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ABSTRACT BOOK

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ON THE TORIC SYMMETRY OF TURBULENT CHANNEL FLOWS

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Summary In the last century, the study of turbulence has been approached following the great Kolmogorov's physical insights on the inertial energy cascade and, more recently by investigating the geometry of the state space of the Navier-Stokes equations treated as a dynamical system. Such novel geometric approach arises from the evidence that what is observed in physical space sometimes is not always suggestive of the hidden laws of physics of the turbulent motion. Thus, looking at the turbulent dynamics in state space may lead to new understanding of the associated physical processes. In particular, vortices in a channel flow change shape as they are transported by the mean flow at the Taylor speed, or dynamical velocity. Reducing the translational or Toric symmetry in state space reveals that the shape-changing dynamics of vortices influences their own motion and it induces an additional self-propulsion velocity, the so-called geometric velocity. Thus, in strong turbulence the Taylor's hypothesis of frozen vortices is not satisfied because the geometric velocity can be significant.

RESULTS

Symmetry reduction approaches [1, 2, 3] provide a new way to understand the vortical motion of turbulence. This depends on the inertia of the flow and on its own shape-changing form over time. Because of the inertia of the flow, vortices are transported at roughly the Taylor speed, the so-called dynamical velocity V_{dyn} [2]. If turbulent fluctuations are significant, vortices change shape over time as they are transported by the mean flow. Their shape-changing dynamics induces an additional self-propulsion velocity, the so-called geometric velocity V_{geom} [2]. Thus, in strong turbulence the Taylors hypothesis of frozen vortices is not satisfied since the geometric velocity is not negligible over the Kolmogorov's inertial range.

The Navier-Stokes (NS) equations for channel flows have translational symmetry, that is if the fluid velocity $u(x,t)$ and associated pressure $p(x,t)$ fields are solutions, so are the space-shifted $u(x+L,t)$ and $p(x+L,t)$, where L is the shift. From a dynamical system perspective, if symmetry is present, the velocity V of the vortical motion can be uncoupled in the sum of dynamical and geometric components [2]

$$V = V_{dyn} + V_{geom}. \tag{1}$$

Such uncoupling is clearly observed in state space, and symmetry can be reduced or quotiented out to reveal the pure motion solely due to turbulence. The desymmetrized state space reveals how i) vortices change shape over time and how ii) their shape-changing influences their own motion. The Fourier representation of the velocity field is

$$u(x,t) = \sum_{-N}^{N} z_n(t) e^{ik_0 nx} \tag{2}$$

and the NS state space is $\mathbb{C}^N = \mathbb{R}^{2N}$, where N is the number of Fourier modes. We collect the Fourier modes z_n in a vector z and the NS equations can be rewritten as

$$\dot{z} = F(z), \tag{3}$$

where $F(z)$ is a nonlinear operator of z. Translational symmetry in physical space becomes a Toric symmetry (T-symmetry) in Fourier space, i.e. if the set (z_n) of Fourier modes is a solution of Eq. (4), so is $(z_n \exp(ik_0 L))$, for any shift L. The state space $\mathbb{C}^N$ has the geometric structure of a fiber bundle (see [2] and references therein). The bundle is made of a base or shape manifold $B = \mathbb{C}^N|\mathbb{R}$ of dimensions $\mathbb{R}^{2N-1}$ and 1D fibers along the direction of T-symmetry that attach to any point of B as shown in Figure 1a.

The T-symmetry can be reduced by devising a reduction map π that maps trajectories or orbits $z(t)$ of $\mathbb{C}^N$ onto desymmetrized orbits $Z_D(t)$ of the base manifold B, as illustrated in Figure 1a. The map π is invariant under the T-symmetry and it can be interpreted as a coordinate change transformation that allows for a map representation of the abstract base manifold. There are infinite ways to represent such manifold, i.e Mercatore projection, stereographic projections, etc. The desymmetrized orbit $Z_D(t)$ depends only on the shape-changing dynamics of vortices as in the motion of a jellyfish. Geometric and dynamical velocities allow studying the motion of the vortical motion in the desymmetrized frame. Figure 1b shows the mean velocity profile V of a turbulent channel flow at $Re = 3300$ $(Re_\tau = 180)$. The regime of turbulence is strong and the Taylor hypothesis is not satisfied. The dynamical velocity V_{dyn} is estimated as if vortices are transported by and frozen in the flow. V_{dyn} underestimates the observed mean velocity and the geometric component V_{geom} is not negligible, indicating that the shape-changing of vortices affects their own speed.

