

**The finite element code NOSA  
Version 2.0**

**User's manual**

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## **PRESENTATION**

The finite element code NOSA (Non-linear Structural Analysis) has been developed by the team of Mechanics of Materials and Structures of the CNUCE. The work's primary goal has been the development of a suitable set of tools for experimentation on engineering problems involving material constitutive equations and algorithms for integrating the equation of motion as well as other numerical techniques of research interest to the group.

A first version of the code, written in the early 1980's, included plane, three-dimensional and axisymmetric isoparametric elements [1] and permitted elastic-plastic analysis in the presence of infinitesimal strains with the hardening model described in [2]. Subsequently, the possible applications of NOSA were extended to include cases of finite strains based on the studies carried out on both constitutive equations [3, 4, 5, 6] and methods of numerical integration of the equation of motion [7, 8, 9] in the presence of follower forces and contact problems. At the same time, the element library was broadened by the adding of the shell elements (thin and thick) [10].

More recently, elastic material, with low resistance to traction, has been added in order to permit static analysis of masonry solids [11, 12, 13, 14, 15].

Finally, the capability of performing heat transfer and dynamic analysis has been added.

Over these years many engineering students of the University of Pisa have collaborated in the development of the NOSA code as part of their thesis research.

The authors

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## A. INTRODUCTION

Preparation of input data and display of numerical results may be performed using the pre and post-processing code MENTAT II<sup>1</sup>.

PRE-PROCESSING

MENTAT II

F.E.M. ANALYSIS

NOSA

POST-PROCESSING

MENTAT II

The code MENTAT II can process two- and three-dimensional meshes; the main operations that can be carried out are:

- automatic mesh generation,
- definition of nodal coordinates,
- definition of element connectivities,
- definition of material properties of elements,
- definition of geometrical properties of elements,
- definition of loads and boundary conditions,
- plotting NOSA output .

The F.E.M. code NOSA can be used for solving linear and non-linear equilibrium problems; the output of NOSA is the stress, strain and displacement field, as well as the reactions forces. Two kinds of non-linearities can be taken into account: material non-linearities due to the material's constitutive equation and geometric non-linearities due to finite strains or contact conditions.

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<sup>1</sup> MENTAT is a product of the MARC Analysis Research Corporation.





## **A1. MATERIAL MODELS**

- ELASTICITY:** static and/or dynamic analysis is performed for structures made up of a linear elastic material, using all elements of the library.
- COMPOSITE:** the shell elements (type 5, 10 and 17) and the beam element (type 9) allow modeling of the structures consisting of sections of different mechanical and/or thermal properties, such as composite materials.
- ELASTO-PLASTICITY:** static and/or dynamic analysis is performed for structures made up of elastic-plastic materials under the assumption of infinitesimal strains and finite strains in the presence of isotropic, cinematic and combined hardening, using all elements except shells and beams (the finite strain elasto-plasticity is available only for the elements with linear interpolating functions).
- MASONRY-LIKE** static and/or dynamic analysis is performed, using all element types, for structures made up of masonry-like materials non-resistant to traction and infinitely resistant to compression, as well as for masonry-like materials with bounded tensile and compressive strength.
- HEAT TRANSFER** non-linear heat transfer analysis is performed for plane, axisymmetric, shells and 3d elements.

The elements available in the NOSA code are listed in the following Table.

Structural type	Element identifier number	Interpolating functions	Remarks
three-dimensional element	1	quadratic	20 nodes isoparametric element
plane stress element	2	quadratic	8 nodes isoparametric element
plane strain element	3	quadratic	8 nodes isoparametric element
axisymmetric element	4	quadratic	8 nodes isoparametric element
thin shell element	5	linear for displacements, quadratic for rotations	8 nodes isoparametric element
plane strain element	6	linear	4 nodes isoparametric element
axisymmetric element	7	linear	4 nodes isoparametric element
three-dimensional element	8	linear	8 nodes isoparametric element
straight beam element	9	linear	2 nodes isoparametric element
thick shell element	10	linear for displacements and rotations	4 nodes isoparametric element
plane heat transfer element	11	quadratic	8 nodes isoparametric element
plane heat transfer element	12	linear	4 nodes isoparametric element
axisymmetric heat transfer element	13	quadratic	8 nodes isoparametric element
axisymmetric heat transfer element	14	linear	4 nodes isoparametric element
3D heat transfer element	15	linear	8 nodes isoparametric element
3D heat transfer element	16	quadratic	20 nodes isoparametric element
heat transfer shell element	17	linear	4 nodes isoparametric element

**Table A1.**

## B. BIBLIOGRAPHIC REFERENCES

For a complete and detailed description of the theories and algorithms used in NOSA, the interested reader is referred to the following:

- [1] HINTON E., OWEN D. R. J., Finite Element Programming, Academic Press, 1977.
- [2] GUIDOTTI P, LUCCHESI M, PAGNI A., PASQUINELLI G., Elastic-Plastic Behavior with Work Hardening: an Appropriate Model for Structural Software, *Meccanica* **19**, 1984.
- [3] LUCCHESI M., PODIO GUIDUGLI P., Materials with Elastic Range: a Theory with a view toward Applications. Part I, *Arch. Rat. Mech. Anal.*
- [4] LUCCHESI M., PODIO GUIDUGLI P., Materials with Elastic Range: a Theory with a view toward Applications. Part II, *Arch. Rat. Mech. Anal.*
- [5] LUCCHESI M., PODIO GUIDUGLI P., Materials with Elastic Range: a Theory with a view toward Applications. Part III, *Arch. Rat. Mech. Anal.*
- [6] LUCCHESI M., PODIO GUIDUGLI P., Materials with Elastic Range and the Possibility of Stress Oscillations in Pure Shear , Proc. Int. Conf. on Comp. Plasticity, Model, Software and Applications, Barcelona, 6-10 aprile 1987.
- [7] GUIDOTTI P., LUCCHESI M., A Numerical Method for Solving Boundary-Value problems in Finite Plasticity, *Meccanica*, **28**, 1988.
- [8] DEGL'INNOCENTI S., PADOVANI C., PASQUINELLI G., An improved numerical method to integrate the equation of motion in finite elastoplasticity problems, *Complas II*, Second International Conference on Computational Plasticity, Barcelona, Settembre 1989.
- [9] PASQUINELLI G., Simulation of Metal-Forming Processes by the Finite Element Method, *Int J. Plasticity*, Vol. **11**, No. 5 (1995), pp. 623-651.
- [10] GUIDOTTI P., LUCCHESI M., PAGNI A., PASQUINELLI G., Application of Shell Theory to Structural Problem Using the Finite Element Method , *Quaderni de "La Ricerca Scientifica"*, **115**, 1986.
- [11] LUCCHESI M., PADOVANI C. and PAGNI A., A numerical method for solving equilibrium problems of masonry-like solids. *Meccanica*, **24** (1994), pp. 175-193.
- [12] LUCCHESI M., PADOVANI C. and PASQUINELLI G., On the numerical solution of equilibrium problems of elastic solids with bounded tensile strength. *Comput. Methods Appl. Mech. Engrg.* **127** (1995), pp. 37-56
- [13] LUCCHESI M., PADOVANI C. and ZANI N., Masonry-like materials with bounded compressive strength. *Int. J. Solids Structures* **33** (1996) pp. 1961-1994

- [14] LUCCHESI M., PADOVANI C., PAGNI A. and ZANI N., Un metodo numerico per lo studio degli archi in muratura. Atti del VII Convegno Italiano di Meccanica Computazionale, Trieste 1-3 Giugno 1993, pp. 239-244.
- [15] LUCCHESI M., PADOVANI C., PASQUINELLI G. and ZANI N., Un metodo numerico per le volte in muratura. Atti del VIII Convegno Italiano di Meccanica Computazionale, Torino 15-17 Giugno 1994, pp. 44-49.
- [16] PADOVANI C., PAGNI A. and PASQUINELLI G., Glie elementi guscio nel codice agli elementi finiti NOSA. Internal report CNUCE-B4-1998-012, Luglio 1998.
- [17] PASQUINELLI G., Un elemento trave introdotto nel codice agli elementi finiti NOSA. Internal report CNUCE B4-2000-029, Dicembre 2000.
- [18] PASQUINELLI G., Conduzione non lineare del calore nel codice agli elementi finiti NOSA. Internal report CNUCE B4-2000-030, Dicembre 2000.

## **C. NOSA ELEMENT LIBRARY**

At present, the elements available in NOSA are seventeen and they are described in the next sections.



