuracy obtained for the attitude has always been well within mission constraints.

As the spacecraft spin axis has different orientations during different mission phases, bias determination is a most delicate task.

The best situation for bias determination is the intermediate attitude (INTA) phase, when the attitude lies halfway between the orbit co-planar position and the orbit normal position. This situation is carefully examined here.

The results of the bias determinations discussed here make it possible to evaluate the reliability of the determination method and the uncertainty of the results obtained in attitude determination, at the present point in the mission.

§ 1 INTRODUCTION

The SIRIO attitude determination method here described is implemented in the Attitude Determination Program (ADP), one of the programs composing the Flight Dynamics System (FDS), running at CNUCE, which is used for SIRIO flight control.

This paper first describes the technique used to determine the SIRIO attitude and certain systematic errors which affect the onboard sensor measurements.

The major components of the method are:

a) the statistical filter used by the program to calculate the state vector, formed by the attitude components and by certain systematic errors;

b) the deterministic method which calculates the attitude using the previously computed systematic errors and which makes it possible to compare this calculated attitude with that calculated previously ;

c) the simulator which, using the previously calculated state vector, simulates the telemetry data and makes it possible to compare graphically the real data (transmitted from the satellite) and the simulated data.

In the second part of the paper, we have attempted to verify the onboard sensor performances and the accuracy of certain mathematical hypotheses used by the ADP program on the basis of an analysis of the more significant mission phases.

This verification is necessary as we have observed that the simulated earth width values disagree slightly with the values effectively measured and this discrepancy is not recoverable by a change in the state vector.

We have deduced that this fact is mainly due to the adoption, in the mathematical model, of a simplifying hypothesis which has been proved to be not completely exact.

The analysis has enabled a better estimate of the attitude determination uncertainty caused by the probable existence of residual systematic errors in the telemetry measurements.

However, from our experience, acquired during the mission, and in view of the considerations made in this paper, we feel able to affirm that the attitude determination system gives more accurate results than those required by the mission specifications.

§ 2 DEFINITIONS AND NOTATIONS

In this paper spacecraft attitude is denoted by the (α, δ) vector, where: α = spin axis right ascension; δ = spin axis declination. Only those measurements which interest the attitude determination are specified here: the sun angle β; the earth width W ; the dihedral angle Φ_i (Fig. 1), see reference (1) for a complete description of the onboard sensors.

The satellite transmits a set of data, the major frame, every 16 seconds. The β, W, Φ_i values are obtained from this set of data.

Fig. 1 represents the celestial sphere centered on the spacecraft. We have:

A - unit vector coinciding with the spin axis;

T - unit vector coinciding with the narrow field earth sensor;

$\underline{S}$ - "sun vector", unit vector from the spacecraft to the sun;

$\underline{N}$ - "nadir vector", unit vector from the spacecraft to the earth center;

β - sun angle, measured by the appropriate onboard sensor;

η - nadir angle, obtained from the ephemeris data;

γ - earth sensor mounting angle (narrow field);

ρ - earth angular radius;

W - earth width as measured from the narrow field earth sensor;

Φ_i - dihedral angle as measured from the narrow field earth sensor.

Experience has shown that the more important systematic errors in attitude determination are: $\Delta\beta$, $\Delta\gamma$, $\Delta\bar{\Phi}$, $\Delta\rho$. It is, thus, necessary to realize a mathematical model which will give an estimation of the vector (α, δ, $\Delta\beta$, $\Delta\gamma$, $\Delta\Phi$, $\Delta\rho$), the state vector. This vector will be always assumed constant in time.

§ 3 THE FILTER IMPLEMENTED IN THE ADP PROGRAM

The general mathematical method used to estimate the state vector is briefly described here.

Let $\underline{X}$, the generic state vector, constant in time, be estimated and let s be its dimension.

Let $\underline{X0}$ be an a priori value of $\underline{X}$, and S0 be the inverse of the covariance matrix relative to the $\underline{X0}$ estimate of $\underline{X}$.

Let $\underline{Y}$ be the vector, of dimension p, of the measurements obtained from the satellite (i.e. $\underline{Y}$ is the column vector formed with a p number of sun angle measurements).

Let $\underline{YC} = \underline{YC}(\underline{X}, \underline{\Psi}(t))$ a function, continuous and with continuous partial derivatives in the neighbourhood of $\underline{XC}$, which makes it possible to calculate the $\underline{Y}$ measurements as functions of the state vector $\underline{X}$ and of the ephemeris data $\underline{\Psi}(t)$.

In this way, the $\underline{YC}$ vector remains associated with the $\underline{Y}$ vector which contains the real measurements from the satellite. If the $\underline{Y}$ measurements are without error (systematic or accidental), $\underline{Y}-\underline{YC} = \underline{0}$. However, as errors are normally present, we have the residual vector indicated by $\underline{Z}=\underline{Y}-\underline{YC}$.

