Thresholds, localization and centrality in epidemic spreading on networks

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Epidemics

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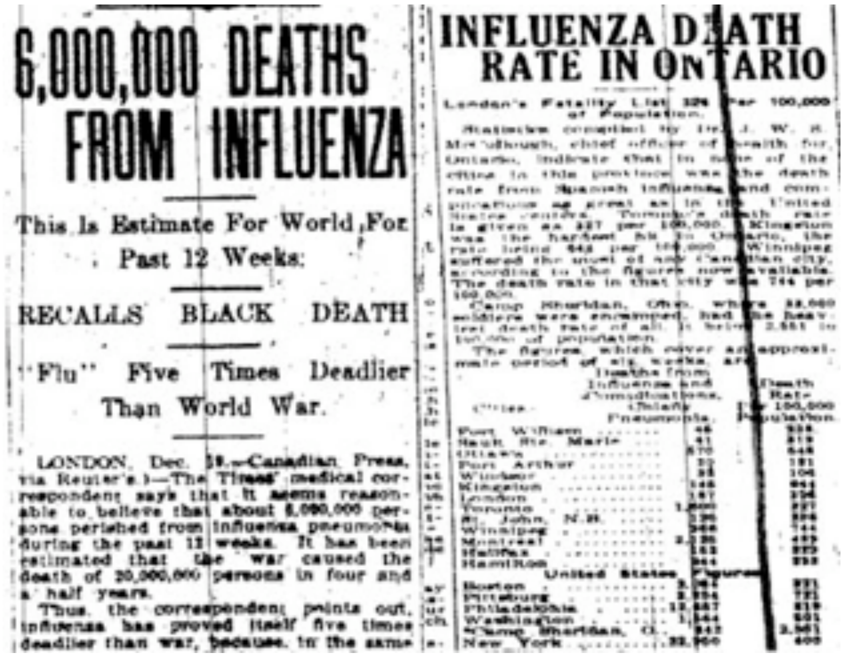

6,000,000 DEATHS FROM INFLUENZA

This Is Estimate For World For Past 12 Weeks.

RECALLS BLACK DEATH

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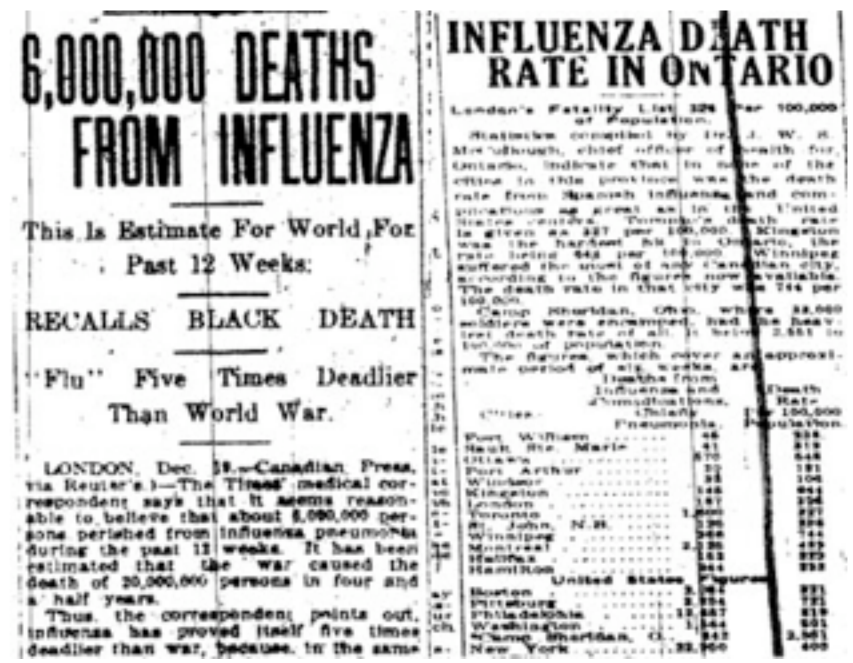

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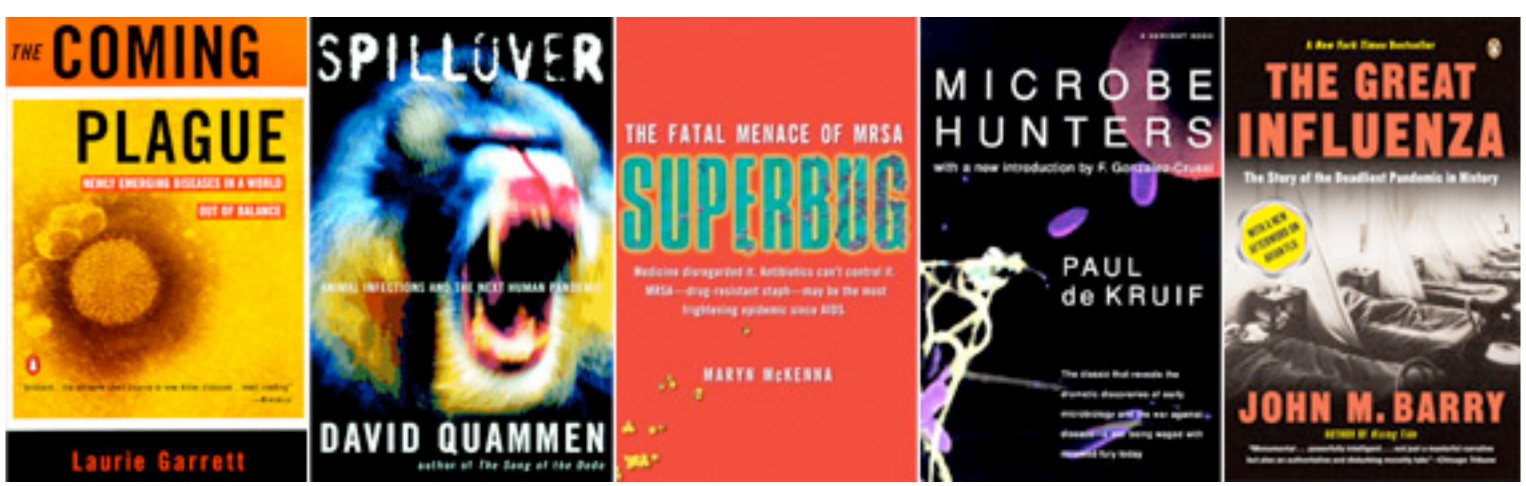
- New epidemics constantly appear (HIV, SARS, Ebola...)



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- .....

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Fast and long-range travel is crucial

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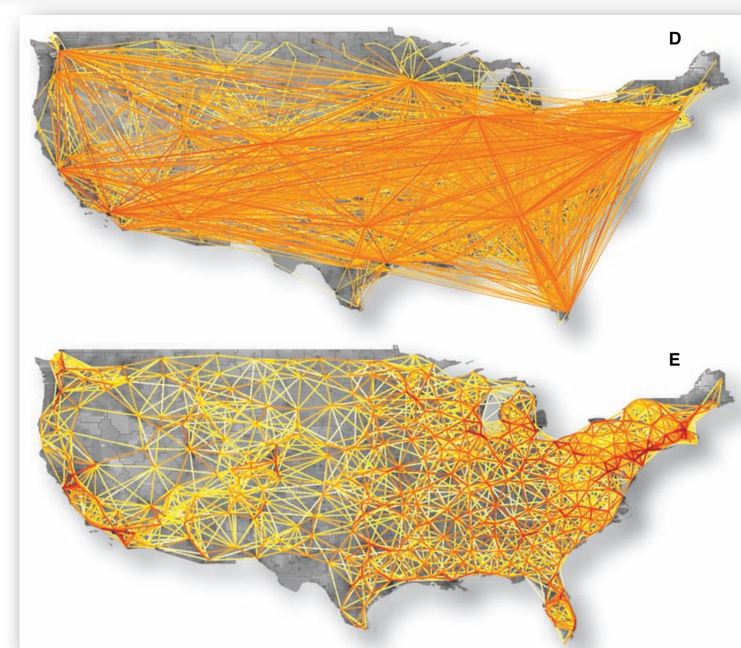

