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PAPER

Aspiration-based coevolution of node weights promotes cooperation in the spatial prisoner's dilemma game

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Abstract

Coevolution is considered as an effective means to optimize the conditions for the survival of cooperation. In this work, we propose a coevolution rule between individuals' node weights and aspiration, and then explore how this mechanism affects the evolution of cooperation in the spatial prisoner's dilemma game. We show that there is an optimistic amplitude of node weights that guarantees the survival of cooperation even when temptation to antisocial behavior is relatively large. An explanation is provided from a microscopic point of view by dividing nodes into four different types. What is interesting, our coevolution rule results in spontaneous emergence of cyclic dominance, where defectors with low weight become cooperators by imitating cooperators with high weight.

1. Introduction

Cooperation behavior emerges not only in nature but also in human society [1–3]. It is interesting and challenging to explain the emergence and evolution of cooperation among selfish individuals. The thesis has attracted much attention across a wide circle of fields, such as sociology, physics, information science and evolutionary biology [4, 5]. Evolutionary game theory is a potent mathematic tool to analyze diverse dilemmas in nature and human societal systems, which combines game theoretical analysis and dynamic evolutionary process analysis [6–8]. Relative to traditional game theory, evolutionary game theory is more concerned with the dynamic equilibrium. The prisoner's dilemma game (PDG), which forms a prototype of illustrating the dilemma owing to pairwise interactions, has been frequently applied to theoretical and experimental analysis on such an important issue. In the PDG, two individuals choose their strategies, namely, cooperation (C) or defection (D) at the same time. When a cooperator meets a defector, he receives S while the latter receives T. Besides, they both obtain R under mutual cooperation and P under mutual defection. The payoffs are ordered as $T > R > P > S$. It is easy to see that despite of the opponent's strategy, defection is the optimal strategy which lead cooperation to extinction [9, 10].

In the landmark discovery of Nowak, spatial topology has been proved to be effective in promoting the coevolution of cooperation by the mechanism which is widely called spatial reciprocity [11]. Enlightened by this, a large number of spatial topologies were tested to study the dynamics of cooperation in evolution [12–27]. Besides, coevolution is an effective way to resolve social dilemmas which means strategies evolve synchronously with other properties, such as the links between players [28, 29], teaching ability [30, 31], and immigration [32].

In this paper, we consider the coevolution of node weights and strategy within a PDG. In previous studies, an individual updates its strategy by comparing its payoff with its neighborhood. However, in real society, individuals have different sensitivities to benefits. For instance, a player with higher satisfaction often tends to keep the same strategy, even when the payoff is lower. Therefore, we introduce an A-based coevolution rule in which the satisfaction of the player is defined as the node weight, which influences its fitness. In detail, when a player's payoff is larger (smaller) than its aspiration, its satisfaction increases (decreases). Enforcing such a coevolution rule, the result is that cooperation can still be maintained and promoted also with a high level of temptation to defect. The paper is organized as follows. First, we give a detailed description of our model. Then, we show the results by graphs and try to give reasons. Lastly, we summarize and discuss the main conclusions.

2. Methods

All individuals occupy the nodes of a square lattice of size $L \times L$, and connect with four neighbors respectively. At the beginning of the game, each individual is assigned either as a cooperator $S_x = C$ or as a defector $S_x = D$ with equal probability. Players obtain their payoffs by means of pairwise interactions with all their neighbors. Following the standard PDG model, mutual cooperation leads to the reward R, while mutual defection conduces to punishment P. Under the mixed case, the cooperator obtains the sucker's payoff and the defector gets the temptation T. Hence, the payoff can be described by the following payoff matrix with only one parameter b $(1 < b < 2)$.

$$\begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}. \tag{1}$$

At each time step of the Monte Carlo simulation, a player x is selected randomly from N players and gets his payoff p_x by the following equation:

$$p_x = \sum_{y=1}^{k_x} p_{xy}, \tag{2}$$

where $k_x = 4$ is the degree of player x and p_{xy} is the payoff that player x obtains from its neighbor y. The fitness of player x is determined by the equation:

$$F_x = w_x \times p_x, \tag{3}$$

where w_x is defined as the node weight of player x. The node weight w_x is initially set to 1 and evolves within the interval [0, 2] as follows:

$$\begin{cases} w_x = w_x + \delta, & p_x > A_x \\ w_x = w_x, & p_x = A_x, \\ w_x = w_x - \delta, & p_x < A_x \end{cases} \tag{4}$$

where δ is the adjustment amplitude of node weights, and A_x denotes the aspiration of player x. A larger value of *delta* corresponds to a faster change in w_x and a lower heterogeneity of this parameter.