### C1. Element 1 (Three-dimensional 20-node brick)

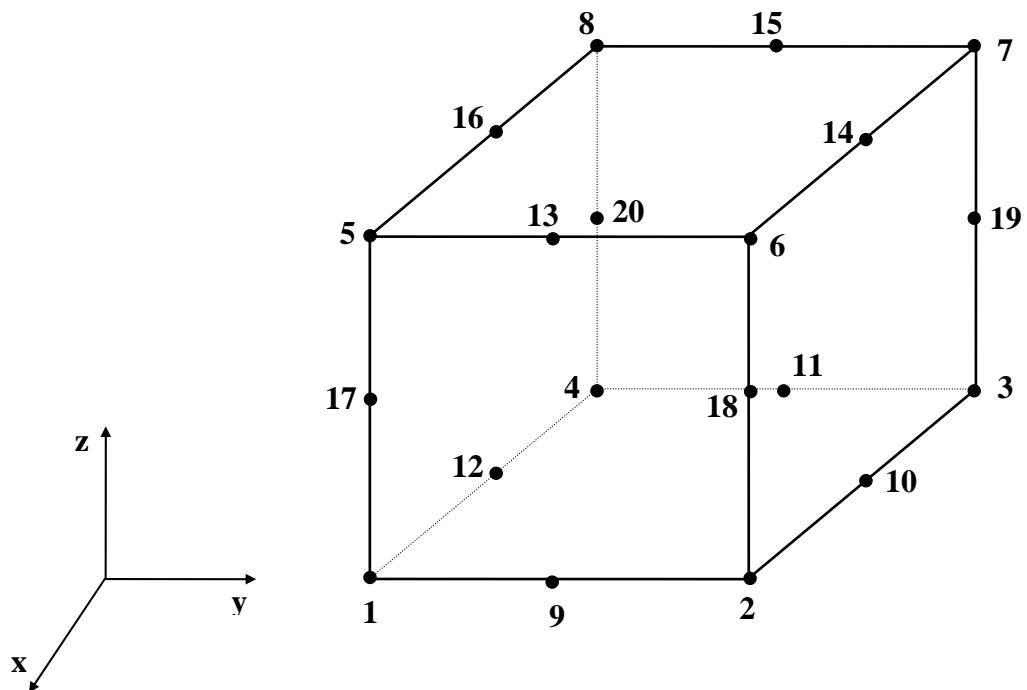
Twenty-node isoparametric element with quadratic interpolating functions. This element is a rapidly converging element for three-dimensional analysis. For thick-shell situations, one element through the thickness will usually provide an acceptable solution for both displacement and stress. Nonetheless, it is advisable that the length to thickness ratio not exceeds a value of 20.

#### Connectivity

Twenty nodes numbered as shown in Figure 1.1.

#### Integration

The element is integrated numerically using twenty-seven points (Gaussian quadrature). Integration points are shown in Figure 1.2.



**Figure 1.1** Element 1.





- 13 Pressure on the 1-2-3-4 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 14 Pressure on the 1-2-3-4 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 21 Pressure on the 5-8-7-6 face (force per unit area) in the global reference system.
- 22 Pressure on the 5-8-7-6 face in the local reference system. The first component of the force per unit area is tangential to the face in the 5-8 direction; the third component is orthogonal to the face and is positive if directed towards the interior of the element. The second component has the direction given by the vector product of the first and third directions.
- 23 Pressure on the 5-8-7-6 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 5-8-7-6 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 1-5-6-2 face (force per unit area) in the global reference system.
- 32 Pressure on the 1-5-6-2 face in the local reference system. The first component of the force per unit area is tangential to the face in the 1-5 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 33 Pressure on the 1-5-6-2 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 34 Pressure on the 1-5-6-2 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 2-6-7-3 face (force per unit area) in the global reference system.
- 42 Pressure on the 2-6-7-3 face in the local reference system. The first component of the force per unit area is tangential to the face in the 2-6 direction, the third component is orthogonal to the face and is positive if directed towards the

element's interior; the second component has the direction given by the vector product of the first and third directions.

- 43 Pressure on the 2-6-7-3 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 44 Pressure on the 2-6-7-3 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 51 Pressure on the 3-7-8-4 face (force per unit area) in the global reference system.
- 52 Pressure on the 3-7-8-4 face in the local reference system. The first component of the force per unit area is tangential to the face in the 3-7 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 53 Pressure on the 3-7-8-4 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 54 Pressure on the 3-7-8-4 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 61 Pressure on the 4-8-5-1 face (force per unit area) in the global reference system.
- 62 Pressure on the 4-8-5-1 face in the local reference system. The first component of the force per unit area is tangential to the face in the 4-8 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 63 Pressure on the 4-8-5-1 face in the global reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.
- 64 Pressure on the 4-8-5-1 face in the local reference system. The three components of the force per unit area on the eight nodes of the face are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangular prism or a tetrahedron.

### Coordinates

Three global coordinates in the x, y and z directions.

### Degrees of freedom

Three degrees of freedom u, v, w (the displacements along the global coordinate directions).

### Output of strains and stresses

1 = xx, 2 = yy, 3 = zz, 4 = xy, 5 = yz, 6 = xz.

### Analysis types

- Infinitesimal elasto-plasticity.
- Masonry-like materials.



## C2. Element 2 (plane stress)

Eight node isoparametric element with quadratic interpolating functions.

### Connectivity

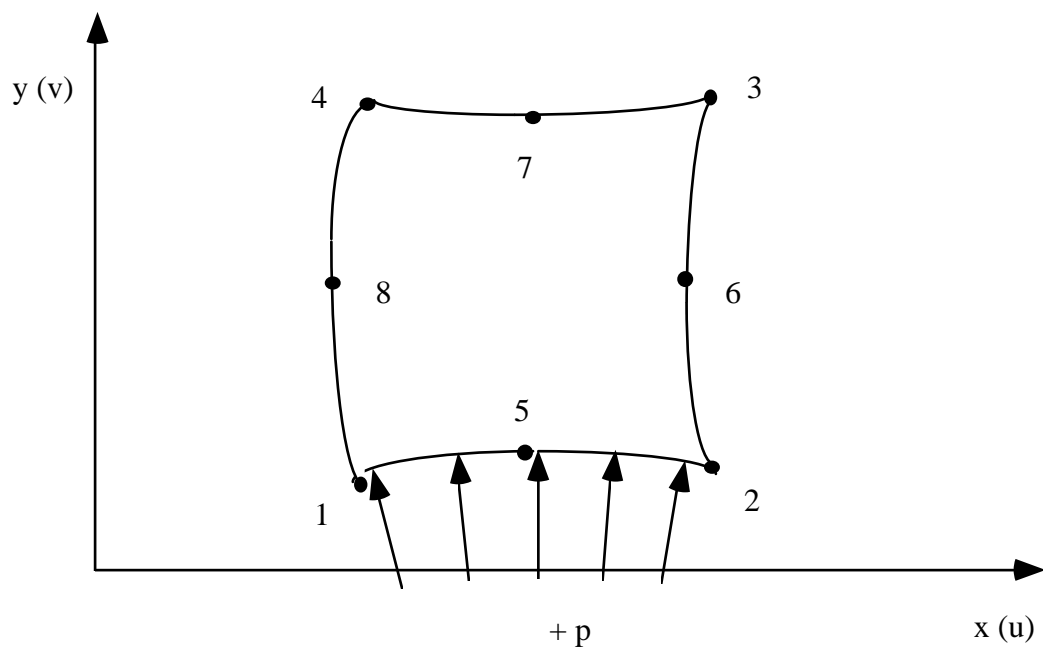
Eight nodes, numbered as shown in Figure 2.1.

### Integration

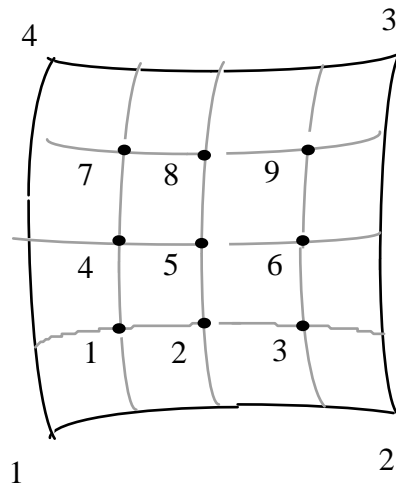
The element is integrated numerically using nine points (Gaussian quadrature). Integration points are shown in Figure 2.2.

### Geometry

The thickness of the element can be specified (default 1).