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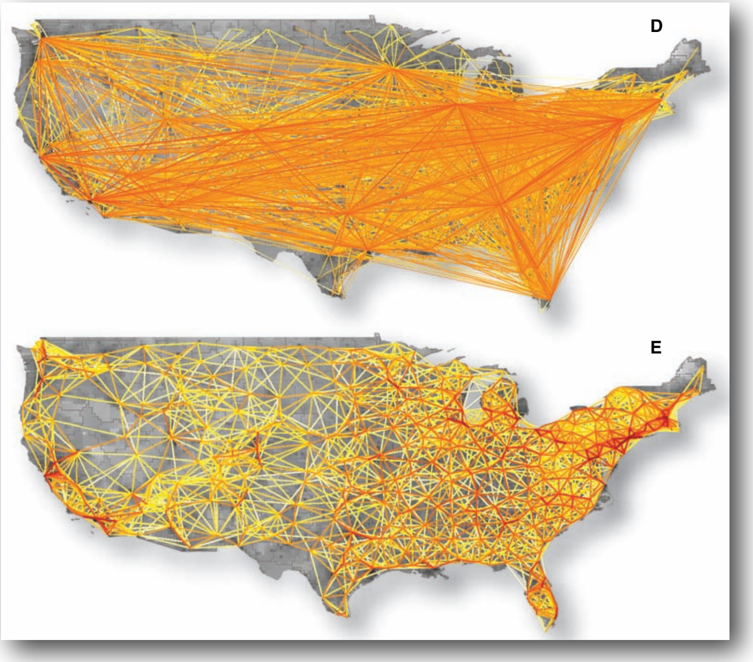

Fast and long-range travel is crucial

Large-scale heterogeneous transportation networks are relevant

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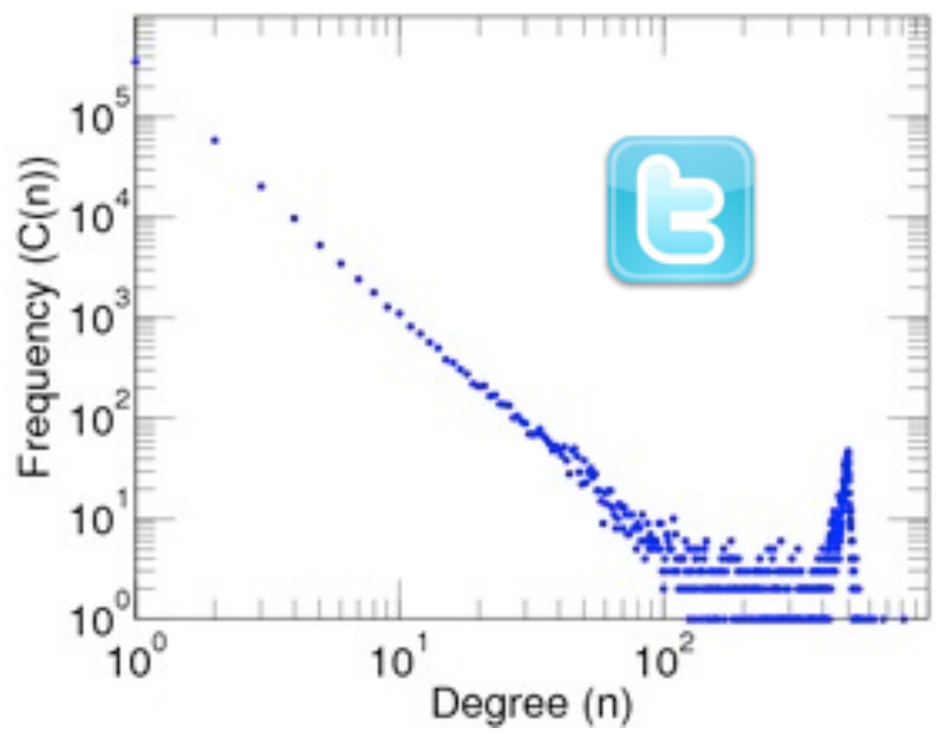

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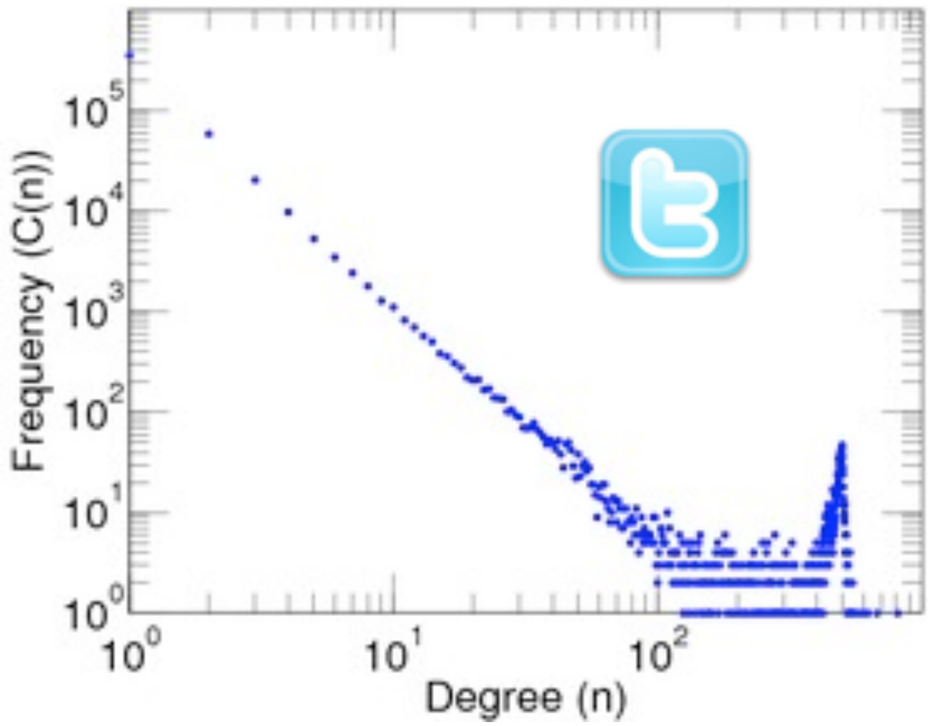

Heterogeneous networks are relevant

HIV "patient zero" infected 40 of the 284 cases of AIDS in the USA

Modeling epidemics on networks

- Practical interest

  Crucial problem throughout human history

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- How can the origin of an outbreak be reconstructed?

Classes of epidemics

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SIR class

SIS class

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SIR class | **SIS class**

- Permanent immunity
- Individuals are infected at most once
- Outbreaks have finite duration

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SIS class

- Temporal/no immunity
- Individuals can be infected many times
- Outbreaks can persist forever

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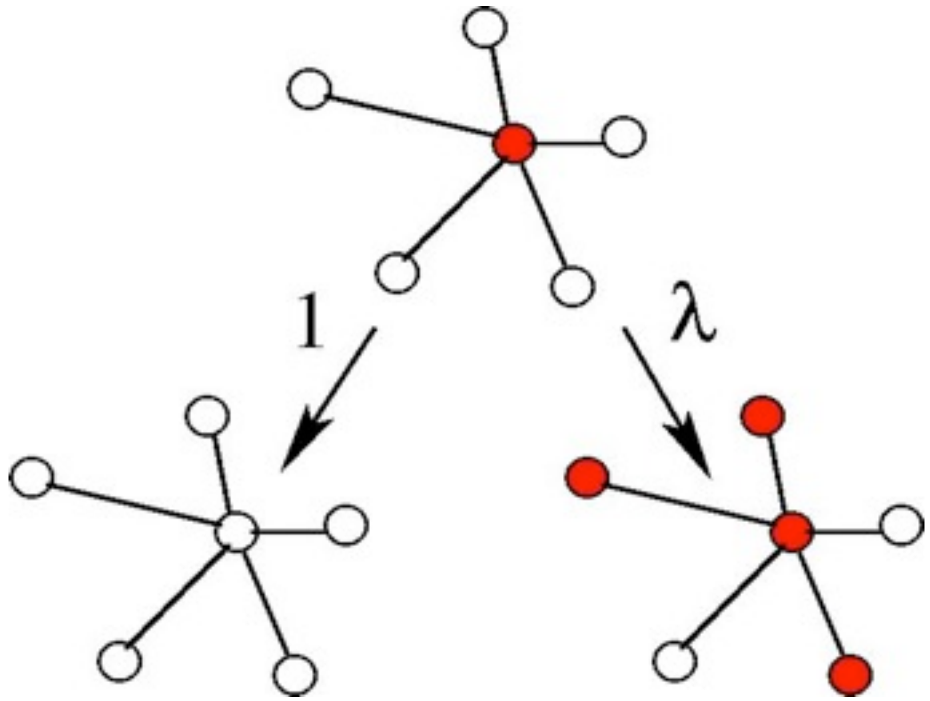