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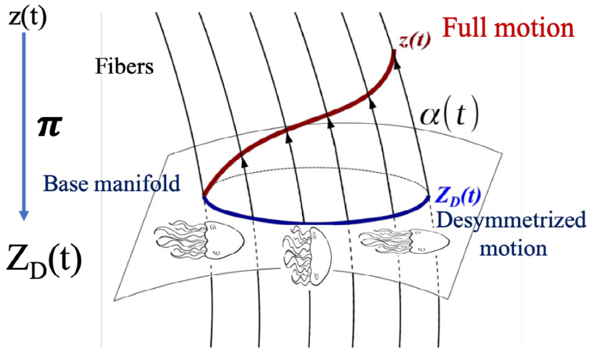
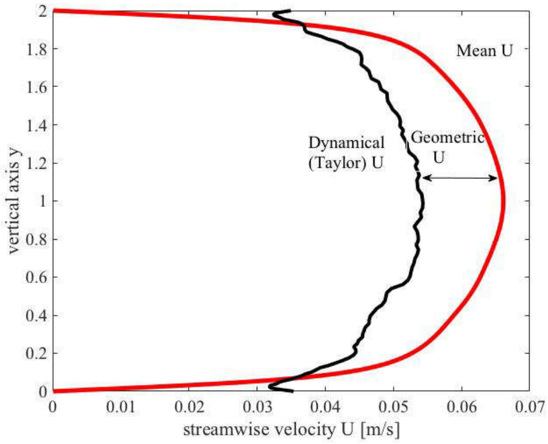


(a) Desymmetrization concepts

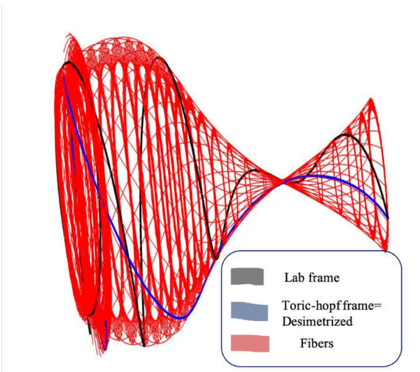


(b) Dynamic and geometric velocity components

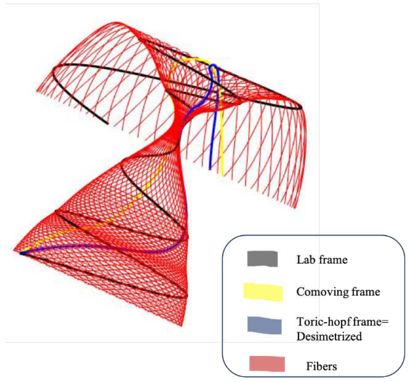
The group orbit of a trajectory $z(t)$ is a sheet of the fiber bundle and it is defined as

$$\begin{cases} G(z) = \{G_\alpha(z(t)) \quad \forall\, t\} \\ G_\alpha(z(t)) = \{z_n(t)e^{in\alpha}, \quad \alpha \in [0\ 2\pi]\} \end{cases} \tag{4}$$

Typical group orbits of $\mathbb{C}^N$, visualized in a 3D subspace, are depicted in Figures 2a and 2b. The simulated full trajectory or orbit $z(t)$ (black line), the orbit in the comoving frame (yellow line) and the desymmetrized orbit $Z_D(t)$ (blu line) are also shown. The comoving frame trajectory is that seen from a frame that moves with the dynamical velocity V_{dyn}. The desymmetrized trajectory is seen from a frame that moves at the total speed $V = V_{dyn} + V_{geom}$, sum of the dynamical and geometrical velocities.



(a) U streamwise velocity component



(b) Spanwise vorticity component

Figure 2: Group orbit of DNS turbulent channel flow

CONCLUSIONS

Symmetry reduction approaches are new ways to study the anatomy of the vortical motion in channel flows. Because of the inertia of the flow, vortices are transported at the Taylor speed, the so-called dynamical velocity [2]. When the vortex shape changes over time, it induces an additional self-propulsion velocity, the so-called geometric velocity [2]. Thus, in strong turbulence the Taylor's hypothesis of frozen vortices is not satisfied since the geometric velocity is not negligible.

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