**Figure 2.1** Element 2.



**Figure 2.2** Integration points of element 2.

### Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The two components of the force per unit volume in the global reference system must be assigned.
2	Body force. The two global components of the force per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis is orthogonal to the plane of the element; the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on the 1-2-5 edge (force per unit area) in the global reference system.
12	Pressure on the 1-2-5 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 1-2 direction.
13	Pressure on the 1-2-5 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
14	Pressure on the 1-2-5 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

- 21 Pressure on the 2-3-6 edge (force per unit area) in the global reference system.
- 22 Pressure on the 2-3-6 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 2-3 direction.
- 23 Pressure on the 2-3-6 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 2-3-6 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 3-4-7 edge (force per unit area) in the global reference system.
- 32 Pressure on the 3-4-7 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 3-4 direction.
- 33 Pressure on the 3-4-7 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 34 Pressure on the 3-4-7 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 4-1-8 edge (force per unit area) in the global reference system.
- 42 Pressure on the 4-1-8 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 4-1 direction.
- 43 Pressure on the 4-1-8 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 44 Pressure on the 4-1-8 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the x and y directions.

### Nodal degrees of freedom

Two degrees of freedom u, v (the displacements along the global coordinate directions).

### Output of strains and stresses (at integration points)

1 = xx, 2 = yy, 3 = xy.

### Analysis types

- Infinitesimal elasto-plasticity.
- Masonry-like materials.



### C3. Element 3 (Plane strain)

Eight-node isoparametric element with quadratic interpolating functions.

#### Connectivity

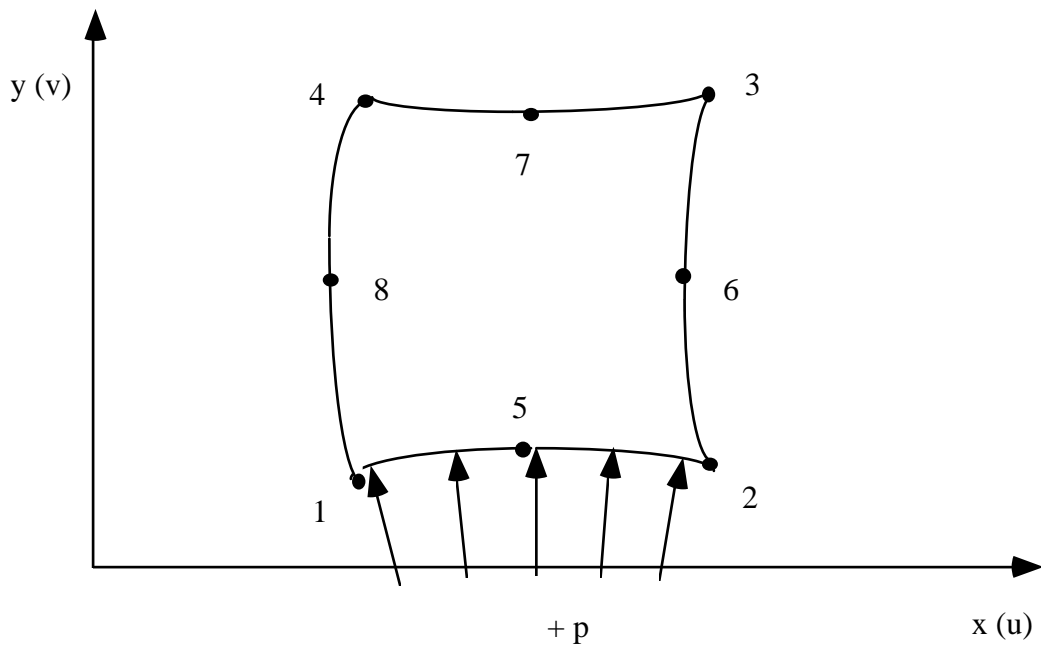
Eight nodes numbered as shown in Figure 3.1.

#### Integration

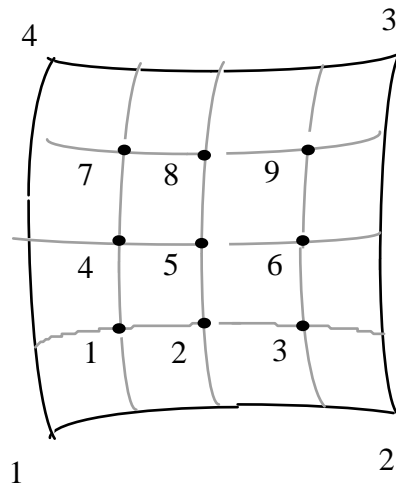
The element is integrated numerically using nine points (Gaussian quadrature). Integration points are shown in Figure 3.2.

#### Geometry

The thickness of the element can be specified (default 1).



**Figure 3.1** Element 3.



**Figure 3.2** Integration points of element 3.

### Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The two components of the force per unit volume in the global reference system must be assigned.
2	Body force. The two global components of the force per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis is orthogonal to the plane of the element; the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on the 1-2-5 edge (force per unit area) in the global reference system.
12	Pressure on the 1-2-5 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 1-2 direction.
13	Pressure on the 1-2-5 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
14	Pressure on the 1-2-5 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

- 21 Pressure on the 2-3-6 edge (force per unit area) in the global reference system.
- 22 Pressure on the 2-3-6 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 2-3 direction.
- 23 Pressure on the 2-3-6 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 2-3-6 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 3-4-7 edge (force per unit area) in the global reference system.
- 32 Pressure on the 3-4-7 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 3-4 direction.
- 33 Pressure on the 3-4-7 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 34 Pressure on the 3-4-7 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 4-1-8 edge (force per unit area) in the global reference system.
- 42 Pressure on the 4-1-8 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 4-1 direction.
- 43 Pressure on the 4-1-8 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 44 Pressure on the 4-1-8 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the x and y directions.

### Nodal degrees of freedom

Two degrees of freedom u, v (the displacements along the global coordinate directions).

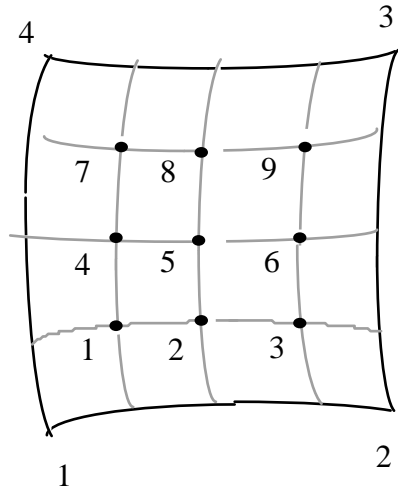
### Output of strains and stresses (at the integration points)

1 = xx, 2 = yy, 3 = xy.

### Analysis types

- Infinitesimal elasto-plasticity.
- Masonry-like materials.





**Figure 4.2** Integration points of elements 4.

### Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The two components of the force per unit volume in the global reference system must be assigned.
2	Body force. The two global components of the force per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis coincides with the axis of symmetry, the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on the 1-2-5 edge (force per unit area) in the global reference system.
12	Pressure on the 1-2-5 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 1-2 direction.
13	Pressure on the 1-2-5 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
14	Pressure on the 1-2-5 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

- 21 Pressure on the 2-3-6 edge (force per unit area) in the global reference system.
- 22 Pressure on the 2-3-6 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 2-3 direction.
- 23 Pressure on the 2-3-6 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 2-3-6 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 3-4-7 edge (force per unit area) in the global reference system.
- 32 Pressure on the 3-4-7 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 3-4 direction.
- 33 Pressure on the 3-4-7 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 34 Pressure on the 3-4-7 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 4-1-8 edge (force per unit area) in the global reference system.
- 42 Pressure on the 4-1-8 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 4-1 direction.
- 43 Pressure on the 4-1-8 edge in the global reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.
- 44 Pressure on the 4-1-8 edge in the local reference system. The two components of the force per unit area on the three nodes of the edge are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the z and r directions.

### Nodal degrees of freedom

Two degrees of freedom u, v (the displacements along the global coordinate directions).

### Output of strains and stresses (at the integration points)

1 = zz, 2 = rr, 3 =  $\vartheta\vartheta$ , 4 = zr.

### Analysis types

- Infinitesimal elasto-plasticity.
- Masonry-like materials.



### C5. Element 5 (Thin shell)

Eight-node isoparametric element, the interpolating functions are linear for displacement and quadratic for rotations.