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  - ☆ Recovery I → S (rate $\mu=1$)
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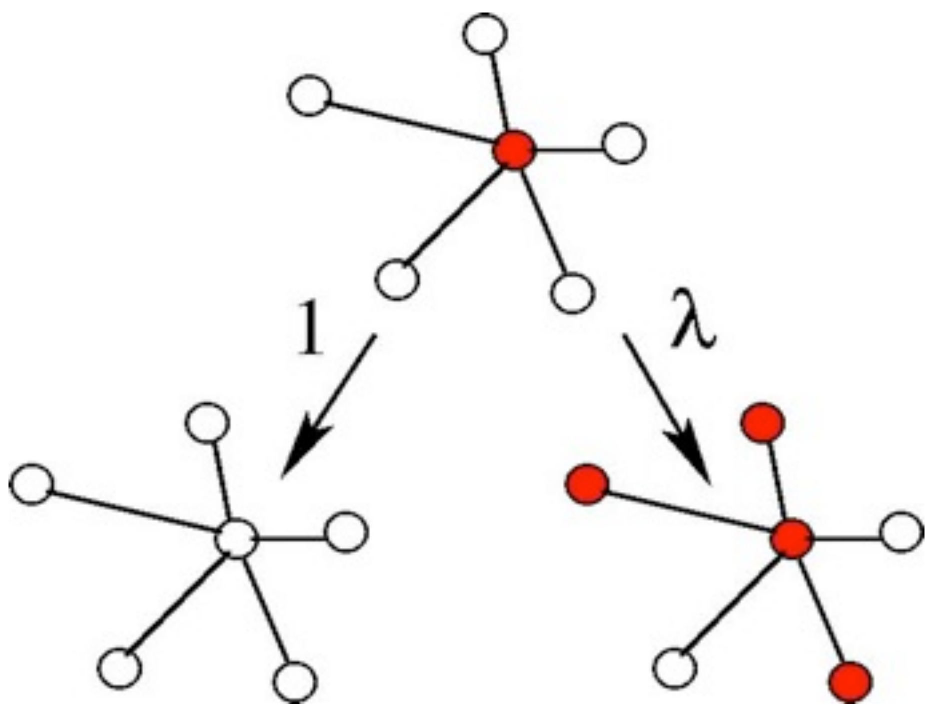

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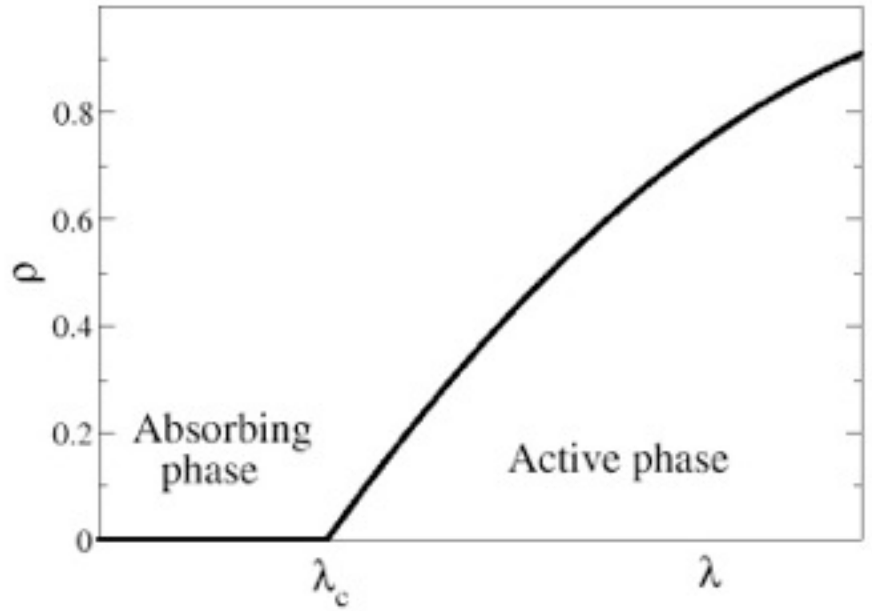
- Order parameter
  ρ = fraction of infected nodes
  in the stationary state



Heterogeneous Mean-Field theory (HMF) for SIS

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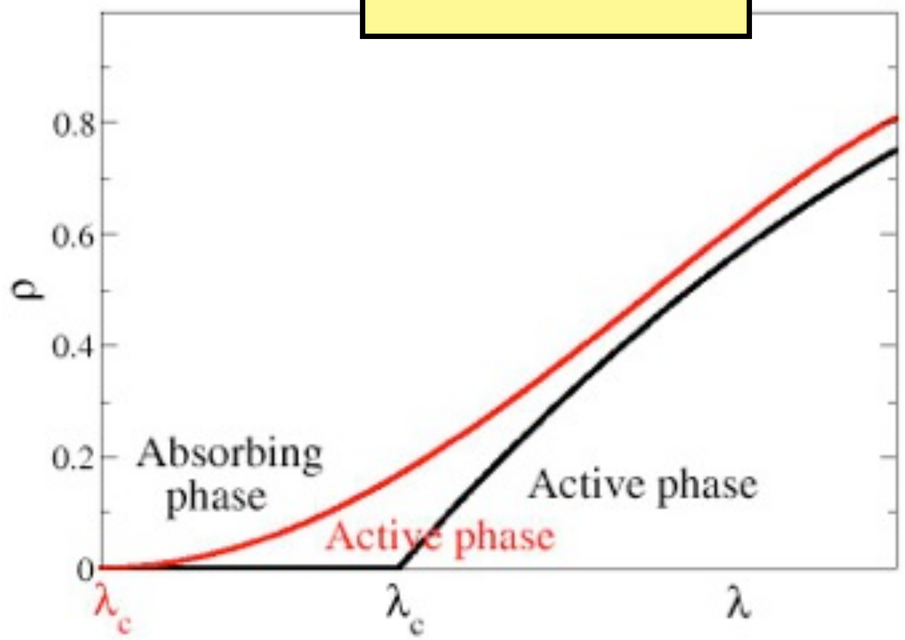
- In the limit of large system size

$$\lambda_c \to \begin{cases} 0 & \gamma \leq 3 \\ \text{finite} & \gamma > 3 \end{cases}$$

Zero threshold for scale-free networks



Beyond HMF: Quenched Mean-Field theory for SIS

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$$\lambda_c = \frac{1}{\Lambda_N}$$

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- QMF + Chung et al. formula

$$\lambda_c \simeq \begin{cases} 1/\sqrt{k_{max}} & \gamma > 5/2 \\ \frac{\langle k\rangle}{\langle k^2\rangle} & 2 < \gamma < 5/2 \end{cases}$$