Let W be the matrix p x p, the inverse of the covariance matrix relative to the $\underline{Y}$ measurements.

Let us consider the following real function of an s number of variables:

$$l(\underline{X}) = \frac{1}{2} \underline{Z}^T W \underline{Z} + \frac{1}{2} (\underline{X}^T - \underline{X}_0^T) S_0 (\underline{X} - \underline{X}_0) \quad (1)$$

the $\underline{X*}$ vector which minimize the $l(\underline{X})$ function is assumed as an estimate of $\underline{X}$.

$\underline{X*}$ minimizes $l(\underline{X})$, when $\underline{X*}$ is a solution for the equation:

$$\frac{\partial l}{\partial \underline{X}} = \underline{0} \quad (2)$$

we must then resolve the equation:

$$\left(\frac{\partial \underline{Z}}{\partial \underline{X}}\right)^T W \underline{Z} + S_0 (\underline{X} - \underline{X}_0) = \underline{0} \quad (3)$$

We next suppose that W and S0 are symmetric, positive defined matrices. Using Taylor's theorem for $\underline{Z}$, starting with $\underline{X}=\underline{X0}$, we obtain:

$$\underline{Z}(\underline{X}) = \underline{Y} - \underline{Y}_c(\underline{X}_o) - \frac{\partial \underline{Y}_c}{\partial \underline{X}}(\underline{X}_o)\left(\underline{X} - \underline{X}_o\right) + O(\underline{X} - \underline{X}_o) \quad (4)$$

where $O(\underline{X} - \underline{X0})$ is a higher order infinitesimal with respect to $|\underline{X} - \underline{X0}|$ when $\underline{X} \rightarrow \underline{X0}$.

If we indicate as G0 the matrix p x s previously indicated as $\frac{\partial \underline{Y}_c}{\partial \underline{X}}(\underline{X}_o)$, disregarding higher order infinitesimals, we obtain:

$$\underline{Z}(\underline{X}) = \underline{Y} - \underline{Y}_c(\underline{X}_o) + G_o \underline{X}_o - G_o \underline{X} \quad (5)$$

$$\frac{\partial \underline{Z}}{\partial \underline{X}} = -G_o \quad (6)$$

Substituting (5) and (6) in (3), we obtain:

$$-G_o^T W\left(\underline{Y} - \underline{Y}_c(\underline{X}_o) + G_o \underline{X}_o - G_o \underline{X}\right) + S_o(\underline{X} - \underline{X}_o) = \underline{O} \quad (7)$$

from which:

$$\left(G_o^T W G_o + S_o\right)(\underline{X} - \underline{X}_o) = G_o^T W\left(\underline{Y} - \underline{Y}_c(\underline{X}_o)\right) \quad (8)$$

so that:

$$\underline{X} = \underline{X}_o + \left(G_o^T W G_o + S_o\right)^{-1} G_o^T W\left(\underline{Y} - \underline{Y}_c(\underline{X}_o)\right) \quad (9)$$

The result thus obtained is affected by an error due to the linearization of the equation (2). We, thus, use an iterative technique: the result obtained $\underline{X} = \underline{X1}$ is assumed as a new a priori value of $\underline{X}^*$ and this process is repeated for $\underline{X0} = \underline{X1}$. The iterative process is stopped when the difference between two consecutive vectors is sufficiently reduced.

Three models are used most frequently during the mission:

1) Sun angle model;
2) Earth width model;
3) Sun to earth mid-scan dihedral angle model.

The table 1 shows the dependence of each of these models on the state vector variables:

Variables	Models 1	2	3
α	*	*	*
δ	*	*	*
$\Delta\beta$	*		
$\Delta\gamma$		*	
$\Delta\Phi$			*
$\Delta\rho$		*	

Table 1

As an example, the first two models are briefly described below (*):

a) Sun angle model: $\underline{Y} = \underline{\beta}$.

In order to express the YC function, the $\underline{A}$ components are indicated by (u1, u2, u3) and the $\underline{S}$ components by (s1, s2, s3), obtained from the ephemeris data. We obtain

$$Y_c = -\Delta\beta + \cos^{-1}(u_1 s_1 + u_2 s_2 + u_3 s_3) \; ; \; u_3 = \sqrt{1 - s_1^2 - s_2^2} \quad (10)$$

(The attitude here defined is the (u1, u2, u3) unit vector instead of the (α, δ) vector).

b) Earth width model: $\underline{Y} = \underline{W}$ YC is obtained from the spheric

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(*) For a complete description of all the models see reference (3).