#### Connectivity

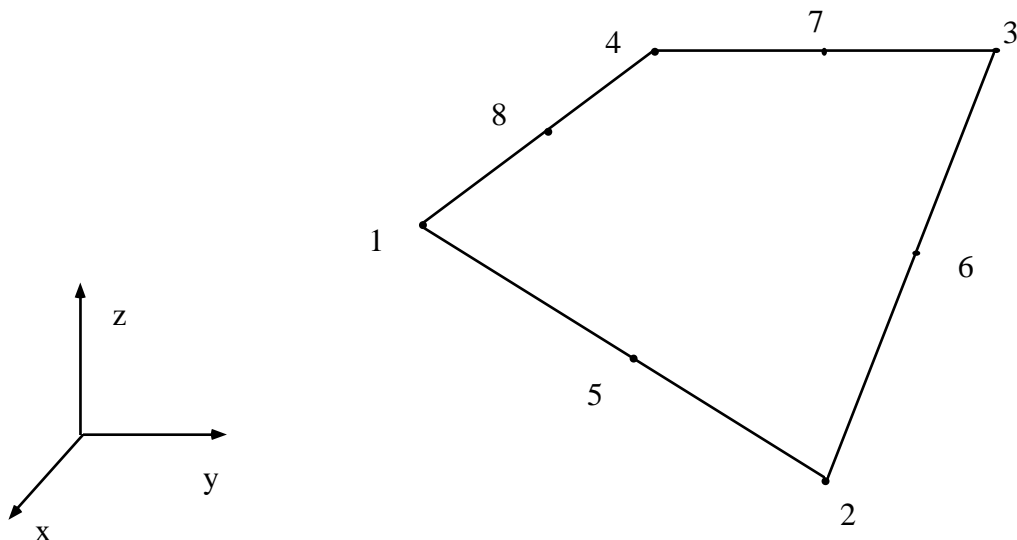
Eight nodes numbered as shown in Figure 5.1.

#### Integration

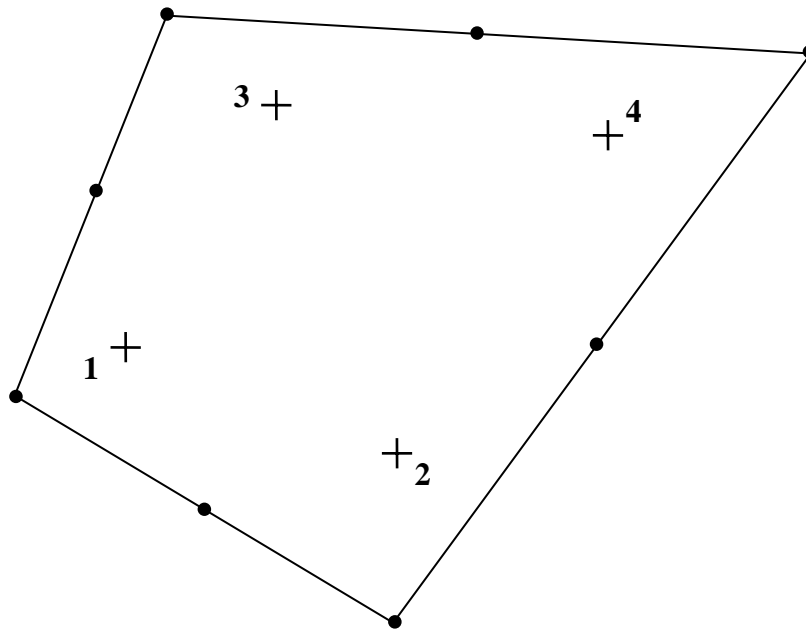
Integrating on the element surface is performed numerically using four-point Gaussian quadrature. The relative integration points are shown in Figure 5.2. Integration within the thickness is also performed numerically using instead the Simpson method, the number of sections into which it is divided being defined in card SHELL SECT (the default value is 1, the maximum number of sections is 99).

#### Geometry

For homogeneous shells the shell thickness must be specified; for non-homogeneous shells the thickness of each section must be specified (COMPOSITE option). The thickness of the whole shell or of each layer can be specified at each corner node by using the user routine UGEOM.



**Figure 5.1** Element 5.



**Figure 5.2** Integration points of element 5.

### Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The three components of the force per unit volume in the global reference system must be assigned.
2	Body force. The three global components of the force per unit volume for each section are calculated in the user subroutine FORCEM.
11	Pressure on the 1-2 edge (force per unit length) in the global reference system.
13	Pressure on the 1-2 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM.
21	Pressure on the 2-3 edge (force per unit length) in the global reference system.
23	Pressure on the 2-3 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM.

- 31            Pressure on the 3-4 edge (force per unit length) in the global reference system.
- 33            Pressure on the 3-4 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM
- 41            Pressure on the 4-1 edge (force per unit length) in the global reference system.
- 43            Pressure on the 4-1 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM
- 51            Pressure on the surface of the element (force per unit area) in the global reference system.
- 52            Pressure on the surface of the element in the local reference system. The first component of the force per unit area is directed tangential to the surface in the 1-2 direction; the second component is directed tangential in the 2-3 direction; the third is directed perpendicular to the element's surface in the positive direction determined by the right-hand rule.
- 53            Pressure on the surface of the element in the global reference system. The three components of the force per unit area on the four nodes of the element are to be specified in the user subroutine FORCEM.
- 54            Pressure on the surface of the element in the local reference system. The three components of the force per unit area on the four nodes of the element are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on corner nodes, concentrated moments on mid-side nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle. In this case the collapsed edge has zero stiffness.

### Nodal coordinates

Three global coordinates in the x, y and z directions. It is not necessary to specify the coordinates of mid-side nodes.

### Nodal degrees of freedom

The corner nodes have three degrees of freedom u, v and w (the displacements along the global coordinate directions); the degree of freedom of the mid-side nodes is the rotation q about the corresponding edge (positive if counterclockwise).

### Output of strains and stresses (at the integration points)

Strains are printed for the four Gauss points of the mean surface of the shell. The components of the strain are given in the local orthonormal reference system s, t, n in the following order:

$$1 = ss, 2 = tt, 3 = st.$$

The three values of stress are printed for the four integration points for each section of the element. The stress components are given in the local orthonormal reference system s, t, n in the following order:

$$1 = ss, 2 = tt, 3 = st.$$

### Analysis types

- Linear elasticity.
- Masonry-like materials.

## C6. Element 6 (Plane strain)

Four-node isoparametric element with bilinear interpolating functions.

### Connectivity

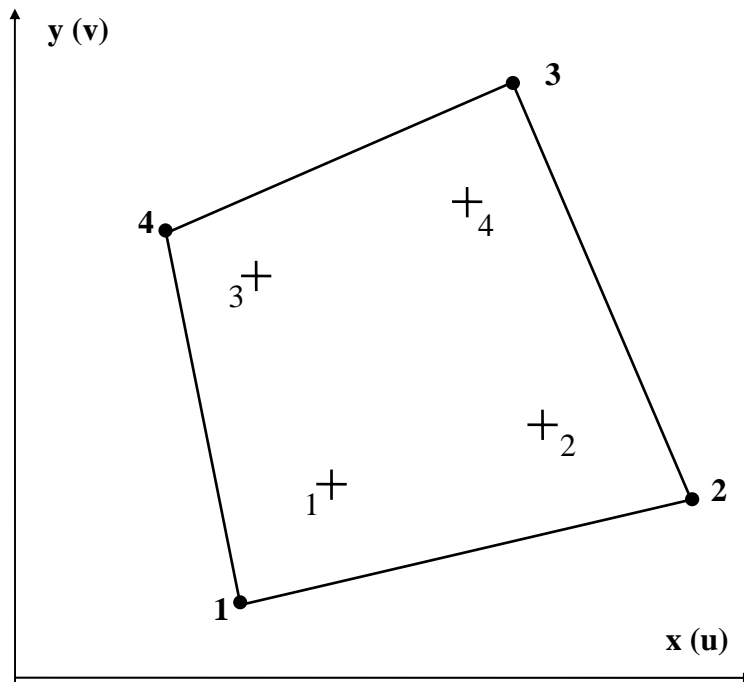
Four nodes numbered as shown in Figure 6.1.

### Integration

The element is integrated numerically using four points (Gaussian quadrature). Integration points are shown in Figure 6.1. If selective reduced integration is required, the hydrostatic part of the strain is calculated using one integration point, namely the centroid of the element.

### Geometry

The thickness of the element must be specified (default 1).



**Fig. 6.1** Element 6.

## Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The two components of the force per unit volume in the global reference system must be assigned.
2	Body force. The two global components of the force per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis is orthogonal to the plane of the element; the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on the 1-2 edge (force per unit area) in the global reference system.
12	Pressure on the 1-2 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 1-2 direction.
13	Pressure on the 1-2 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
14	Pressure on the 1-2 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
21	Pressure on the 2-3 edge (force per unit area) in the global reference system.
22	Pressure on the 2-3 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 2-3 direction.
23	Pressure on the 2-3 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
24	Pressure on the 2-3 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
31	Pressure on the 3-4 edge (force per unit area) in the global reference system.