The epidemic threshold always goes to zero

Zero threshold has not to do with the scale-free property

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- QMF is **not** exact

$$\rho_{ij} \neq \rho_i \rho_j$$

Triggering mechanisms

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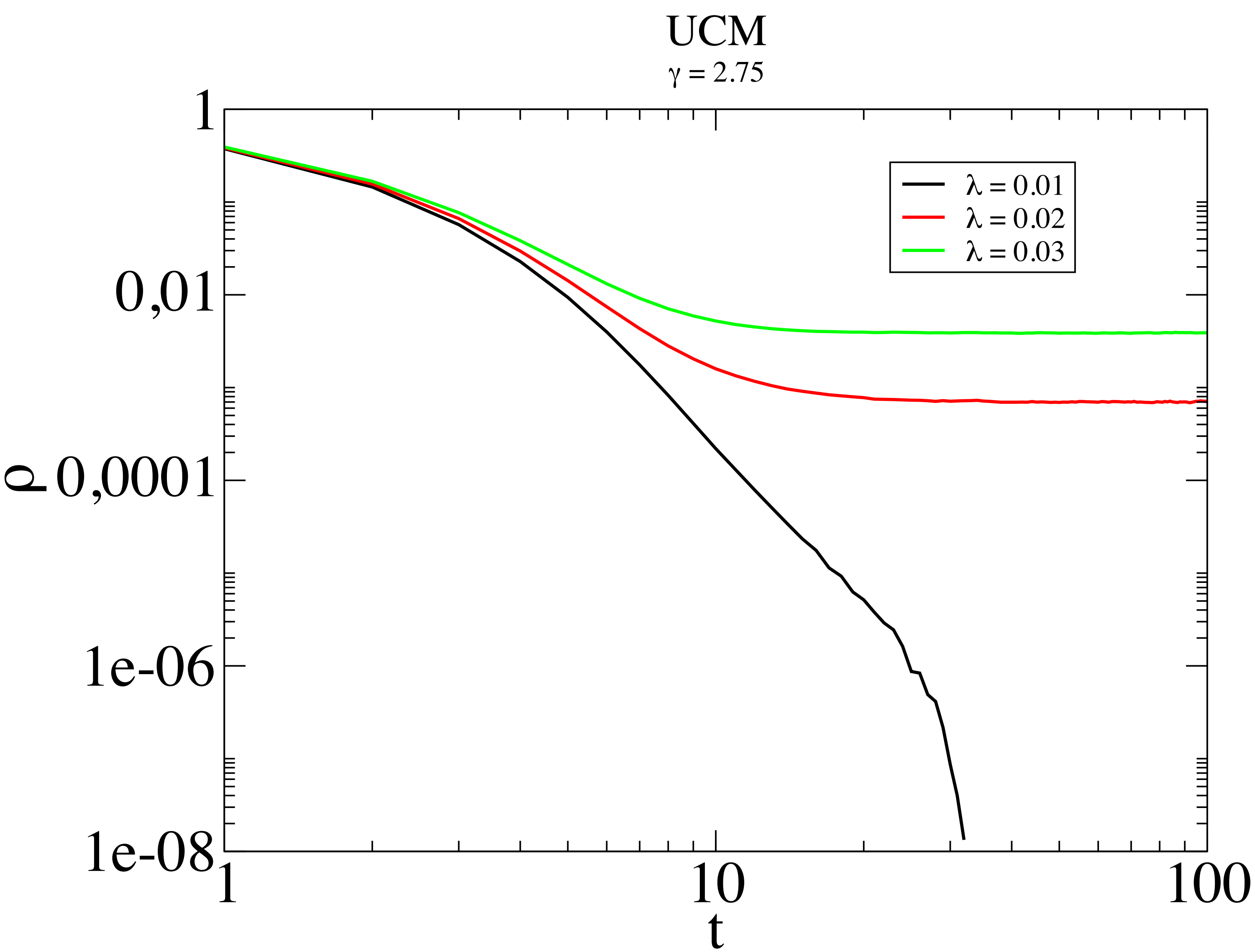
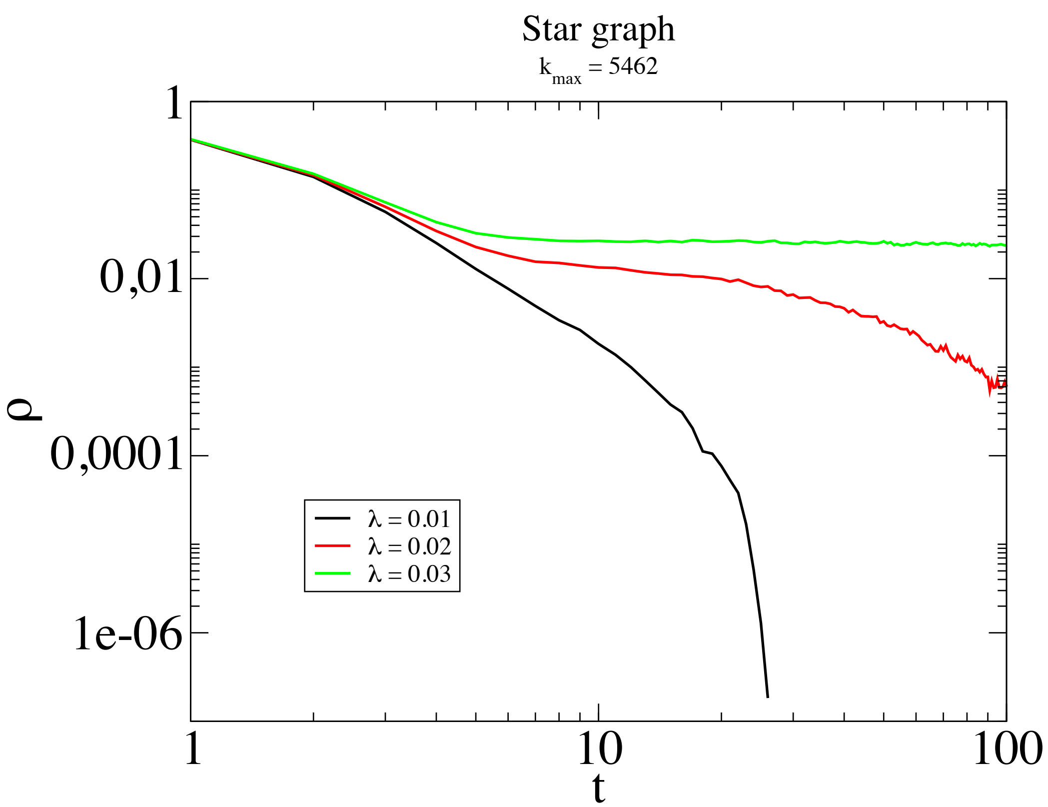
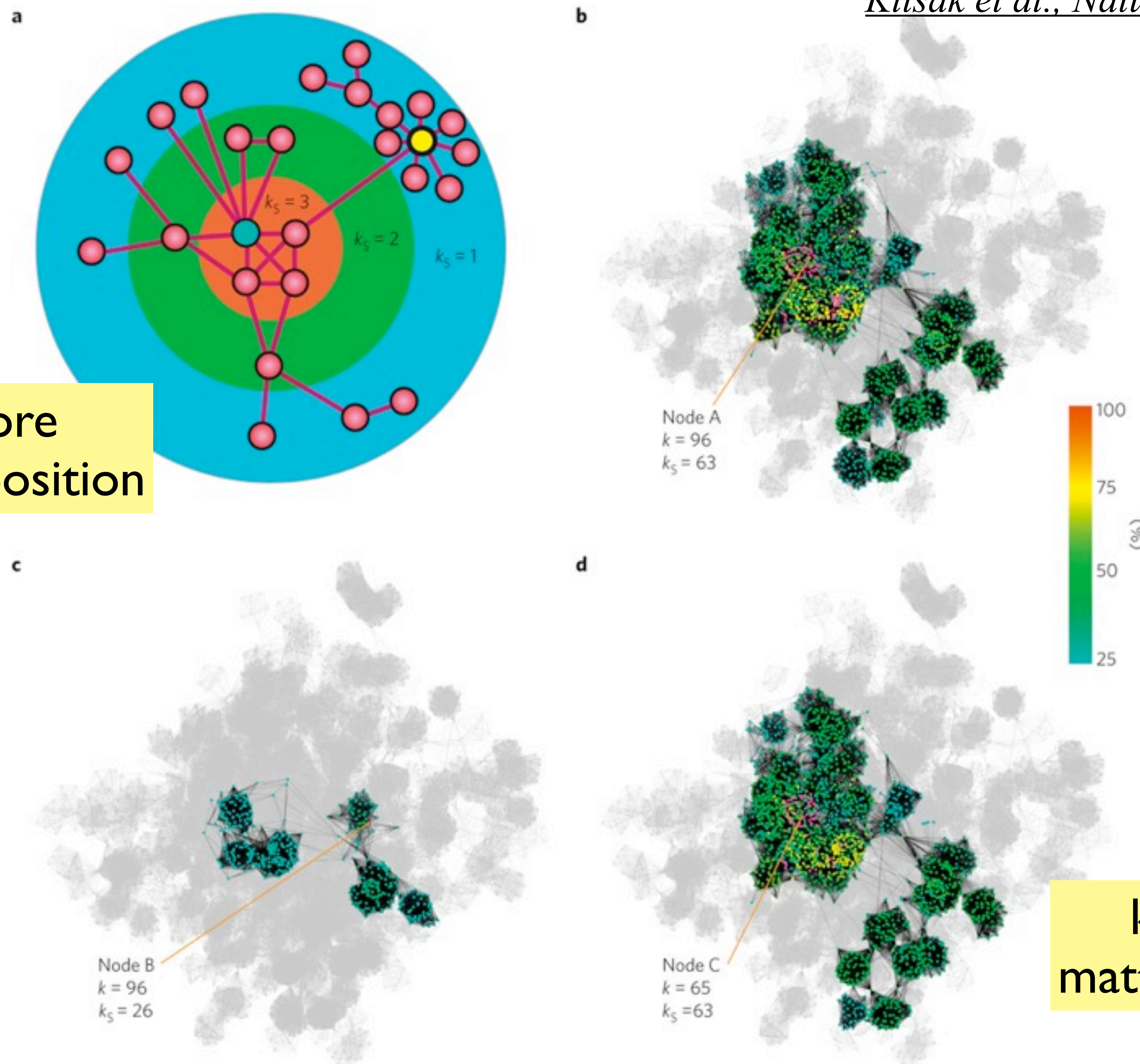
- For $\gamma > 5/2$ it is the largest hub