triangle $\underline{TNA}$ as shown in Fig. 1 :

$$\gamma_c = 2 \cos^{-1}\left[\frac{\cos(\rho+\Delta\rho) - \cos(\gamma+\Delta\gamma)\cos\eta}{\sin(\gamma+\Delta\gamma)\sin\eta}\right] \qquad (11)$$

where, indicating $\underline{N}$ by (n1, n2, n3), we have:

$$\eta = \cos^{-1}(n_1 u_1 + n_2 u_2 + n_3 u_3) \qquad (13)$$

In order to determine the attitude and the biases for a single model, at the same time, there must be a substantial variation in the geometric configuration during the observation period. Therefore, if ϑ is the angle between the normals to the initial and final "constant measurement" surfaces, it must be $\vartheta >> 0$. See reference (4) for an accurate analysis of these geometric conditions during the various mission phases in relation to the delicate problem of the state vector estimate accuracy. A number of these conditions are specified below.

a) $\Delta\beta$ can never be determined, using model 1, at the same time as the attitude. Model 1 can only be used after α and δ have been determined using a different model.

b) $\Delta\gamma$ can be calculated at the same time as the attitude, using model 2, only if the observation period includes measurements made both in apogee and in perigee.

c) $\Delta\rho$ can be calculated at the same time as the attitude, using model 2, if the observations contain both $\eta > \gamma$ and $\eta < \gamma$ measurements.

d) $\Delta\bar{\Phi}$ can only be calculated at the same time as the attitude in particular mission phases; this is impossible in the geostationary phase.

In the transfer orbit phase, condition b) is not verified, therefore $\Delta\gamma$ cannot be determined and is thus set zero. The other state vector elements can be well determined at the same time using the measurements obtained during an apogee pass.

After apogee motor firing, during station acquisition phase (SA), the spin axis is nearly parallel to the orbit plane. Under these conditions, spin axis declination cannot be well determined. In this case the $\Delta\bar{\Phi}$ value obtained during the preceding mission phase and model 3 are generally used for attitude determination.

The best geometric configuration for an accurate bias determination (and the only configuration in which all the biases can be determined at the same time) occurs during the INTA phase, which is characterized by an equatorial, nearly circular orbit, with a radius very close to the geostationary radius, and with a spin axis declination of approximatively -75 degrees. In this configuration all the conditions (a-d) are verified.

During geostationary phase, with the spin axis normal to the orbit plane, it is impossible to determine the attitude and the $\Delta\rho$ at the same time. The $\Delta\rho$ calculated during the INTA phase is thus used and α, δ, $\Delta\gamma$ are determined at the same time using the earth width model. When α and δ are known, $\Delta\beta$ is calculated using model 1 and $\Delta\Phi$ using model 3.

The conditions (a-d) for the bias determinability are, in practice, equivalent to the conditions for the convergence of the calculus method implemented in the ADP program. When convergence is obtained under conditions (a-d), the result is reasonably reliable. The system is quite complex to use as all the different geometric conditions encountered in the various mission phases must be satisfied. Consequently, another attitude determination method is essential to verify the results obtained with the filter.

§ 4 THE DETERMINISTIC METHODS

A number of deterministic methods are used to verify the attitude calculated using the filter.

These methods give an attitude value for each telemetry frame. As the information contained in a frame is redundant, the different methods use different subsets of the frame content. In this way, each method produces a different attitude value (obtained by averaging the frame by frame results).

The different behaviour of one method with respect to the others may, in some cases, indicate which data subset should be controlled for wrong data.

In practice, four methods are used. The table 2 shows the telemetry data utilized by each method (**) :

(**) The Φ_i data is utilized both by the filter mathematical models and by the deterministic methods. In SIRIO, even when both the narrow field sensors see the earth, only one of them gives the Φ_i data, i.e. the sensor seeing the bigger earth width.
Thus, the telemetry data cannot be fully explcited as data from only one sensor at time can be utilized. Consequently, different methods, not described in the table, but used successfully in other missions, cannot be used here, as these methods utilize the earth width and the dihedral angle measured from both sensors at the same time.

METHOD	Φ_i	β	W
1	*	*	
2		*	*
3	*	*	*
4	*	*	*

Table 2

As an example , a brief description of the second method is given.

For each telemetry frame, η is determined from the spherical triangle <u>TNA</u>, (Fig. 1) using the equation:

$$\cos(\rho+\Delta\rho) = \cos(\gamma+\Delta\gamma)\cos\eta + \sin(\gamma+\Delta\gamma)\cos W \sin\eta \quad (13)$$

where ρ is obtained from the satellite ephemeris data; $\Delta\rho$ and $\Delta\gamma$ have been previously calculated using the statistical filter.

The Φ_i value, obtained from the telemetry data, is utilized in this method only in order to resolve the ambiguity arising in expressing the equation (13) with respect to η , thus it is unnecessary to know the Φ_i value with great precision. Therefore, in the preceding table, the second method is no way dependent on Φ_i. From Fig. 1, we see that the attitude lies on a cone which has the <u>N</u> vector (obtained from the ephemerides) as its axis and a cone angle of 2η.