- 32        Pressure on the 3-4 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 3-4 direction.
- 33        Pressure on the 3-4 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 34        Pressure on the 3-4 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 41        Pressure on the 4-1 edge (force per unit area) in the global reference system.
- 42        Pressure on the 4-1 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 4-1 direction.
- 43        Pressure on the 4-1 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 44        Pressure on the 4-1 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

#### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

#### Nodal coordinates

Two global coordinates in the x and y directions.

#### Nodal degrees of freedom

Two degrees of freedom u, v, the displacements along the global coordinate directions.

Output of strains and stresses (at the integration points)

1 = xx, 2 = yy, 3 = xy.

Analysis types

- Infinitesimal elasto-plasticity.
- Finite elasto-plasticity.
- Masonry-like materials.



### C7. Element 7 (Axisymmetric quadrilateral)

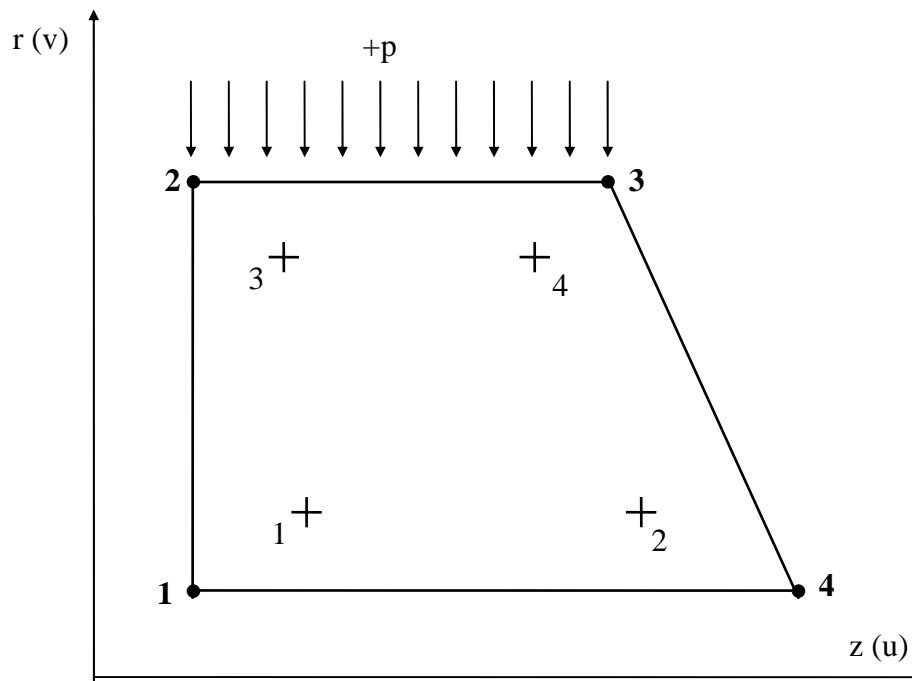
Four-node isoparametric element with bilinear interpolating function.

#### Connectivity

Four nodes numbered as shown in Figure 7.1.

#### Integration

The element is integrated numerically using four points (Gaussian quadrature). Integration points are shown in Figure 7.1. If selective reduced integration is required, the hydrostatic part of the strain is calculated using one integration point, namely the centroid of the element.



**Fig. 7.1** Element 7

#### Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The two components of the force per unit volume in the global reference system must be assigned.

- 2 Body force. The two global components of the force per unit volume are calculated in the user subroutine FORCEM.
- 3 Centrifugal force. The rotation axis is the axis of symmetry, the angular velocity must be specified in the field reserved for the force magnitudes.
- 11 Pressure on the 1-2 edge (force per unit area) in the global reference system.
- 12 Pressure on the 1-2 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 1-2 direction.
- 13 Pressure on the 1-2 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 14 Pressure on the 1-2 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 21 Pressure on the 2-3 edge (force per unit area) in the global reference system.
- 22 Pressure on the 2-3 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 2-3 direction.
- 23 Pressure on the 2-3 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 2-3 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 3-4 edge (force per unit area) in the global reference system.
- 32 Pressure on the 3-4 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 3-4 direction.
- 33 Pressure on the 3-4 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.

- 34 Pressure on the 3-4 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 4-1 edge (force per unit area) in the global reference system.
- 42 Pressure on the 4-1 edge in the local reference system. The first component of the force per unit area is orthogonal to the edge and is positive if directed towards the element's interior; the second component is tangential to the edge in the 4-1 direction.
- 43 Pressure on the 4-1 edge in the global reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.
- 44 Pressure on the 4-1 edge in the local reference system. The two components of the force per unit area on the two nodes of the edge are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

#### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

#### Nodal coordinates

Two global coordinates in the z and r directions.

#### Nodal degrees of freedom

Two degrees of freedom u, v, the displacements along the global coordinate directions.

#### Output of strains and stresses (at the integration points)

1 = zz, 2 = rr, 3 =  $\theta\theta$ , 4 = zr.

### Analysis types

- Infinitesimal elasto-plasticity.
- Finite elasto-plasticity.
- Masonry-like materials.

### C8. Element 8 (three-dimensional element)

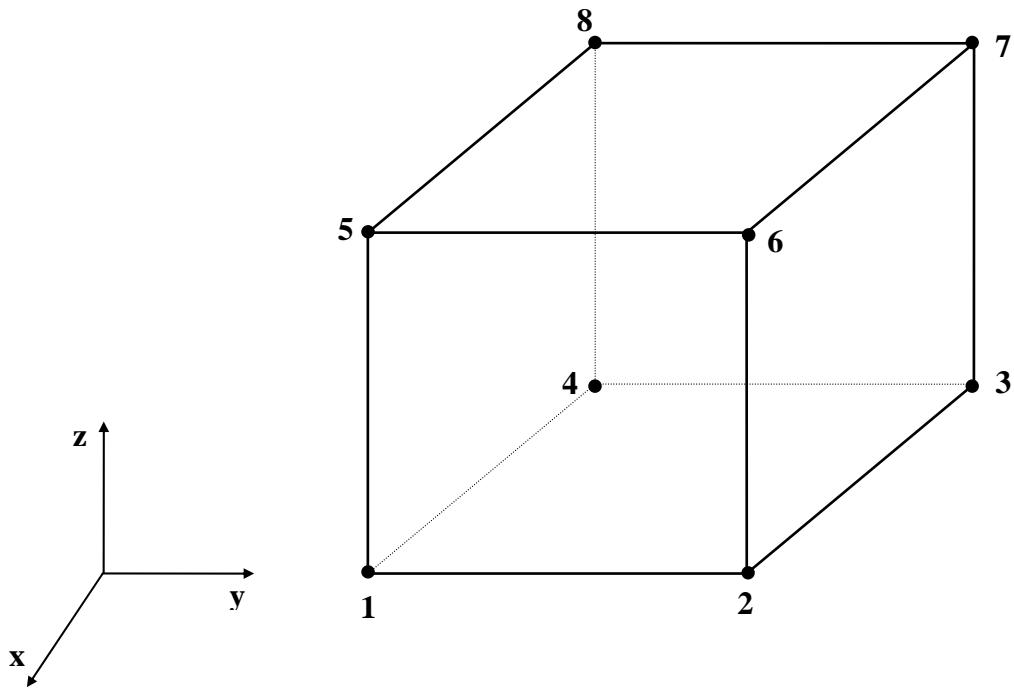
Eight-node isoparametric element with bilinear interpolating functions. This element is suitable for three-dimensional analysis with finite strains.

#### Connectivity

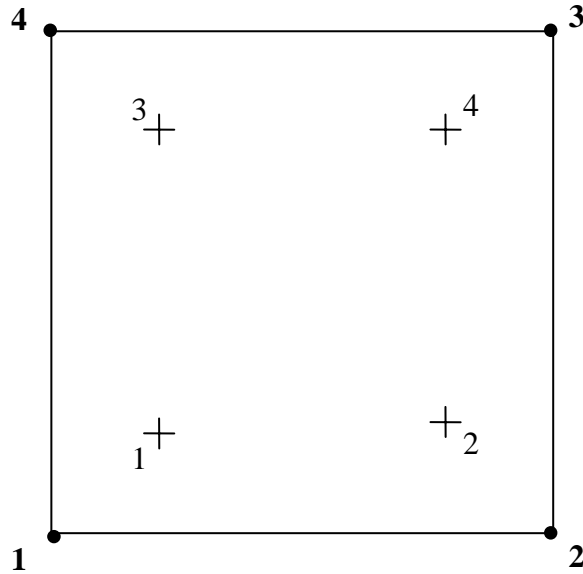
Eight nodes numbered as shown in Figure 8.1.