Different δ values are set in different simulations. In conventional PDG evolution games, $\delta = 0$; in contrast, we introduce our coevolution mechanism by setting δ in the range of (0, 1]. Considering the heterogeneity of each individual's aspiration level in reality, A_x is initially drawn randomly from the interval $[0, 4b]$. Clearly, 0 is the minimum payoff one player can gain in a round of games, while $4b$ is the maximum. The aspiration changes along with the player's strategy. Since our goal is to explore the impact of δ on the evolution of cooperation, for simplicity, we do not consider the different types of aspiration distribution, such as the power-law or exponential distribution, which have been already investigated in previous work.

By choosing a neighbor y randomly, the local player x imitates the strategy and aspiration of y with the following probability:

$$P_{(x\to y)} = \frac{1}{1 + \exp[(F_x - F_y)/K]}, \tag{5}$$

where K is the intensity of selection which denotes the amplitude of noise. Without loss of generality, we set $K = 0.5$.

We carry out the simulation results by setting square lattice with $L = 100$ and scale-free network with 10 000 nodes. Besides, we set the relaxation time equals to 5×10^4 Monte Carlo steps. To ensure suitable accuracy, the final results is the average number of 20 independent repeated experiments for each set of parameters.

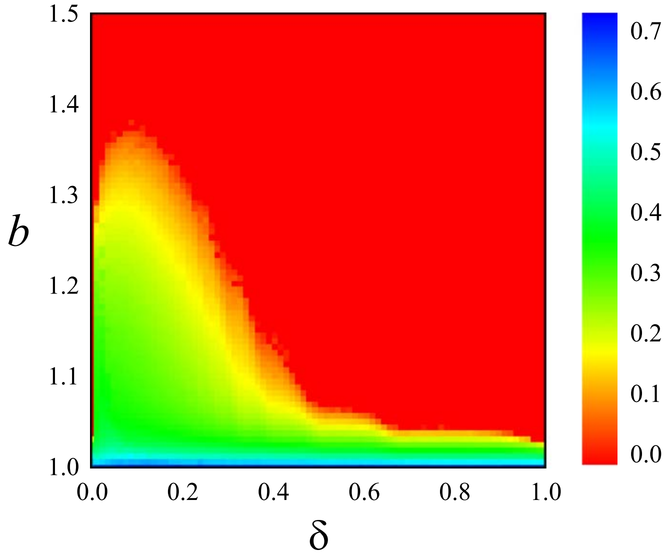

Figure 1. Fraction of cooperation ρ_c depending on a definite b for different values of δ on square lattice . For $\delta = 0$, the condition simplifies to the traditional PDG. When $\delta > 0.1$, as δ increases, ρ_c decreases. The panels show that the optimal value, where cooperation is best promoted, lies in the range of approximately 0.1 irrespective of the temptation to defect b.

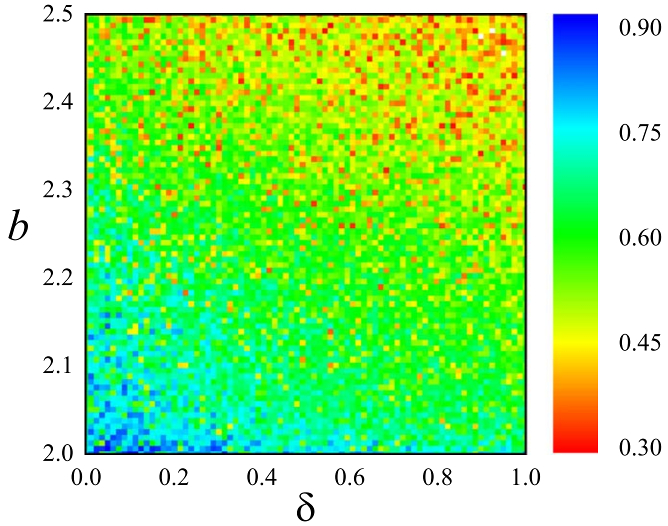

Figure 2. Fraction of cooperation ρ_c depending on a definite b for different values of δ on square lattice . For $\delta = 0$, the condition simplifies to the traditional PDG. When $\delta > 0.1$, as δ increases, ρ_c decreases. The panels show that the optimal value, where cooperation is best promoted, lies in the range of approximately 0.1 irrespective of the temptation to defect b.

