Article

Acoustic Plasmons in Graphene Sandwiched between Two Metallic Slabs

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Abstract: We study the effect of two metallic slabs on the collective dynamics of electrons in graphene positioned between the two slabs. We show that if the slabs are perfect conductors, the plasmons of graphene display a linear dispersion relation. The velocity of these acoustic plasmons crucially depends on the distance between the two metal gates and the graphene sheet. In the case of generic slabs, the dispersion relation of graphene plasmons is much more complicated, but we find that acoustic plasmons can still be obtained under specific conditions.

Keywords: graphene; quantum many-body theory; acoustic plasmons

1. Introduction

In 2004, Graphene, a single layer of carbon atoms arranged in a two-dimensional honeycomb lattice, was isolated and characterized [1]. Since then, many electrical, thermal, chemical, optical, and mechanical properties of graphene have been studied, both experimentally and theoretically [2–5]. Quite remarkably, under appropriate conditions, the electrons in graphene behave as viscous fluids, exhibiting peculiar hydrodynamic effects [6]. In particular, it has been shown that the plasmons of graphene display a linear dispersion relation when, in the proximity of the graphene, a metallic slab screens the Coulomb potential of electrons in graphene [7,8].

In this brief communication, we extend the predictions obtained in [7] by considering a graphene sheet sandwiched between two metallic slabs. We find that, in this case, the electrons of graphene are characterized by acoustic modes whose dispersion relation is linear in the long-wavelength regime. We obtain a simple analytical formula for the speed of these acustic modes.

2. Graphene Sandwiched between Two Materials

The monolayer graphene is a honeycomb lattice of carbon atoms in two spatial dimensions. Quasiparticles in graphene have the dispersion relation

$$E_{\mathbf{k}} = \pm v_F \hbar |\mathbf{k}| - \mu \,, \tag{1}$$

where v_F is the Fermi velocity, $\mathbf{k}$ is the two-dimensional (2D) quasiparticle wavevector, and μ the chemical potential. The Fermi wavenumber k_F depends on the chemical potential μ through the relation $k_F = \mu/(\hbar v_F)$. Note that, in 2D, $k_F = \sqrt{4\pi n/g}$, with n as the electron number density and g as the degeneracy. In graphene, $g = 4$: 2 for spin and 2, for inequivalent valleys in the Brillouin zone, and the chemical potential $\mu = \hbar v_F k_F$ is usually $\mu \simeq 10^2$ meV, while the Fermi velocity is $v_F \simeq 10^6$ m/s [2,3,9].

We initially assume that the graphene is sandwiched between two slabs made of generic materials, where L is the distance between the two slabs and d the distance between

the lower slab and the graphene sheet. The Coulomb potential of charges in graphene is influenced by the the two slabs. We choose the z axis perpendicular to the graphene sheet, such that $z = 0$ fixes the position of graphene sheet. It follows that the lower slab is located at $z = -d$ and the upper slab at $z = L - d$.

Within the Random Phase Approximation (RPA) [10], the relative dielectric function of graphene is given by

$$\epsilon_g(q,\omega) = 1 - \tilde{V}(q,\omega)\,\Pi_0(q,\omega)\,, \tag{2}$$

where $\tilde{V}(q,\omega)$ is the Fourier transform of the screened (by the presence of the two slabs) Coulomb potential between quasiparticles of graphene and $\Pi_0(q,\omega)$ is the first-order dynamical polarization of non-interacting quasiparticles in graphene. Note that, for a very small wavenumber, $q = \sqrt{q_x^2 + q_y^2}$ and a frequency ω, such that $v_F q \ll \omega \ll 2\mu/\hbar$ the dynamical polarization reads [11,12]

$$\Pi_0(q,\omega) = \frac{\mu}{\pi\hbar^2}\frac{q^2}{\omega^2}\,. \tag{3}$$

The collective mode of plasmons in graphene is then obtained from the resonance condition [10]