#### Integration

The element is integrated numerically using eight points (Gaussian quadrature). Integration points are shown in Figure 8.2. If selective reduced integration is required, the hydrostatic part of the strain is calculated using one integration point, namely the centroid of the element.



**Fig. 8.1** Element 8.



**Figure 8.2** Integration points for element 8.

Distributed loads

Distributed loads chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The three components of the force per unit volume in the global reference system must be assigned.
2	Body force. The three global components of the force per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis must be defined in the card ROTATION AXIS, the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on the 1-2-3-4 face (load per unit area) in the global reference system.
12	Pressure on the 1-2-3-4 face in the local reference system. The first component of the force per unit area is tangential to the face in the 1-2 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
13	Pressure on the 1-2-3-4 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.

- 14 Pressure on the 1-2-3-4 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 21 Pressure on the 5-8-7-6 face (load per unit area) in the global reference system.
- 22 Pressure on the 5-8-7-6 face in the local reference system. The first component of the force per unit area is tangential to the face in the 5-8 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 23 Pressure on the 5-8-7-6 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 24 Pressure on the 5-8-7-6 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 31 Pressure on the 1-5-6-2 face (load per unit area) in the global reference system.
- 32 Pressure on the 1-5-6-2 face in the local reference system. The first component of the force per unit area is tangential to the face in the 1-5 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 33 Pressure on the 1-5-6-2 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 34 Pressure on the 1-5-6-2 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 41 Pressure on the 2-6-7-3 face (load per unit area) in the global reference system.
- 42 Pressure on the 2-6-7-3 face in the local reference system. The first component of the force per unit area is tangential to the face in the 2-6 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 43 Pressure on the 2-6-7-3 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.

- 44 Pressure on the 2-6-7-3 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 51 Pressure on the 3-7-8-4 face (load per unit area) in the global reference system.
- 52 Pressure on the 3-7-8-4 face in the local reference system. The first component of the force per unit area is tangential to the face in the 3-7 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 53 Pressure on the 3-7-8-4 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 54 Pressure on the 3-7-8-4 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 61 Pressure on the 4-8-5-1 face (load per unit area) in the global reference system.
- 62 Pressure on the 4-8-5-1 face in the local reference system. The first component of the force per unit area is tangential to the face in the 4-8 direction, the third component is orthogonal to the face and is positive if directed towards the element's interior; the second component has the direction given by the vector product of the first and third directions.
- 63 Pressure on the 4-8-5-1 face in the global reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.
- 64 Pressure on the 4-8-5-1 face in the local reference system. The three components of the force per unit area on the four nodes of the face are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads on nodes and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a tetrahedron.



### Nodal coordinates

Three global coordinates in the x, y and z directions.

### Nodal degrees of freedom

Three degrees of freedom u, v, w, the displacements along the global coordinate directions.

### Output of strains and stresses (at the integration points)

1 = xx, 2 = yy, 3 = zz, 4 = xy, 5 = yz, 6 = xz.

### Analysis types

- Infinitesimal elasto-plasticity.
- Finite elasto-plasticity.
- Masonry-like materials.



## C9. Element 9 (Beam)

Two-node isoparametric element, the interpolating functions are linear for displacements and rotations.

### Connectivity

Two nodes numbered as shown in Figure 9.1. Moreover, a local reference system  $e_1$ ,  $e_2$ ,  $e_3$ , have to be specified, by using the AXIS option ( $e_1$  and  $e_2$  define the principal directions on the cross section and  $e_3$  is in the axial direction, from node 1 to node 2).

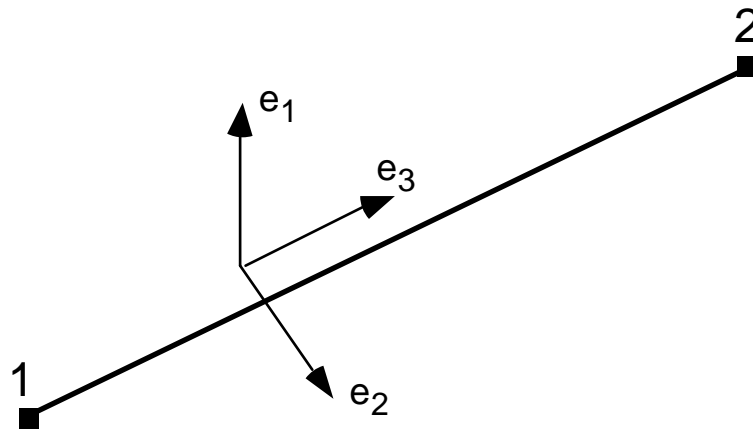


Fig. 9.1 Layout of the beam element type 9.

### Integration

The integration along the element axis is performed numerically using two-point Gaussian quadrature. Integration on the cross section is also performed numerically using instead the Simpson method, the number of fibers into which it is divided being defined in card BEAM SECT (the default value is 3x3, the maximum number of fibers is 21x21).

### Geometry

By default, the beam is supposed to have a rectangular cross section, so for homogeneous beams the thickness, along  $e_1$  and  $e_2$  directions, must be specified; for non-homogeneous beams the thickness of each fiber must be specified following the order shown in Fig. 9.2. Is

to be remarked that it is possible to give the fiber thicknesses on a nodal basis by using the user routine UGEOM.

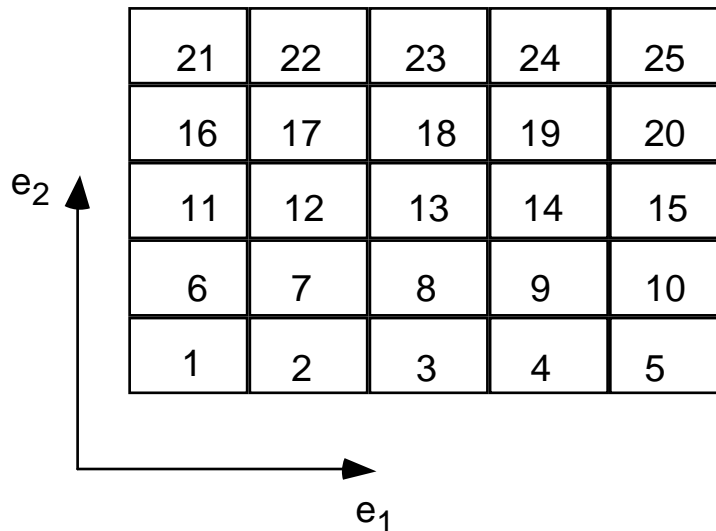


Fig. 9.2. Numbering scheme for the fibers of the beam on a 5x5 example.

### Distributed loads

Distributed loads chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force and/or moment. The three components of the force and moment per unit volume in the global reference system must be assigned.
2	Body force and/or moment. The three global components of the force and moment per unit volume are calculated in the user subroutine FORCEM.
3	Centrifugal force. The rotation axis must be defined in the card ROTATION AXIS, the angular velocity must be specified in the field reserved for the force magnitudes.
11	Pressure on beam axis (load per unit length) in the global reference system.
12	Pressure on the beam axis in the local reference system $e_1, e_2, e_3$ .
13	Pressure on the beam axis in the global reference system. The components of the load per unit length at the two nodes of the beam are to be specified in the user subroutine FORCEM.

- 14 Pressure on the beam axis in the local reference system. The components of the load per unit length at the two nodes of the beam are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads and/or concentrated moments on nodes. Thermal loads are not yet available.

#### Nodal coordinates

Three global coordinates in the x, y and z directions.

#### Nodal degrees of freedom

Six degrees of freedom  $u, v, w$  (the displacements along the global coordinate directions) and  $q_x, q_y, q_z$  (the rotations, counterclockwise positive, around the global coordinate directions).

#### Output of strains and stresses characteristics (at the integration points)

Strains and stress characteristics are printed for the two Gauss points. The components of the strain characteristics are given in the local reference system  $e_1, e_2, e_3$  in the following order :

- 1)  $\varepsilon$  axial stretch,
- 2)  $k_1$  curvature change around  $e_1$  axis,
- 3)  $k_2$  curvature change around  $e_2$  axis,
- 4)  $\psi$  twist around  $e_3$  axis,
- 5)  $\gamma_1$  shear deformation on the  $e_1$ - $e_3$  plane,
- 6)  $\gamma_2$  shear deformation on the  $e_2$ - $e_3$  plane.