Influential spreaders for SIR

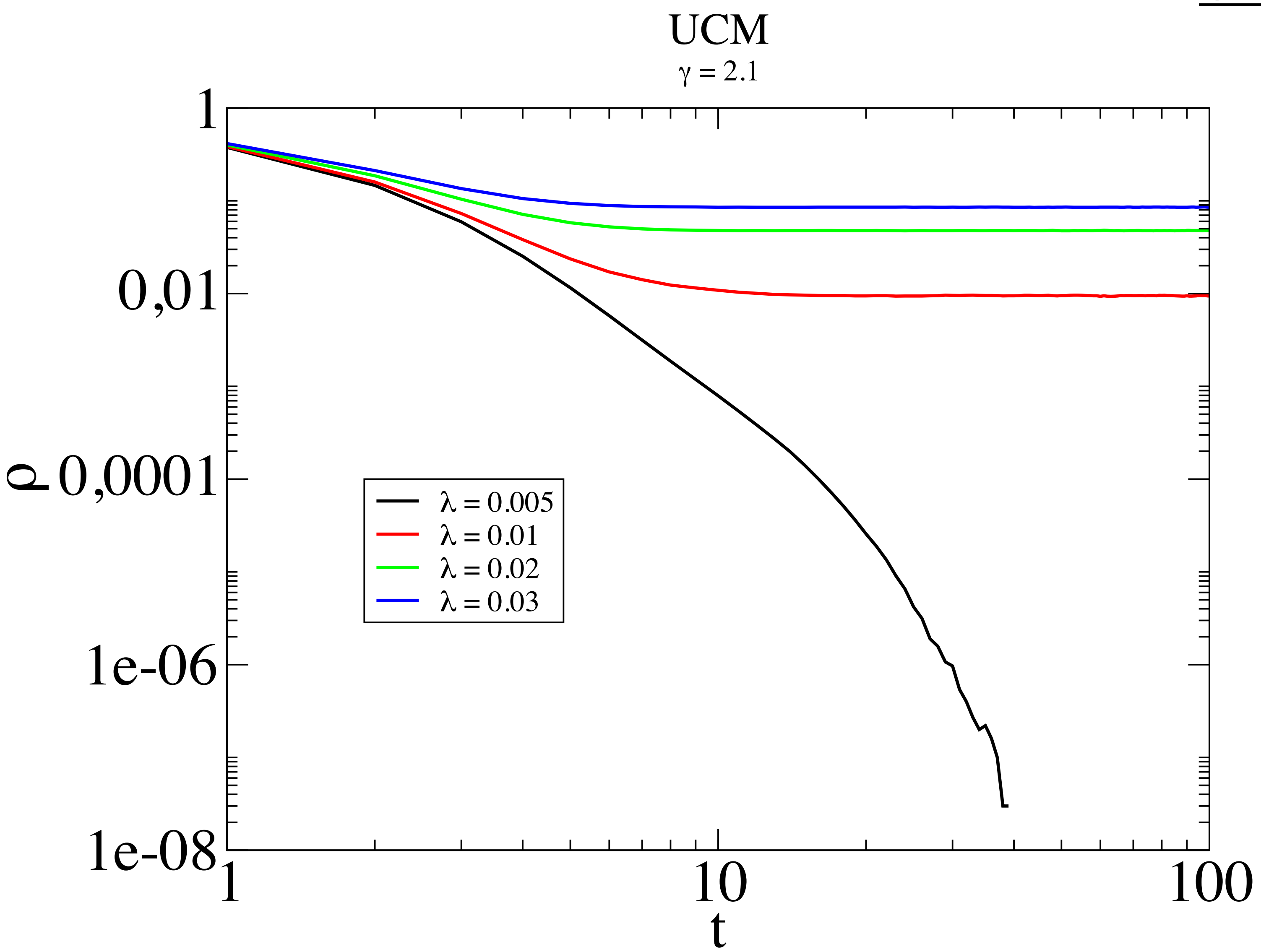
Kitsak et al., Nature Physics (2010)



k-core decomposition

k-cores matter for SIR

Castellano and Pastor-Satorras, Sci. Rep. (2012)



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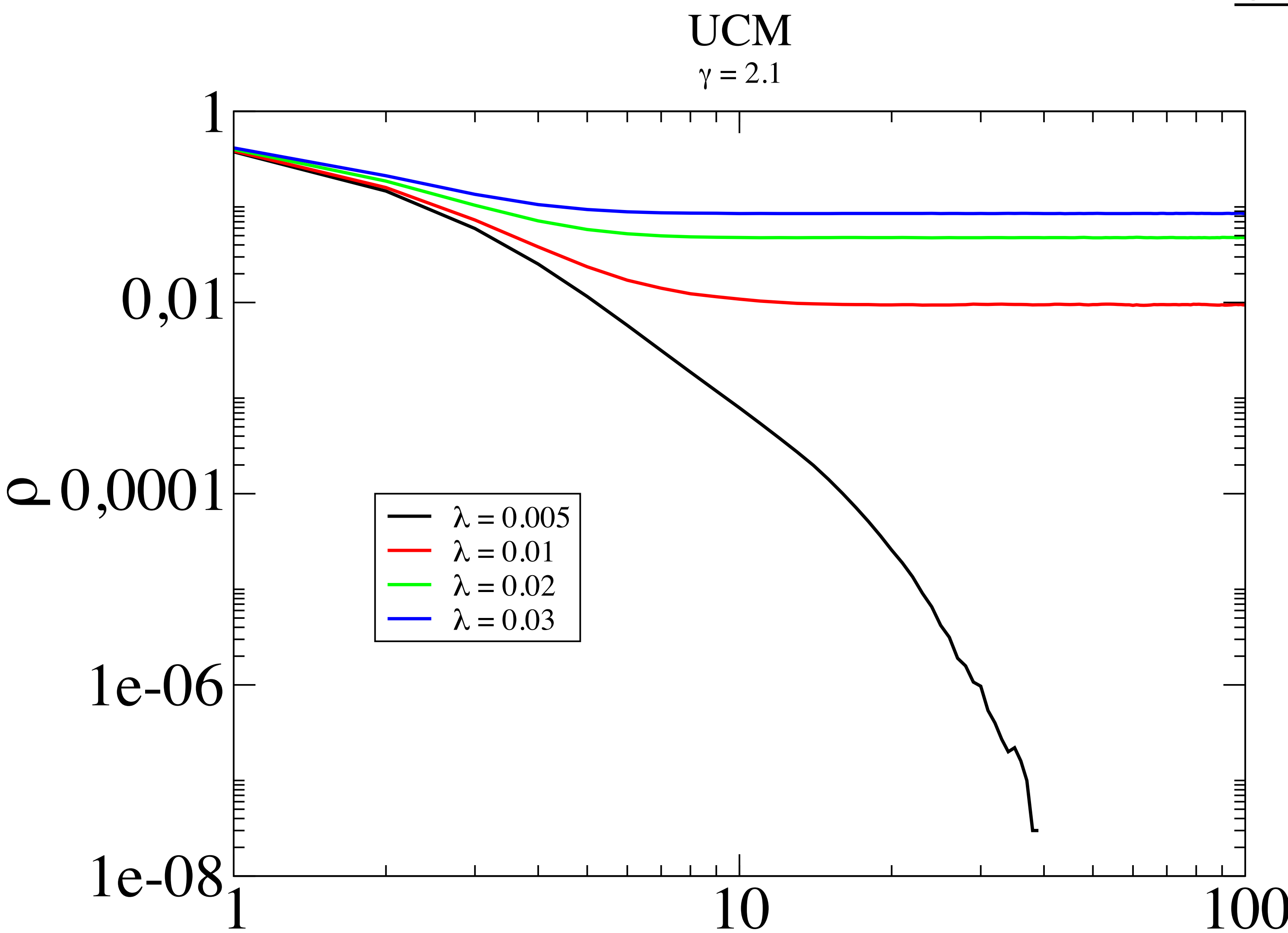

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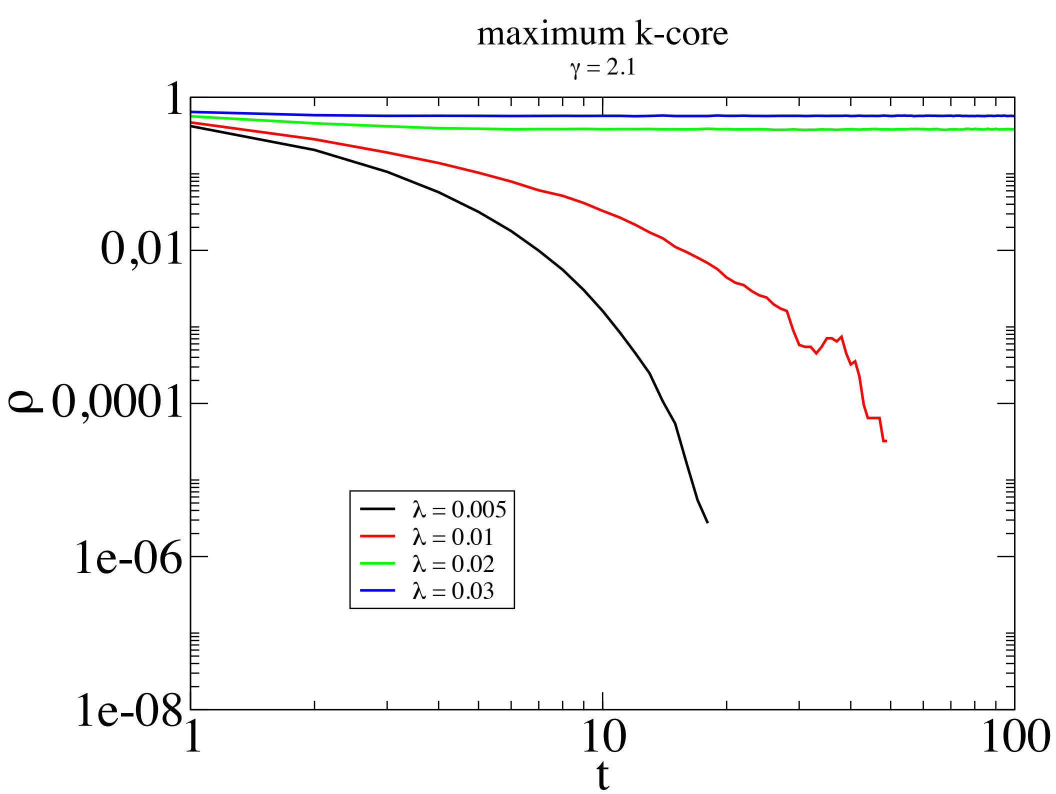