3. Results

First, we consider the influence of parameters δ and b on the coevolution. Figure 1 presents a contour plot encrypting the fraction of cooperation ρ_c on the δ-b parameter plane. When $\delta = 0$ the weight of each individual is fixed and the model turns into the traditional case, where cooperation soon die out (approximately 1.0375). By setting $\delta > 0$, the coevolution takes place and promotes cooperation. It is obvious that an intermediate δ value ($\delta = 0.1$) promotes cooperation optimally. When $\delta > 0.1$, as δ increases, the fraction of cooperation decreases. On the whole, our coevolution mechanism regarding the node weights and strategy enhances the survival of cooperation significantly. The scenario is similar for scale-free networks as showed in figure 2.

Figure 3 shows the evolution of cooperation over time (Monte Carlo steps) for different δ values, which lead to different evolutionary curves. When $\delta = 0.001$, the fraction of cooperation represented by a black curve keeps decreasing; after approximately 10^2 rounds, cooperation is completely extinct. This result occurs because the cooperators are in a disadvantageous payoff position in the game with respect to defectors, which leads to a lower fitness of the cooperators. When $\delta = 0.6$, the situation is the same as with $\delta = 0.001$. However, when $\delta = 0.01$,

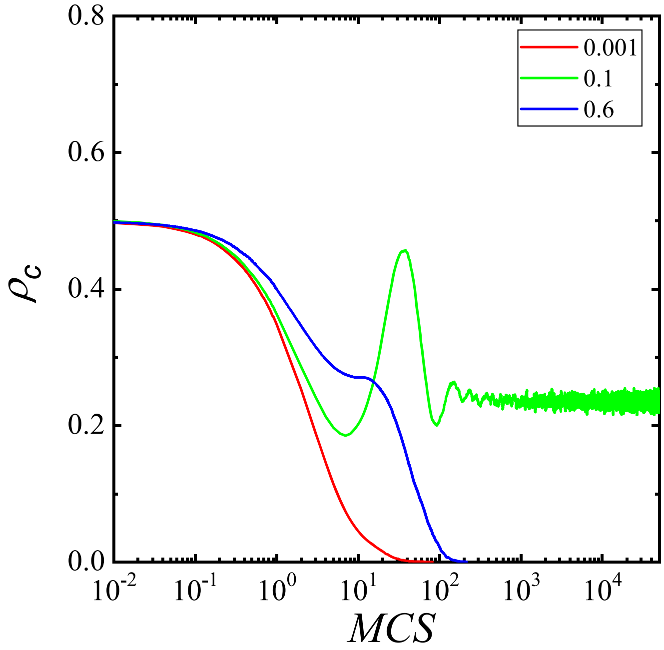

Figure 3. Fraction of cooperation given the time step t for different values of δ. The black, red and blue lines denote $\delta = 0.001, 0.1$, and 0.6, respectively, and the temptation to defect b is fixed at 1.2. A too-large or too-small value of δ inhibits the evolution of cooperation; however, an optimal value of δ exists that can enable the survival of cooperation.

the situation becomes quite different. Specifically, the fraction of cooperation decreases to its minimum (approximately 0.1) until the 10th MC step. It then increases, until reaching its maximum (approximately 0.7) and ultimately maintains dynamic stability at a value of approximately 0.33. Thus, relative to the case δ is very small ($\delta = 0$), our mechanism promotes cooperation when δ is set to a moderate value. Our next step is then explaining this phenomenon from a microscopic perspective.

To gather information on the origins of the observed phenomenon, we show the characteristic snapshots in both a strategy-specific manner and a weight-specific manner in figure 4. All the nodes on the square lattice are divided into four types: cooperators with high node weights, cooperators with low node weights, defectors with high node weights and defector with low node weights, as shown in blue, cyan, magenta, and red, respectively. High weight means that the weight of nodes is higher than the average weight of all individuals at a certain time, while low weight means the weight of nodes is lower than or equal to the average weight of all individuals. For different values of δ, from top to bottom on figure 4, the processes of evolution and stability are completely different. The top four snapshots show the evolutionary process for $\delta = 0.001$, from which we can observe that due to the disadvantages of payoffs, the cooperators die out gradually. Because of too slow change of nodes weight, a too small δ will make the situation consistent with the traditional situation. However, when the value of δ is set to 0.1, the situation becomes very different. As shown in the middle four snapshots, the process of coevolution can be divided into several stages, which is similar to the results shown in figure 3.