Analogously, for the stress characteristics we have:

- 1)  $N$  axial force,
- 2)  $M_1$  moment around  $e_1$  axis,
- 3)  $M_2$  moment around  $e_2$  axis,
- 4)  $M_3$  torque around  $e_3$  axis,
- 5)  $Q_1$  shear force on the  $e_1$ - $e_3$  plane,
- 6)  $Q_2$  shear force on the  $e_2$ - $e_3$  plane.

#### Analysis types

- Linear elasticity.
- Masonry-like materials.



### **C10. Element 10** (Thick shell)

Four-node isoparametric element, the interpolating functions are linear for displacements and rotations.

#### Connectivity

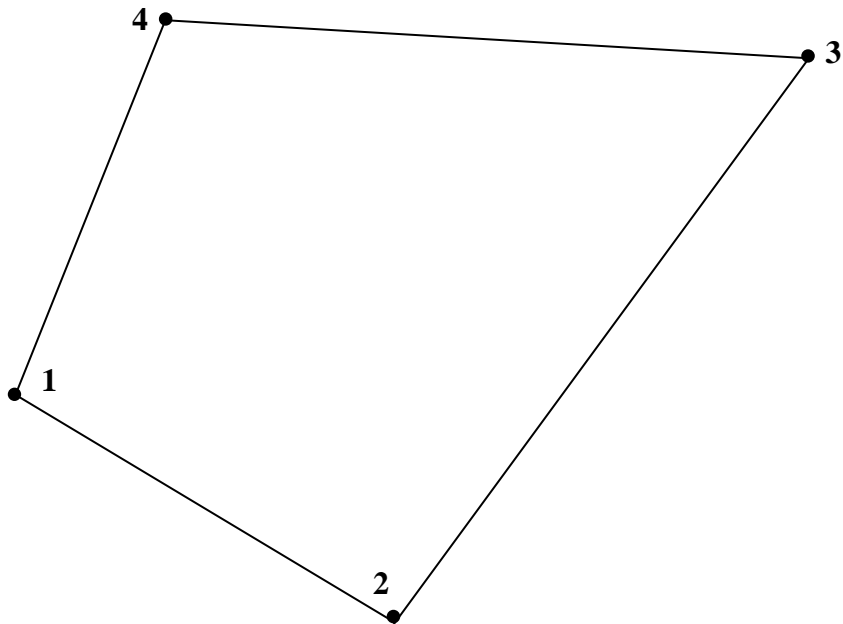
Four nodes numbered as shown in Figure 10.1.

#### Integration

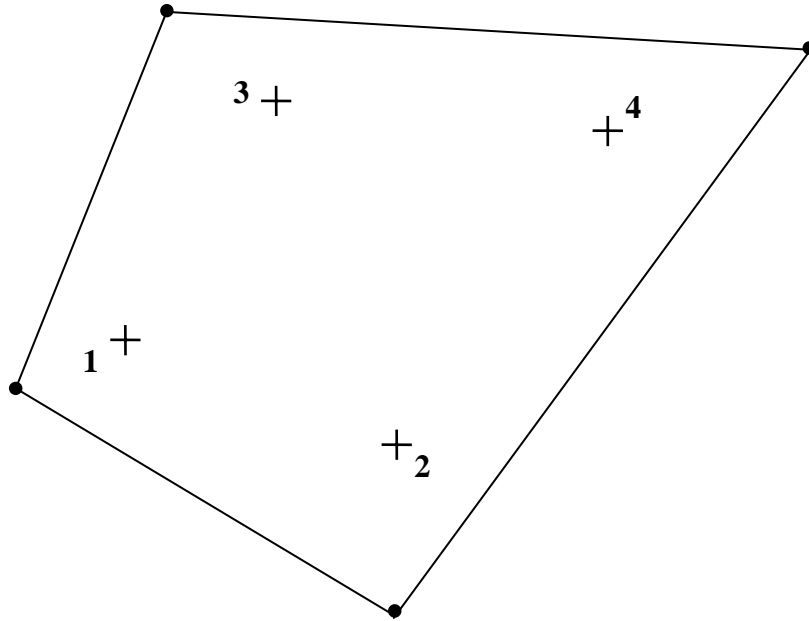
Integrating on the element surface is performed numerically using four-point Gaussian quadrature. The relative integration points are shown in Figure 10.2. Integration within the thickness is also performed numerically using instead the Simpson method, the number of sections into which it is divided being defined in card SHELL SECT (the default value is 3, the maximum number of sections is 99).

#### Geometry

For homogeneous shells the shell thickness must be specified; for non-homogeneous shells the thickness of each section must be specified (COMPOSITE option). The thickness of the whole shell or of each layer can be specified at each node by using the user routine UGEOM.



**Figure 10.1** Element 10.



**Figure 10.2** Integration points of element 10.

Distributed loads

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Body force. The three components of the force per unit volume in the global reference system must be assigned.
2	Body force. The three global components of the force per unit volume for each section are calculated in the user subroutine FORCEM.
11	Pressure on the 1-2 edge (force per unit length) in the global reference system.
13	Pressure on the 1-2 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM.
21	Pressure on the 2-3 edge (force per unit length) in the global reference system.
23	Pressure on the 2-3 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM.
31	Pressure on the 3-4 edge (force per unit length) in the global reference system.



- 33 Pressure on the 3-4 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM
- 41 Pressure on the 4-1 edge (force per unit length) in the global reference system.
- 43 Pressure on the 4-1 edge in the global reference system. The three components of the force per unit length on the two nodes of the edge are to be specified in the user subroutine FORCEM
- 51 Pressure on the surface of the element (force per unit area) in the global reference system.
- 52 Pressure on the surface of the element in the local reference system. The first component of the force per unit area is directed tangential to the surface in the 1-2 direction; the second component is directed tangential in the 2-3 direction; the third is directed perpendicular to the element's surface in the positive direction determined by the right-hand rule.
- 53 Pressure on the surface of the element in the global reference system. The three components of the force per unit area on the four nodes of the element are to be specified in the user subroutine FORCEM.
- 54 Pressure on the surface of the element in the local reference system. The three components of the force per unit area on the four nodes of the element are to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point loads and/or concentrated moments on nodes, and to thermal dilatation loads.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Three global coordinates in the x, y and z directions.

### Nodal degrees of freedom

Six degrees of freedom  $u$ ,  $v$ ,  $w$  (the displacements along the global coordinate directions) and  $q_x$ ,  $q_y$ ,  $q_z$  (the rotations, counterclockwise positive, around the global coordinate directions).

### Output of strains and stresses (at the integration points)

Strains are printed for the four Gauss points of the mean surface of the shell. The components of the strain are given in the local orthonormal reference system s, t, n in the following order :

1 = ss, 2 = tt, 3 = st, 4 = tn, 5 = sn.

The three values of stress are printed for the four integration points for each section of the element. The stress components are given in the local orthonormal reference system s, t, n in the following order :

1 = ss, 2 = tt, 3 = st, 4 = tn, 5 = sn.

### Analysis types

- Linear elasticity,
- Masonry-like materials.

### **C11. Element 11** (Plane heat transfer)

Eight-node isoparametric element with quadratic interpolating functions (geometrically it is identical to elements 2 and 3).

#### Connectivity

Eight nodes numbered in anticlockwise order.

#### Integration

The element is integrated numerically using nine integration points (Gaussian quadrature).

#### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Surface flux. The heat quantity per unit area and unit time must be assigned.
2	Surface flux. The heat quantity per unit area and unit time is calculated in the user subroutine FORCEM.
11	Edge flux on the 1-2-5 edge (heat per unit length and unit time).
13	Edge flux on the 1-2-5 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
21	Edge flux on the 2-3-6 edge (heat per unit length and unit time).
23	Edge flux on the 2-3-6 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
31	Edge flux on the 3-4-7 edge (heat per unit length and unit time).
33	Edge flux on the 3-4-7 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
41	Edge flux on the 4-1-8 edge (heat per unit length and unit time).
43	Edge flux on the 4-1-8 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the x and y directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y.

### Analysis types

- Linear and non-linear transient heat transfer analysis.

## **C12. Element 12** (Plane heat transfer)

Four-node isoparametric element with linear interpolating functions (geometrically it is identical to element 6 ).

### Connectivity

Four nodes numbered in anticlockwise order.

### Integration

The element is integrated numerically using four integration points (Gaussian quadrature).

### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Surface flux. The heat quantity per unit area and unit time must be assigned.
2	Surface flux. The heat quantity per unit area and unit time is calculated in the user subroutine FORCEM.
11	Edge flux on the 1-2 edge (heat per unit length and unit time).
13	Edge flux on the 1-2 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
21	Edge flux on the 2-3 edge (heat per unit length and unit time).
23	Edge flux on the 2-3 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
31	Edge flux on the 3-4 edge (heat per unit length and unit time).
33	Edge flux on the 3-4 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
41	Edge flux on the 4-1 edge (heat per unit length and unit time).
43	Edge flux on the 4-1 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global co-ordinates in the x and y directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y.