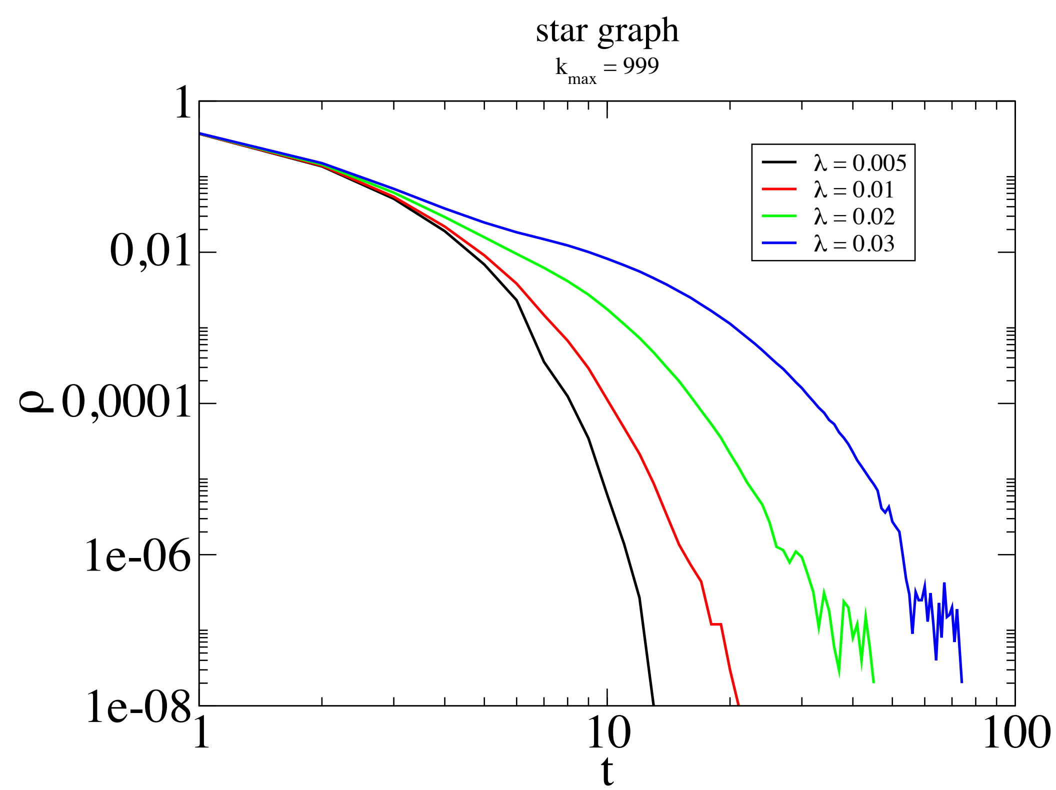

Localized vs endemic activity

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- This picture is derived **within** QMF framework

A Griffiths phase?

Lee, Shim and Noh, Phys. Rev. E. (2013)

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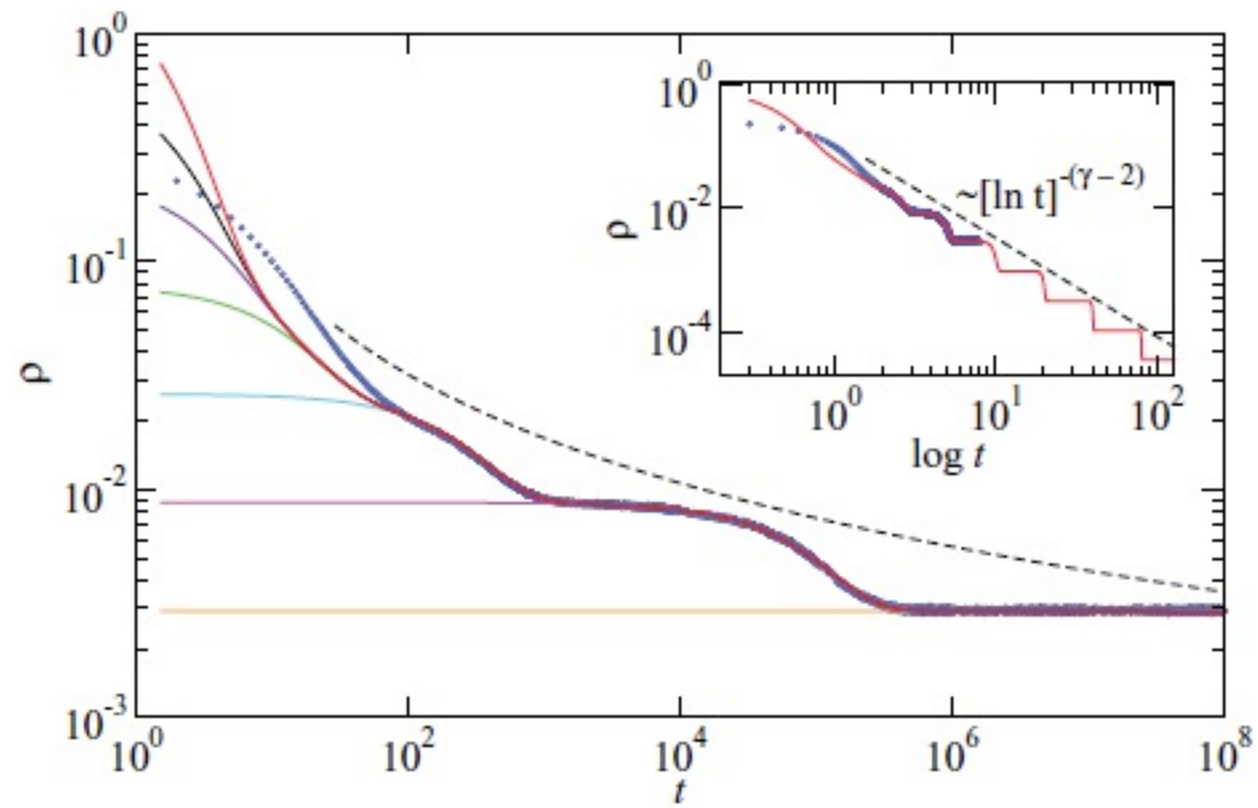

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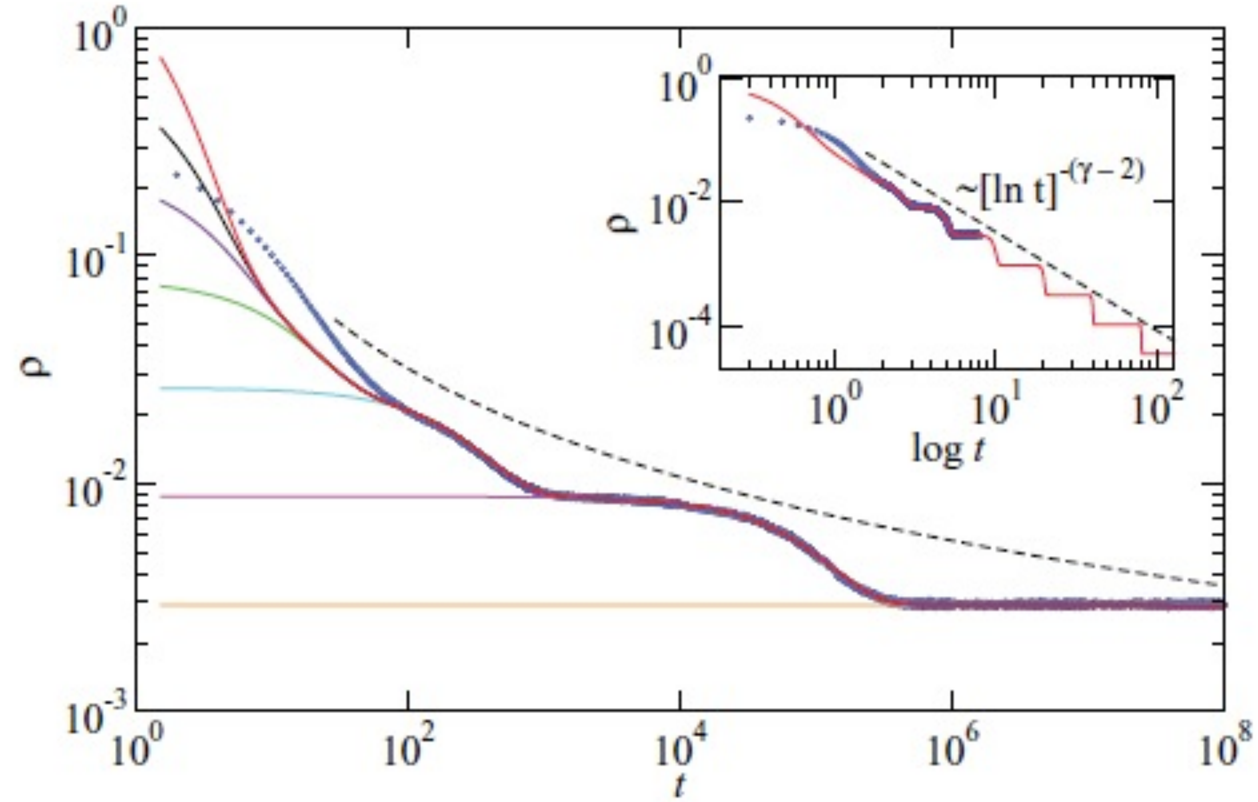