In the first few rounds, the weights of all nodes are almost the same for small δ values, and thus the evolution of cooperation is similar to the traditional case, in which the number of cooperators decreases rapidly. At the end of this stage, the distribution of the strategy and node weights has prominent spatial characteristics. Most of the remaining cooperators have high node weights and are surrounded by defectors with both high weights and low weights, and the remaining areas contain a substantial number of defectors with low node weights. A deeper description can be obtained by looking at figure 5(a). In early stages, even cooperators with high weights cannot resist defecting since the increase of fitness caused by the high weight is much less than the loss caused by being taken advantage of by defectors. However, note that in this stage, the weights of the cooperators in clusters keep increasing, while the weight of the average node of defectors continues to decrease. This stage is very important since it not only protects cooperators by forming clusters but also directs the evolution of strategies into the next stage, where cooperation expands.

In this stage, clusters of cooperators begin to expand following the pattern, as shown in figure 4. The defectors with low weights change to cooperators when they interact with high-weight cooperators. Then, the weight of these cooperators along the boundary of clusters increases continuously because of their satisfaction with their payoff. In this way, the clusters of cooperators expand and dominate the majority of the system that was previously occupied by defectors with low weights, as shown in figure 5. As defectors with high weights take

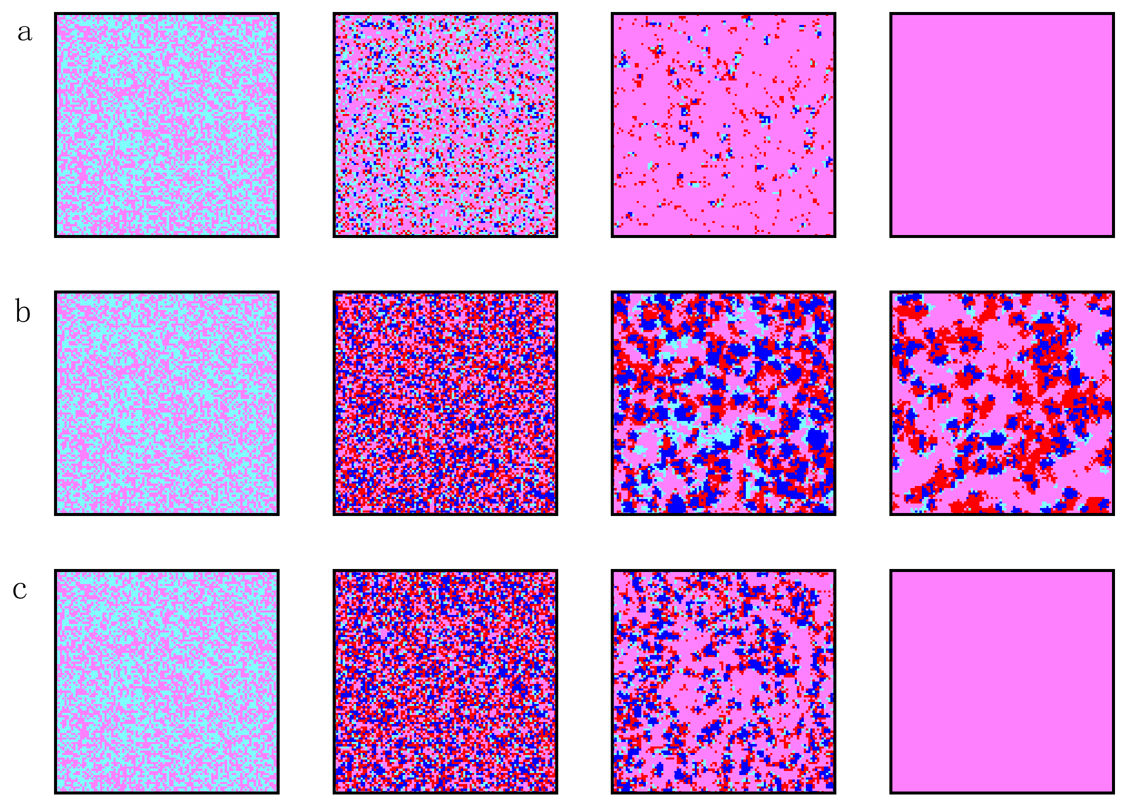

Figure 4. Characteristic snapshots of cooperators with high node weights (blue), cooperators with low node weights (cyan), defectors with high node weights (red) and defectors with low node weights (magenta) on the regular lattice. The top to the bottom correspond to δ values of 0.001, 0.1, and 0.6. From left to right, snapshots are shown of MCS = 0, 5, 50, and 49 999 for the top and bottom and MCS = 0, 10, 100, and 49 999 for the middle. The results are obtained by setting $b = 1.2$.