### Analysis types

- Linear and non-linear transient heat transfer analysis.

### **C13. Element 13** (Axisymmetric heat transfer)

Eight-node isoparametric element with quadratic interpolating functions (geometrically it is identical to element 4).

#### Connectivity

Eight nodes numbered in anticlockwise order.

#### Integration

The element is integrated numerically using nine integration points (Gaussian quadrature).

#### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Surface flux. The heat quantity per unit area and unit time must be assigned.
2	Surface flux. The heat quantity per unit area and unit time is calculated in the user subroutine FORCEM.
11	Edge flux on the 1-2-5 edge (heat per unit length and unit time).
13	Edge flux on the 1-2-5 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
21	Edge flux on the 2-3-6 edge (heat per unit length and unit time).
23	Edge flux on the 2-3-6 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
31	Edge flux on the 3-4-7 edge (heat per unit length and unit time).
33	Edge flux on the 3-4-7 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
41	Edge flux on the 4-1-8 edge (heat per unit length and unit time).
43	Edge flux on the 4-1-8 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the x and y directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y.

### Analysis types

- Linear and non-linear transient heat transfer analysis.



#### **C14. Element 14** (Axisymmetric heat transfer)

Four-node isoparametric element with linear interpolating functions (geometrically it is identical to element 7 ).

##### Connectivity

Four nodes numbered in anticlockwise order.

##### Integration

The element is integrated numerically using four integration points (Gaussian quadrature).

##### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Surface flux. The heat quantity per unit area and unit time must be assigned.
2	Surface flux. The heat quantity per unit area and unit time is calculated in the user subroutine FORCEM.
11	Edge flux on the 1-2 edge (heat per unit length and unit time).
13	Edge flux on the 1-2 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
21	Edge flux on the 2-3 edge (heat per unit length and unit time).
23	Edge flux on the 2-3 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
31	Edge flux on the 3-4 edge (heat per unit length and unit time).
33	Edge flux on the 3-4 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.
41	Edge flux on the 4-1 edge (heat per unit length and unit time).
43	Edge flux on the 4-1 edge. The heat per unit length and unit time on the three nodes of the edge is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Two global coordinates in the x and y directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y.

### Analysis types

- Linear and non-linear transient heat transfer analysis.

### **C15. Element 15** (3D heat transfer)

Eighth-node isoparametric element with linear interpolating functions (geometrically it is identical to element 8 ).

#### Connectivity

8 nodes numbered as in Fig. 8.1.

#### Integration

The element is integrated numerically using 8 integration points (Gaussian quadrature).

#### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Volumetric flux. The heat quantity per unit volume and unit time must be assigned.
2	Volumetric flux. The heat quantity per unit volume and unit time is calculated in the user subroutine FORCEM.
11	Surface flux on the 1-2-3-4 face (heat per unit area and unit time).
13	Surface flux on the 1-2-3-4 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
21	Surface flux on the 5-6-7-8 face (heat per unit area and unit time).
23	Surface flux on the 5-6-7-8 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
31	Surface flux on the 1-5-6-2 face (heat per unit area and unit time).
33	Surface flux on the 1-5-6-2 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
41	Surface flux on the 2-6-7-3 face (heat per unit area and unit time).
43	Surface flux on the 2-6-7-3 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
51	Surface flux on the 3-7-8-4 face (heat per unit area and unit time).
53	Surface flux on the 3-7-8-4 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.

- 61 Surface flux on the 4-8-5-1 face (heat per unit area and unit time).
- 63 Surface flux on the 4-8-5-1 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangular prism or to a tetrahedron.

### Nodal coordinates

Three global coordinates in the x, y and z directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y, 3 = z.

### Analysis types

- Linear and non-linear transient heat transfer analysis.

## **C16. Element 16** (3D heat transfer)

Twenty-node isoparametric element with quadratic interpolating functions (geometrically it is identical to element 1 ).

### Connectivity

Twenty nodes numbered as in Fig. 1.1.

### Integration

The element is integrated numerically using twenty-seven integration points (Gaussian quadrature).

### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Volumetric flux. The heat quantity per unit volume and unit time must be assigned.
2	Volumetric flux. The heat quantity per unit volume and unit time is calculated in the user subroutine FORCEM.
11	Surface flux on the 1-2-3-4 face (heat per unit area and unit time).
14	Surface flux on the 1-2-3-4 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
21	Surface flux on the 5-6-7-8 face (heat per unit area and unit time).
23	Surface flux on the 5-6-7-8 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
31	Surface flux on the 1-5-6-2 face (heat per unit area and unit time).
33	Surface flux on the 1-5-6-2 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
41	Surface flux on the 2-6-7-3 face (heat per unit area and unit time).
43	Surface flux on the 2-6-7-3 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
51	Surface flux on the 3-7-8-4 face (heat per unit area and unit time).

- 53 Surface flux on the 3-7-8-4 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.
- 61 Surface flux on the 4-8-5-1 face (heat per unit area and unit time).
- 63 Surface flux on the 4-8-5-1 face. The heat per unit area and unit time on the four nodes of the face is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangular prism or to a tetrahedron.

### Nodal coordinates

Three global coordinates in the x, y and z directions.

### Nodal degrees of freedom

One degree of freedom T (the nodal temperature).

### Output of temperature gradient (at integration points)

1 = x, 2 = y, 3 = z.

### Analysis types

- Linear and non-linear transient heat transfer analysis.

### **C17. Element 17** (shell heat transfer)

Four-node isoparametric element with linear interpolating functions (geometrically it is identical to element 10 ).

#### Connectivity

Four nodes numbered as in Fig. 10.1.

#### Integration

Integrating on the element surface is performed numerically using four-point Gaussian quadrature. The relative integration points are shown in Figure 10.2. Integration within the thickness is also performed numerically using instead the Simpson method, the number of sections into which it is divided being defined in card SHELL SECT (the default value is 3, the maximum number of sections is 99).

#### Geometry

For homogeneous shells the shell thickness must be specified; for non-homogeneous shells the thickness of each section must be specified (COMPOSITE option). The thickness of the whole shell or of each layer can be specified at each node by using the user routine UGEOM.

#### Distributed fluxes

Distributed loads are chosen by value of IBODY, as follows:

IBODY	DESCRIPTION
1	Volumetric flux. The heat quantity per unit volume and unit time must be assigned.
2	Volumetric flux. The heat quantity per unit volume and unit time is calculated in the user subroutine FORCEM.
11	Edge flux on the 1-2 edge (heat per unit length and unit time).
13	Edge flux on the 1-2 edge. The heat per unit length and unit time on the two nodes of the edge is to be specified in the user subroutine FORCEM.
21	Edge flux on the 2-3 edge (heat per unit length and unit time).
23	Edge flux on the 2-3 edge. The heat per unit length and unit time on the two nodes of the edge is to be specified in the user subroutine FORCEM.
31	Edge flux on the 3-4 edge (heat per unit length and unit time).

- 33 Edge flux on the 3-4 edge. The heat per unit length and unit time on the two nodes of the edge is to be specified in the user subroutine FORCEM.
- 41 Edge flux on the 4-1 edge (heat per unit length and unit time).
- 43 Edge flux on the 4-1 edge. The heat per unit length and unit time on the two nodes of the edge is to be specified in the user subroutine FORCEM.
- 51 Surface flux (heat per unit area and unit time).
- 53 Surface flux. The heat per unit area and unit time on the four nodes is to be specified in the user subroutine FORCEM.

Moreover, the element may be subjected to point fluxes on nodes and convective fluxes by specifying the film coefficients and sink temperatures for each edge.

### Collapse

By suitably repeating node numbers in the connectivity, the element may be reduced as far as a triangle.

### Nodal coordinates

Three global coordinates in the x, y and z directions.

### Nodal degrees of freedom

ITYRD degrees of freedom (temperatures) per node (the value of the ITYRD parameter is specified into the HEAT TRANSFER option)

ITYRD=2 (linear distribution through the thickness)

- 1 Top surface temperature
- 2 Bottom surface temperature

ITYRD=3 (parabolic distribution through the thickness)

- 1 Top surface temperature
- 2 Bottom surface temperature
- 3 Mid surface temperature.

### Output of temperature gradient (at integration points)

The components of the temperature gradient are given in the local reference system of the shell. Namely, the first two components are along the local tangent direction, whereas the third component is along the shell normal unit vector.



### Analysis types

- Linear and non-linear transient heat transfer analysis.