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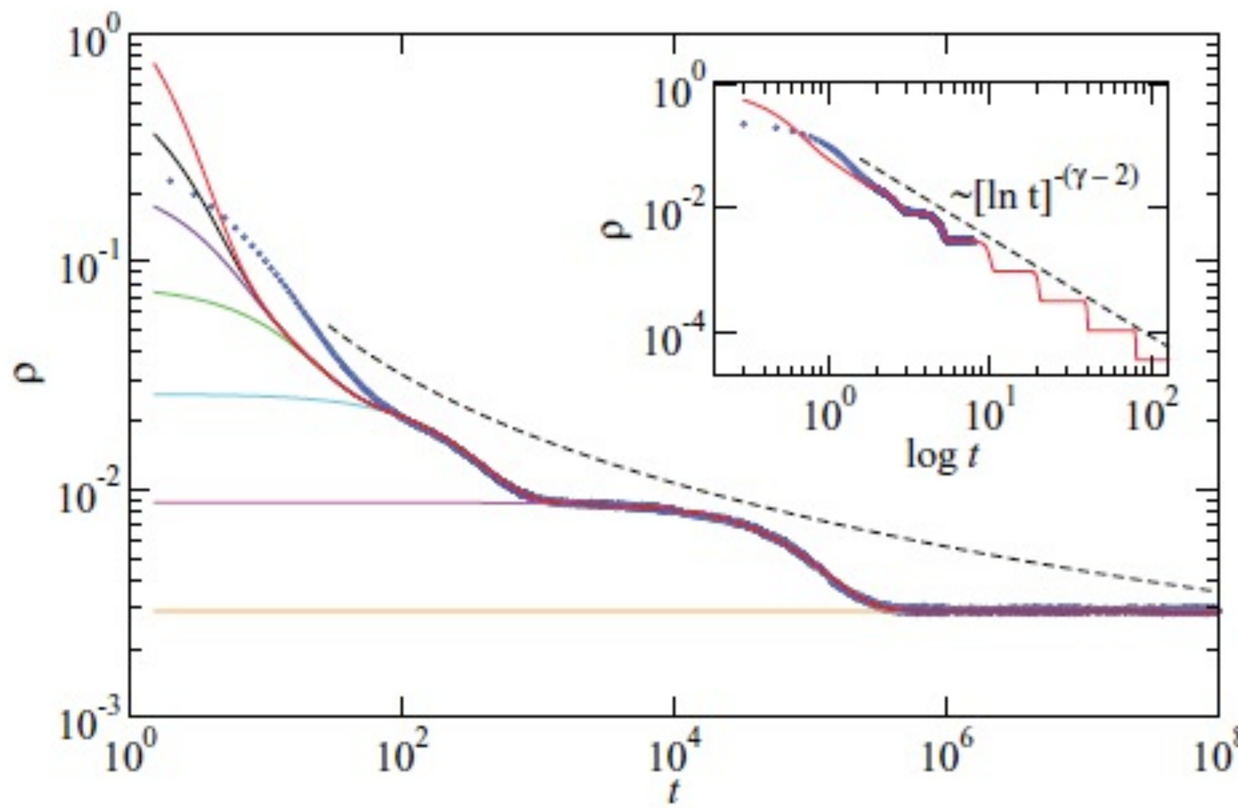

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- For $\gamma > 3$ hubs are **not** in direct contact with each other:
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A new analytical approach: Beyond QMF

Boguna, Castellano and Pastor-Satorras, Phys. Rev. Lett.. (2013)

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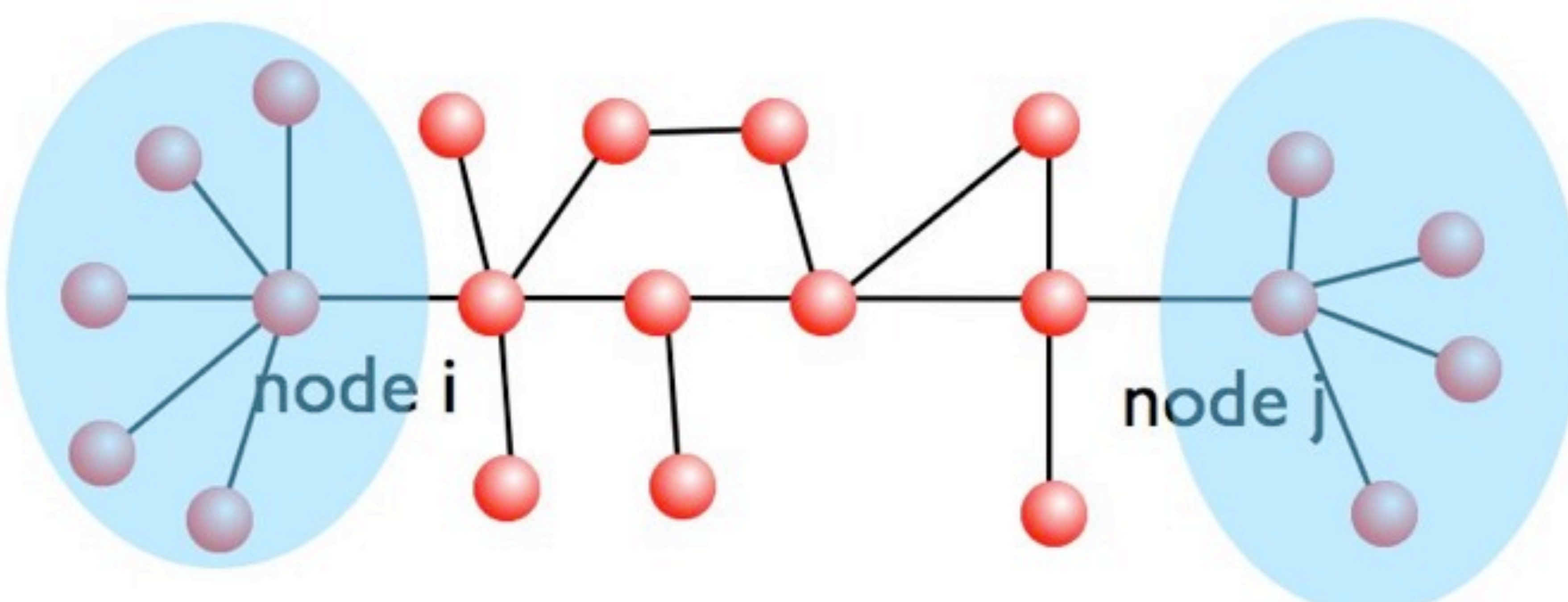
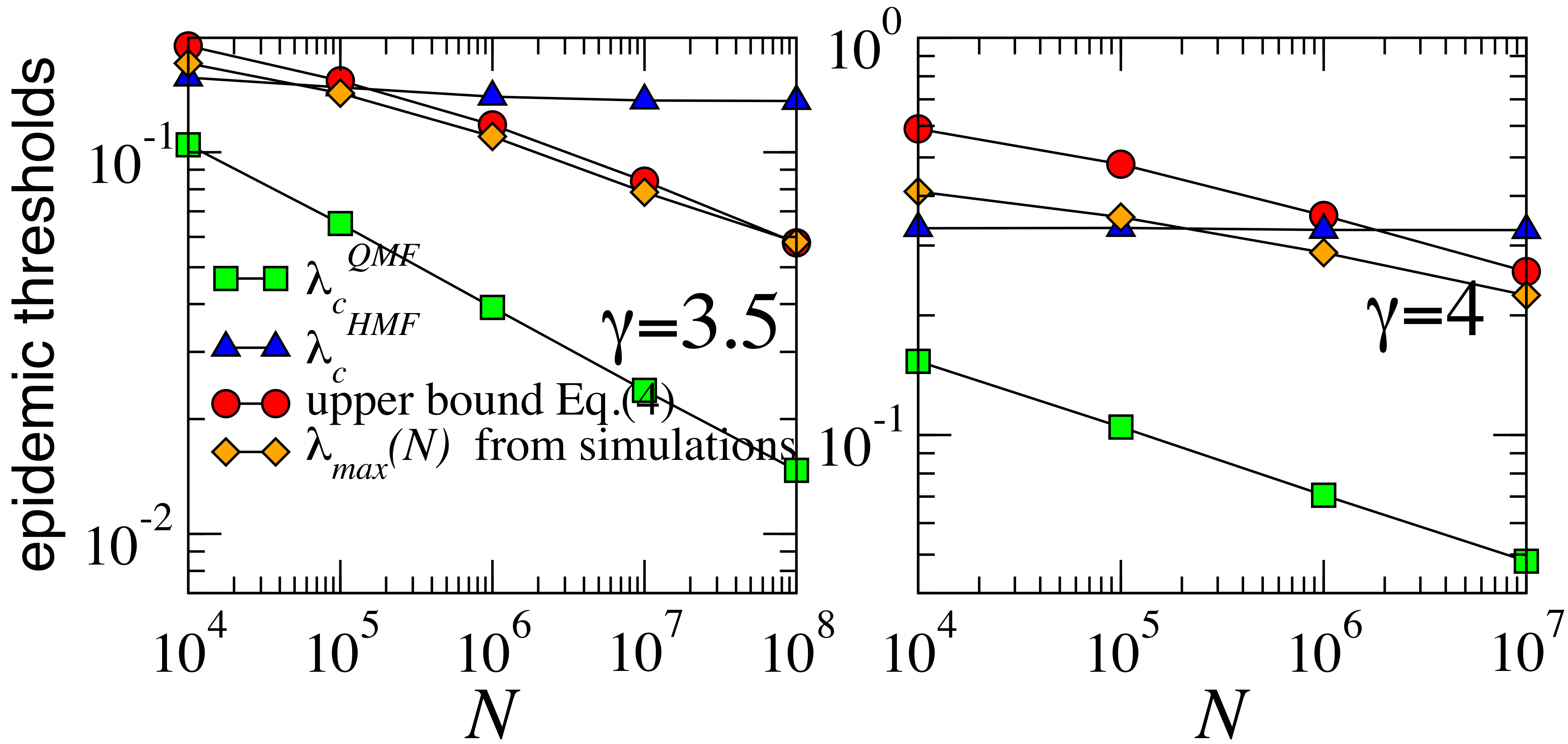
- QMF completely neglects dynamical correlations
- Lee et al. approach includes local dynamical correlations (reinfection between nearest neighboring hubs)
- New approach: take into account also dynamical correlations between distant neighbors (reinfection among distant hubs)