$$\epsilon_g(q,\omega) = 0\,. \tag{4}$$

3. Perfect Conductors

Let us suppose that the two slabs are perfect conductors. A straightforward application of the method of image charges [13] gives the screened Coulomb potential between two particles, with electric charge e located in the plane $z = 0$ at distance $x^2 + y^2$ as

$$V(x,y) = e^2 \sum_{j=-\infty}^{+\infty}\left[\frac{1}{\sqrt{(x^2+y^2)+(2jL)^2}} - \frac{1}{\sqrt{(x^2+y^2)+(2d-2jL)^2}}\right], \tag{5}$$

where x and y are Cartesian coordinates in the plane of graphene. Performing the Fourier transform, we obtain

$$\tilde{V}(q) = \frac{2\pi e^2}{q}\sum_{j=-\infty}^{+\infty}\left[e^{-2q|j|L} - e^{-2q|d-jL|}\right] \tag{6}$$

with $q = \sqrt{q_x^2 + q_y^2}$. The series can be calculated explicity because is the sum of geometric series. After straightforward calculations, we obtain

$$\tilde{V}(q) = \frac{2\pi e^2}{q}\left(1 - \frac{e^{-2qd} - 2e^{-2qL} + e^{-2q(L-d)}}{1 - e^{-2qL}}\right)\,. \tag{7}$$

From Equations (2)–(4) the dispersion relation of plasmons in graphene can be written as

$$\omega^2 = \frac{\mu}{\pi\hbar^2}q^2\tilde{V}(q)\,, \tag{8}$$

or explicitly

$$\omega = \sqrt{\frac{2\mu e^2}{\hbar^2}}\,q^{1/2}\left(1 - \frac{e^{-2qd} - 2e^{-2qL} + e^{-2q(L-d)}}{1 - e^{-2qL}}\right)^{1/2}\,. \tag{9}$$

Thus, we have found an analytical formula for the dispersion relation of plasmons in the graphene sheet.

It is important to observe that, for small q, one obtains

$$\tilde{V}(q) = 4\pi e^2 d\left(1 - \frac{d}{L}\right) - \frac{4\pi e^2}{3}d^2 L\left(1 - 2\frac{d}{L} + \frac{d^2}{L^2}\right)q^2 + \dots \tag{10}$$

Consequently, taking into account Equations (8) and (10), we finally obtain, for small q, the linear dispersion relation

$$\omega = c_p\, q\,,\tag{11}$$

where

$$c_p = \sqrt{\frac{4\mu e^2 d}{\hbar^2}\left(1 - \frac{d}{L}\right)}\tag{12}$$

is the speed of sound of acustic plasmons in graphene sandwiched between two ideal metal gates. Equation (12) is the main result of this brief paper. The velocity c_p can be controlled by varying the chemical potential μ but also the two distances d and L. In the limit $L \to +\infty$ from Equation (12), one finds

$$c_p = \sqrt{\frac{4\mu e^2 d}{\hbar^2}}\tag{13}$$

which is the result of Reference [7], namely, the velocity of acustic plasmons in graphene coupled to a single ideal metal gate.

4. Real Materials

For a generic material, the relative dielectric function ϵ_m depends on frequency ω and wavevector q. We set $\epsilon_{m,1}(q,\omega)$ and $\epsilon_{m,2}(q,\omega)$ as the relative dielectric functions of lower and upper materials, respectively. In this case, the derivation of the screened Coulomb potential is slightly more complicated but still analytically possible [13,14]. We obtain

$$\tilde{V}(q,\omega) = \frac{2\pi e^2}{q}\left(1 - \frac{r_1(q,\omega)\,e^{-2qd} + r_2(q,\omega)\,e^{-2q(L-d)}}{1 - r_1(q,\omega)\,r_2(q,\omega)\,e^{-2qL}} + \frac{2\,r_1(q,\omega)\,r_2(q,\omega)\,e^{-2qL}}{1 - r_1(q,\omega)\,r_2(q,\omega)\,e^{-2qL}}\right),\tag{14}$$

where

$$r_1(q,\omega) = \frac{\epsilon_{m,1}(q,\omega) - 1}{\epsilon_{m,1}(q,\omega) + 1} \qquad \text{and} \qquad r_2(q,\omega) = \frac{\epsilon_{m,2}(q\omega) - 1}{\epsilon_{m,2}(q,\omega) + 1}\,.\tag{15}$$

Note that for two perfect metal gates, where $r_1 = r_2 = 1$, Equation (14) exactly becomes Equation (7).