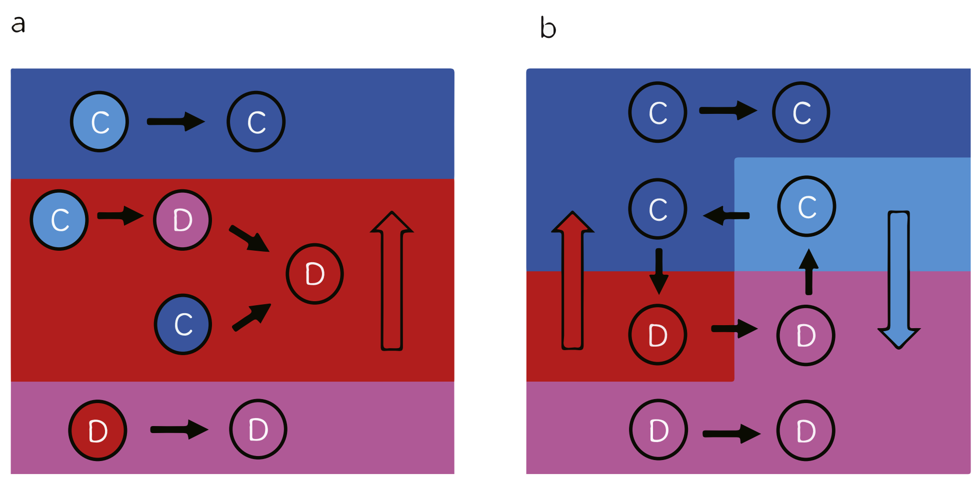

Figure 5. Conversion between different types of nodes at different stages when $\delta = 0.1$. The process of coevolution is divided into two stages, which are shown in (a) and (b), and every stage has different evolutionary strategy directions.

advantage of the cooperators with high weights, the defectors with high weights gradually invade the cooperative clusters. From the evolutionary process, we find that a closed loop exists with four types of nodes of the form DH → DL → CL → CH → DH (DH means defector with high weight). This loop allows cooperators to survive and determines the final distribution of the four types of nodes. Figure 6 shows the evolution of four types of nodes' fraction over time (Monte Carlo steps) when δ value is set to 0.1. To provide direct visual evidence that a closed loop elevates cooperation as shown in figure 5, we start by separating the domains of the four types of nodes and set the CH's weight as 1.78, which is the value of the final weight of high weight nodes when a loop phenomenon exists in our previous simulation. In addition, we respectively set the DH's, CL's, DL's weight as

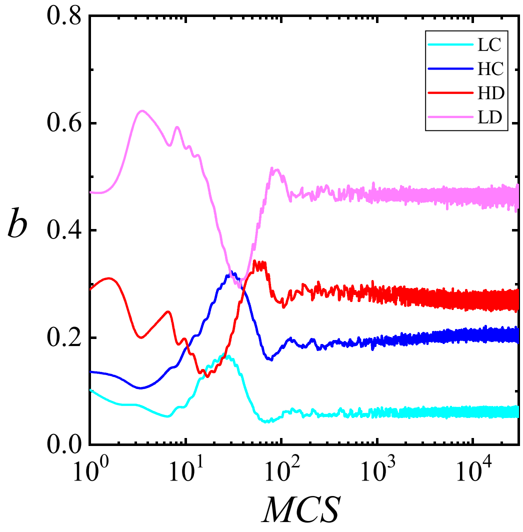

Figure 6. Fraction of four types of nodes given the time step t for $\delta = 0.1$.The temptation to defect b is fixed at 1.2. The line of cooperators with high node weights is draw in blue, the line of cooperators with low node weights is draw in cyan, the line of defectors with high node weights is draw in red, and the line of defectors with low node weights in magenta.