Simultaneously, the attitude vector lies on another cone which has the <u>S</u> vector (known from the ephemerides) as its axis and a cone angle of $2(\beta+\Delta\beta)$ (β is the sun angle measurement given by telemetry and $\Delta\beta$ has been previously determined using the filter).

The attitude is thus determined from the intersection of the two cones. Two attitude values are obtained for each telemetry frame. This ambiguity is suitably eliminated repeating the process for a large number (about 250) of frames, selected from the whole orbit. Only one of the two attitude values remains constant in time (for further explanation on this process see reference (3)).

The attitude accuracy is verified not only by a comparison of the deterministic values with the statistical filter values, but above all by a graphic control.

Plotting the α and δ values obtained using the deterministic methods versus time, curves should be obtained, which, apart from dispersion due to accidental errors, are straight lines parallel to the abscissa axis, as the attitude must be constant during the measurement period (Figs. 2, 3).

If the curves should behave differently, there must be some systematic residual error. One of the basic hypotheses of our mathematical models is that systematic errors are constant in time. This hypothesis is sufficient for our purposes even though it is not completely exact, as it will be shown below.

In order to determine the presence of constant residual errors by observing the plots of the deterministic methods, the geometric configuration must show a sufficient variation during the measurement period.

In different situations, a constant error will have different effects. As all deterministic methods have singular points, the presence of systematic errors leads, in the neighbourhood of these points, to very irregular behaviour in the curves.

Using the graphic control, however, the existence of residual systematic errors is easily verified, but precise identification of the error is much more difficult. It is particularly difficult to establish whether a residual constant systematic error is present or whether a systematic error is slightly variable instead of constant as presumed.

It is, thus, advisable to use the determined state vector to simulate the telemetry data and to compare these simulated data with the real ones (transmitted from the spacecraft).

§ 5 ANALYSIS OF A PARTICULAR MISSION PHASE

The estimated state vector elements are utilized to simulate the telemetry data. A graphic comparison can be made between the simulated data and the measured data. If the measured values are found randomly distributed around the simulated curve, we can reasonably exclude the presence of residual systematic errors on the measurements. For example, Fig. 4 shows the dispersion of the measured earth width values around the simulated values assumed as the reference level. The fact that the measured data do not distribute randomly around the simulated data, indicates the presence of a residual systematic error; this may be $\Delta\rho$.

To analyze this last phenomenon, we have considered a part of the INTA data (see section 3), where a long telemetry period is available, where an accurate state vector determination has been made and where the measured earth width changes sufficiently so that the phenomenon can be observed (Fig. 5).

Thus, if the α and δ values are well known, the disagreement between the measured and simulated W values can be calculated from the functional relation:

$$\Delta W = \frac{\partial W}{\partial \rho} \Delta \rho + \frac{\partial W}{\partial \gamma} \Delta \gamma \qquad (14)$$

and since $\Delta\gamma = 0$ has already been determined with good approximation, we can write:

$$\Delta \rho = \Delta W / \frac{\partial W}{\partial \rho} \qquad (15)$$

with

$$\frac{\partial W}{\partial \rho} = 2 \sin \rho / \left(\sin \gamma \sin \eta \sin \frac{W}{2}\right) \qquad (16)$$

The calculated $\Delta\rho$ values have been interpolated with a fourth degree polynomial and plotted, ten times magnified, in a polar coordinate system (Fig. 6), in order to visualize how the Earth is "seen" by the narrow field sensor.

The disagreement between the simulated data and the measured ones, see Fig. 6, is due above all to the fact that the initial hypothesis $\Delta\rho$ =constant is not completely exact. A certain dependence of $\Delta\rho$ on W has been verified.

The presence of this non-constant systematic error is the most limiting element in attitude determination accuracy.

We have seen that $\Delta\rho$ determination requires a telemetry period with an adequate earth width variation. However, in this case the inexactitude of the mathematical hypothesis that $\Delta\rho$ is constant for all measurements is more evident.

During the geostationary phase, with an equatorial circular orbit and a spin axis declination of -89.9 degrees, the earth width is almost constant and it is not possible to use the second model to find $\Delta\rho$, which must, therefore, be fixed, once and for all, to the known value determined during the INTA phase. As $\Delta\rho$ is, to some extent, a function of W, it must be fixed to that value which most closely corresponds to the earth width value measured during the geostationary phase.

An error in the $\Delta\rho$ estimate affects systematically the attitude value (above all the declination value, because the right ascension is nearly undefined).

As a result of this analysis, the attitude determination uncertainty has been estimated to be about 0.03 degrees (σ value), even though, because of the accuracy of the results and their continuous succession in time, this estimate can be retained as too prudent.