4.1. Materials Sticked to Graphene

Setting $L = 2d$, in the limit $d \to 0$ Equation (14) gives

$$\tilde{V}(q,\omega) = \frac{2\pi e^2}{\epsilon_m(q,\omega)\,q}\,,\tag{16}$$

where

$$\epsilon_m(q,\omega) = \frac{1}{2}\left(\epsilon_{m,1}(q,\omega) + \epsilon_{m,2}(q,\omega)\right).\tag{17}$$

Equation (16) is the screened Coulomb potential in a graphene sheet between two materials stuck onto it.

We adopt Equation (3) again, which is valid for $q \to 0$ and $v_F q \ll \omega \ll 2\mu/\hbar$ [11,12], and Equation (4). Then, for small q and assuming that ϵ_m is constant, we obtain

$$\omega = \sqrt{\frac{2e^2\mu}{\hbar^2\epsilon_m}}\,\sqrt{q}\,,\tag{18}$$

which is the typical dispersion relation of plasmons in graphene exposed to two polar substrates [8].

4.2. Single Material Slab

In the absence of the upper slab, i.e., setting $r_2 = 0$, from Equation (14), we find

$$\tilde{V}(q,\omega) = \frac{2\pi e^2}{q}\left(1 - r(q,\omega)\,e^{-2qd}\right) \tag{19}$$

removing the subindex 1 from $r_1(q,\omega)$. Then, for small q and assuming that r is constant, we obtain

$$\tilde{V}(q,\omega) = \frac{2\pi e^2(1-r)}{q} + 4\pi e^2 rd - 8\pi e^2 rd^2\,q + \dots \tag{20}$$

Clearly, only if $r = 1$ (perfect conductor), the $1/q$ term drops out and one again finds acoustic plasmons with the speed of sound given by Equation (13). More generally, the small-q dispersion relation of plasmons reads

$$\omega = \sqrt{\frac{2\mu e^2(1-r)}{\hbar^2}q + \frac{4\mu e^2 rd}{\hbar^2}q^2}\,, \tag{21}$$

which becomes acoustic-like under the condition

$$q \gg \frac{(1-r)}{2dr}\,. \tag{22}$$

For a real metal gate, the functional dependence of $r(q,\omega)$ is crucial. In this case, the relative dielectric function ϵ_m can be approximated as [15]

$$\epsilon_m(q,\omega) = 1 + \frac{q_{TF}^2}{q^2} - \frac{\omega_p^2}{\omega^2 + i\Gamma\,\omega}\,, \tag{23}$$

where q_{TF} is the Thomas–Fermi wavenumber, ω_p is the plasma frequency, and Γ the damping constant. Notice that the relative dielectric constant of a perfect conductor is $\epsilon_m = -\infty$.

5. Conclusions

We have derived a simple formula for the speed of sound of the acustic modes of electrons in a graphene sheet. The existence of these hydrodynamic effects is due to the presence of metallic slabs, which induce a screening the Coulomb potential of electrons in graphene. Our formula for the graphene sandwiched between two metallic slabs generalizes the one obtained in Reference [7] in the case of graphene coupled to a single metallic slab. In conclusion, it is important to stress that, very recently, acoustic plasmons have been observed, with a real-space imaging, in single graphene sheet over a dielectric-metal slab [14]. This graphene-dielectric-metal configuration is quite different with respect to the one considered in the present paper. However, for the sake of completeness, in the last section of our paper, we have also considered the effect of two generic slabs of the screened Coulomb potential of two electrons in graphene.

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