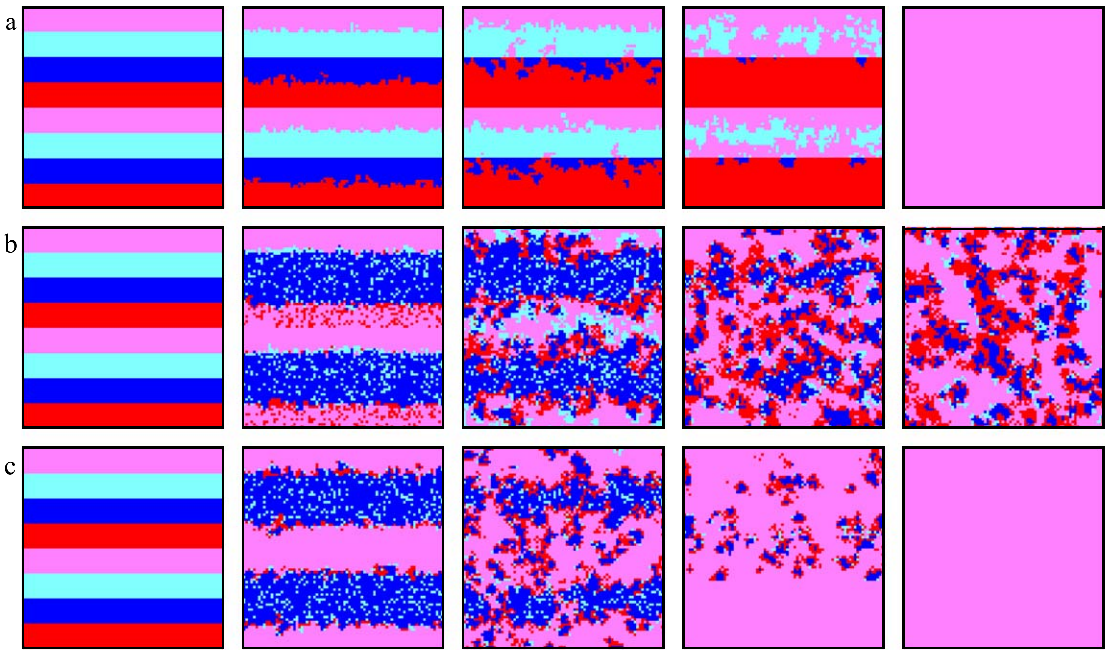

Figure 7. Initial evolution of the prepared scenario. The population structure is represented by an 80×80 square lattice with periodical boundary conditions. The top, medium and bottom correspond to δ values of 0.001, 0.1, 0.6 respectively. From left to right, the snapshots correspond to MCS = 0, 10, 50, 99, and 49 999. Differences clearly exist between the three evolution paths.

1.61, 0.53, 0.20 for the same reason. In figure 7(a), we find a closed loop in the evolution of cooperation, which is proved to promote cooperation in our work.

We now move to study the case of $\delta = 0.6$. The bottom four snapshots in figure 5 indicate that in the early stages, although cooperators form high-weight clusters, these clusters are invaded by defectors with high weights and soon die out. The phenomenon can be explained as follows. A large value of δ can destroy the closed loop mentioned above. In detail, the defectors with low weights can easily change to defectors with high weights, which makes the cooperators with low weights go extinct rapidly. As a result, cooperators die out, and all nodes become defectors with low weights ($w = 0$). The evolution of the four types of nodes in figure 7(b) proceeds in the same way. Here, however, the defectors expand so fast that high-weight cooperators cannot protect low-weight cooperators; hence, the closed loop is destroyed, and cooperation ultimately dies out as shown in figure 7(c).

4. Conclusion

To conclude, we have explored an evolutionary game under coevolution of node weights and strategy: this scenario corresponds to the case which an individuals' social satisfaction denoted by the node weight adaptively changes according to their social performance. Through numerical simulations, we found that our coevolutionary configuration can promote cooperation effectively, while these observations can be attributed to the loop of different types of nodes. Too small or too large values of δ destroy the loop, which can be reflected via the phenomenon of real society. When $\delta = 0$, just as in a society that lacks emotion, people lack expectations of their own social performance and follow popular trends, and thus the death of cooperation is irreversible. However, large δ describes an overly sensitive society where people are moody and impulsive and the decline in cooperation is very rapid once early setbacks are encountered. When δ is intermediate, people seem to be more rational and adapt their aspiration steadily. Under this situation, a stable social structure of cooperative behavior is formed. The above results can give us a comprehensive understanding of the role of node weights on the evolution of cooperation.

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