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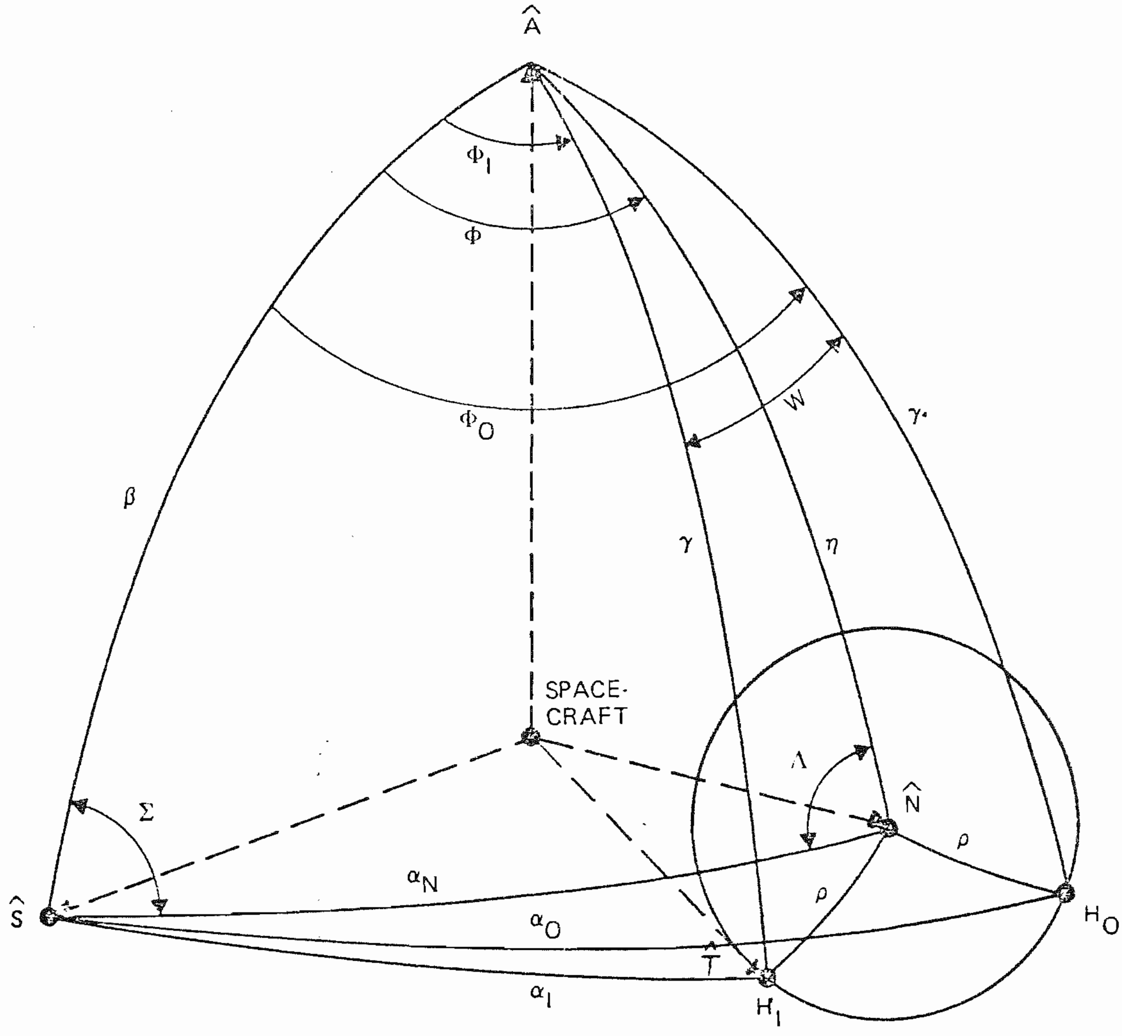


Fig 4

PLOT
IDENTIFY
IDENTIFY ENDED
PLOT

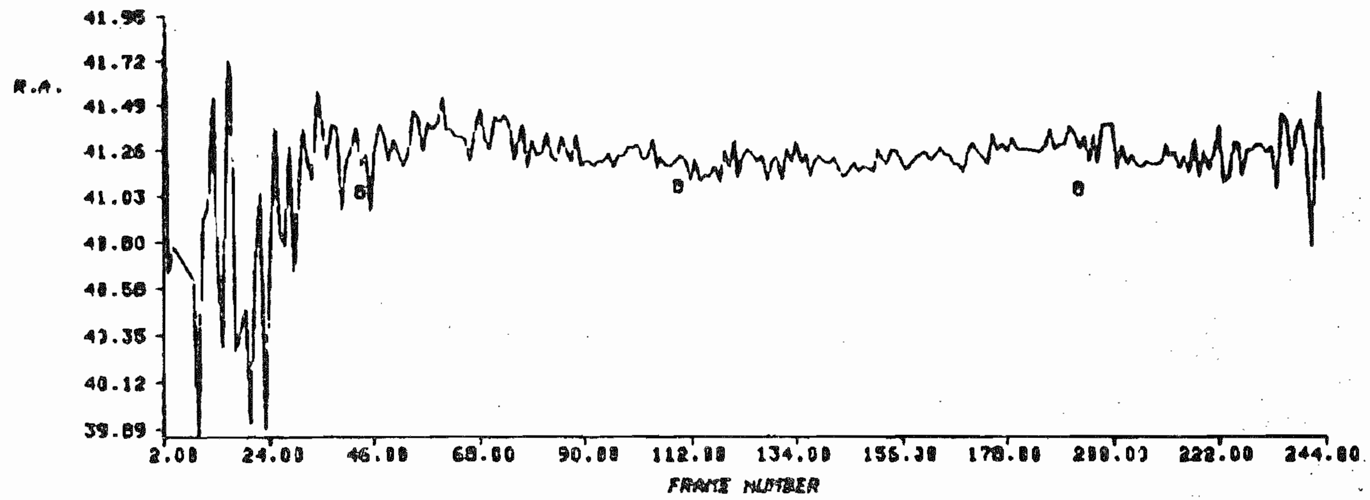
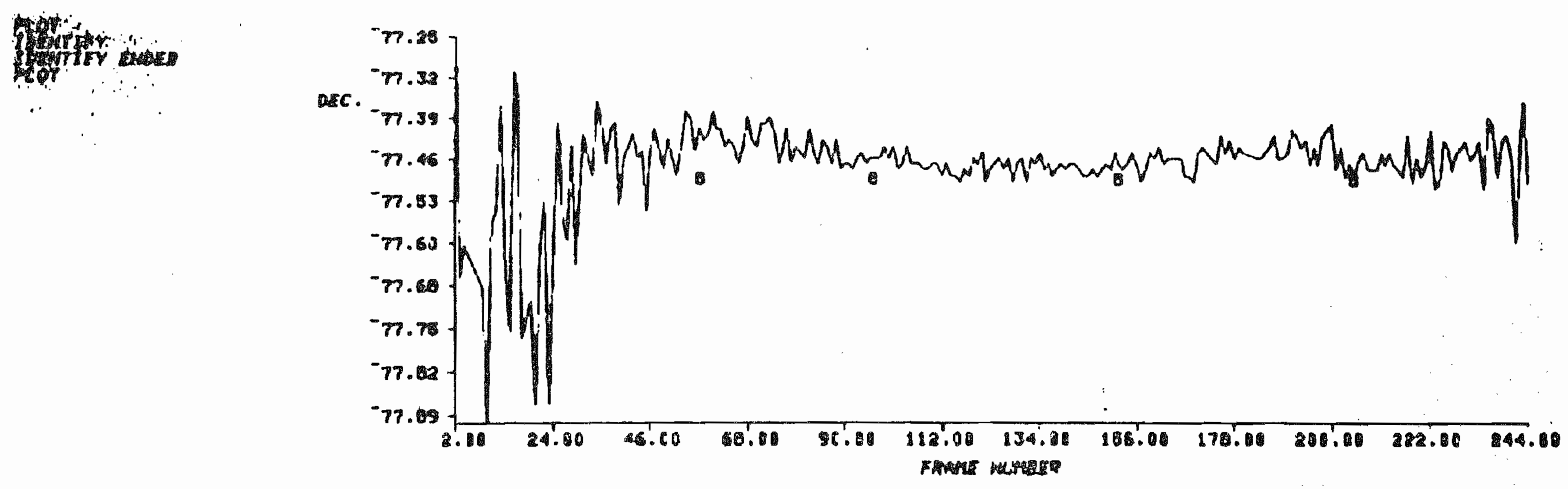


Fig. 2a

Fig 2



PLOT
IDENTIFY
IDENTIFY ENDED
PLOT
DEC.
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-77.32
-77.39
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-77.53
-77.60
-77.68
-77.76
-77.82
-77.89
2.00
24.00
46.00
68.00
90.00
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178.00
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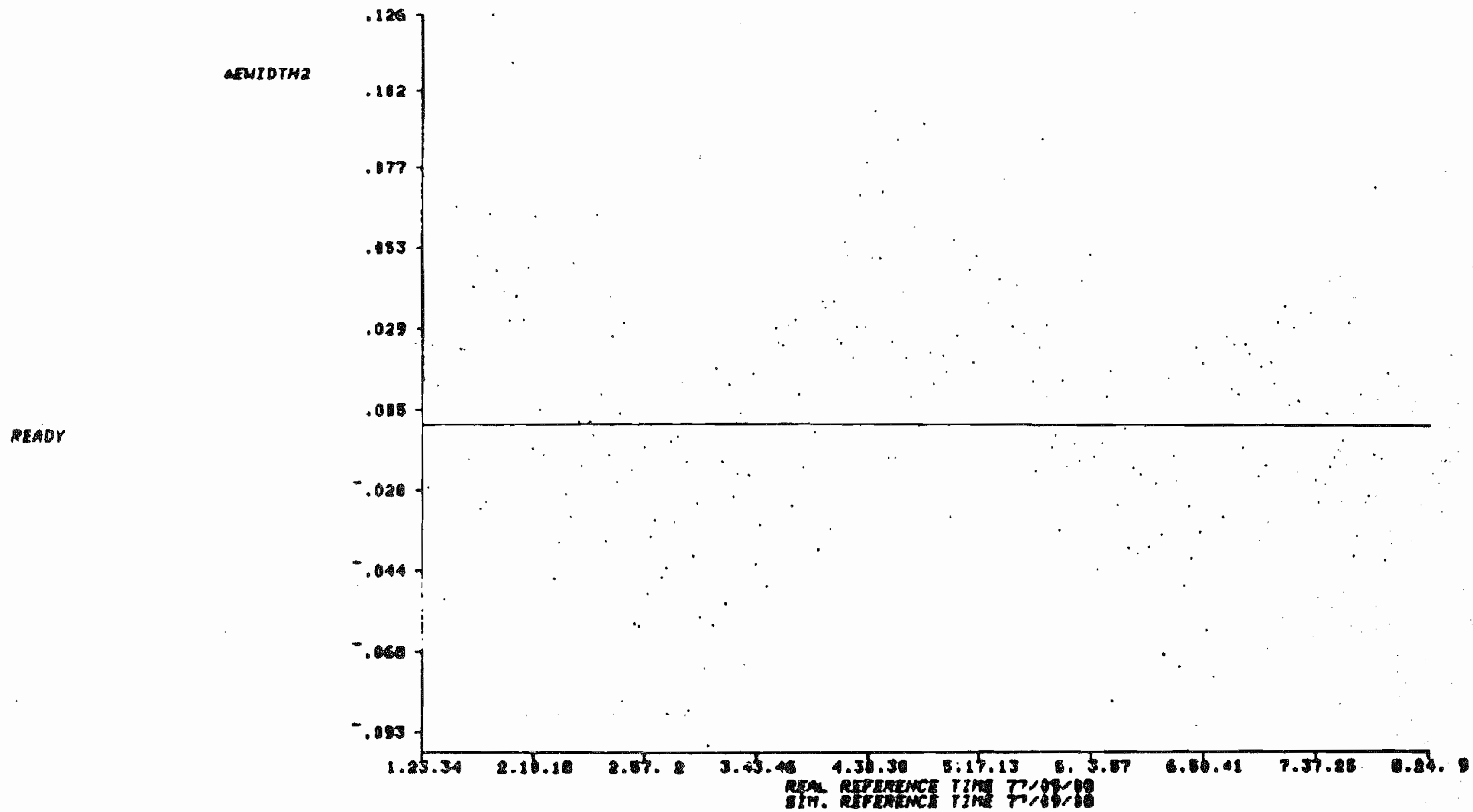
FRAME NUMBER

Fig. 2b

Fig 3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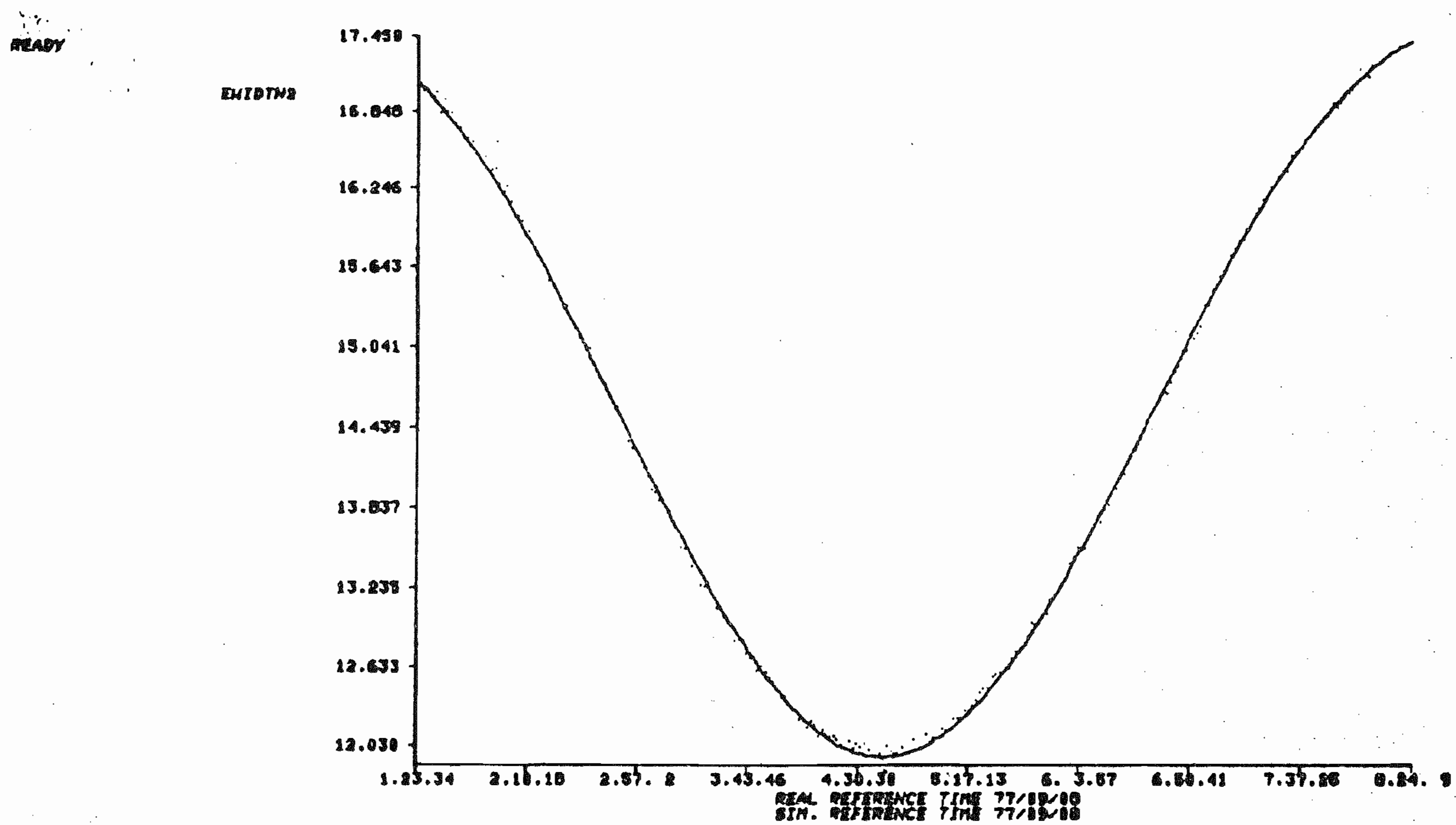
.102
.077
.053
.029
.005
-.020
-.044
-.068
-.093
1.23.34
2.10.18
2.57. 2
3.43.46
4.30.30
5.17.13
6. 3.57
6.50.41
7.37.25
8.24. 9
REAL REFERENCE TIME 77/09/08
SIM. REFERENCE TIME 77/09/08

Fig. 3a

F.p 4



READY
EWIDTH2
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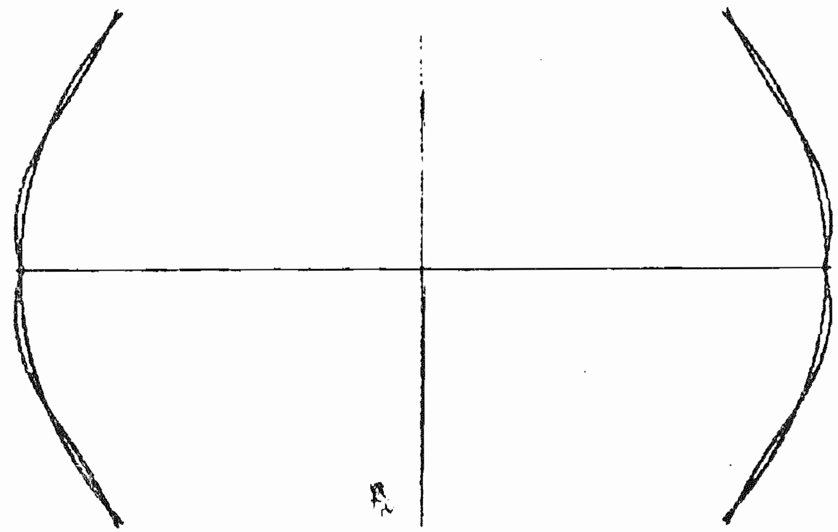
16.848
16.246
15.643
15.041
14.439
13.837
13.235
12.633
12.030
1.23.34
2.10.18
2.57. 2
3.43.46
4.30.38
5.17.13
6. 3.57
6.50.41
7.37.26
8.24. 9
REAL REFERENCE TIME 77/09/08
SIM. REFERENCE TIME 77/09/08

Fig. 36

Fig. 5



POLAR REPRESENTATION OF THE RESIDUAL ERROR ON EARTH RADIUS
WITH RESPECT TO THE SPHERICAL MODEL

PACKAGE A - SENSOR INF2

Fig 6

STAMPATO PRESSO IL
SERVIZIO TECNOGRAFICO
DEL CNUCE