On long time scales reinfections can happen over long distances



New numerical simulations

Boguna, Castellano and Pastor-Satorras, Phys. Rev. Lett.. (2013)



Eigenvector centrality localization

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- For $\gamma > 5/2$: eigenvector localization around hub

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  Inverse participation ratio

$$Y_\Lambda = \sum_i f_i^4(\Lambda)$$

Eigenvector centrality localization

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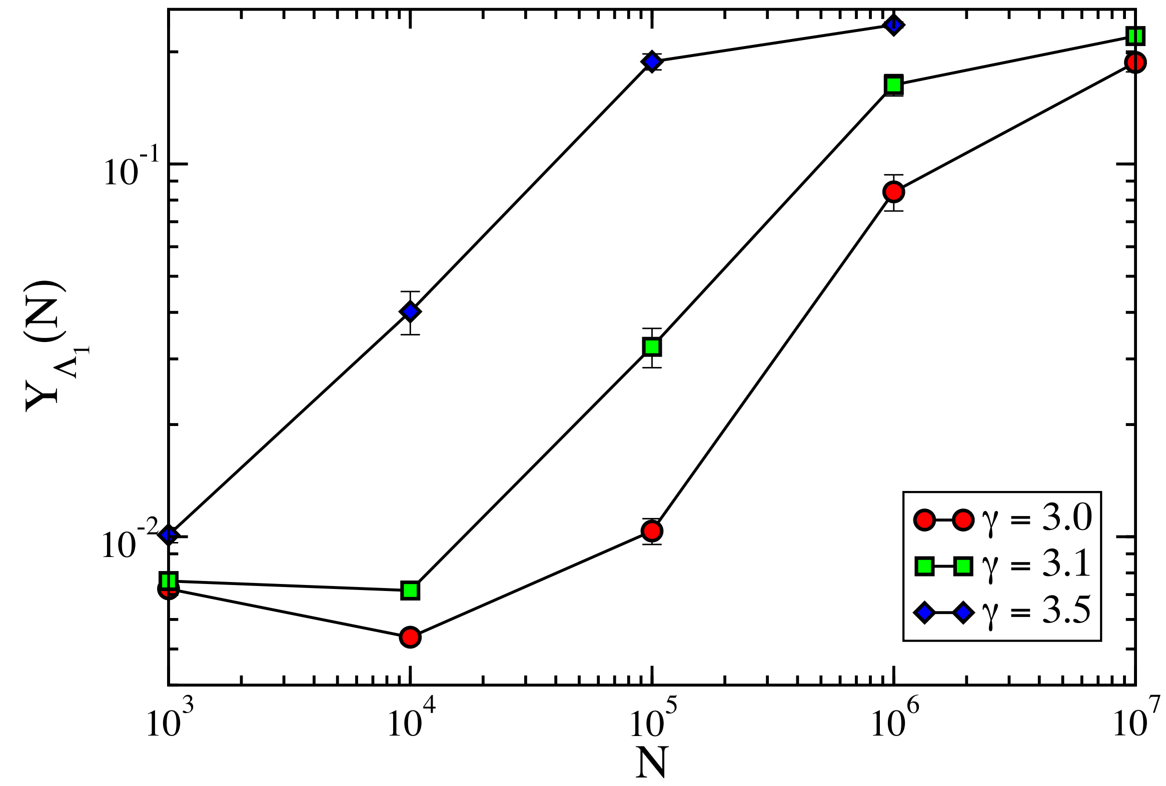

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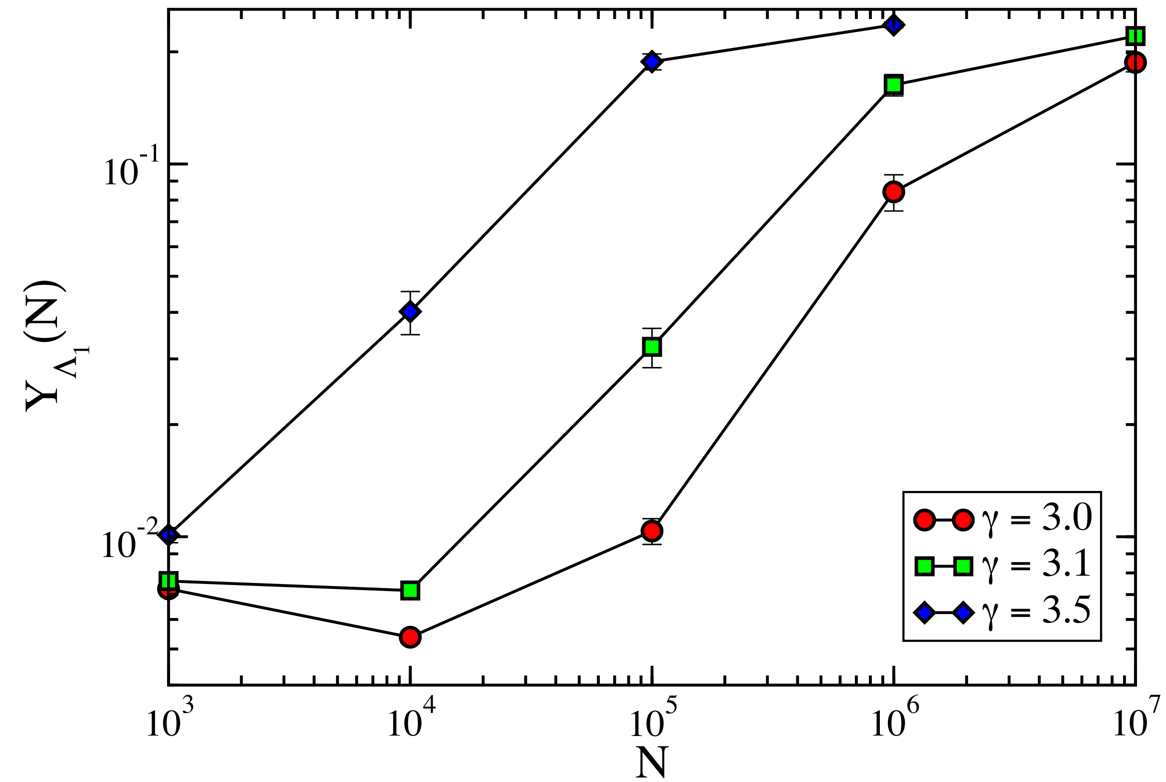

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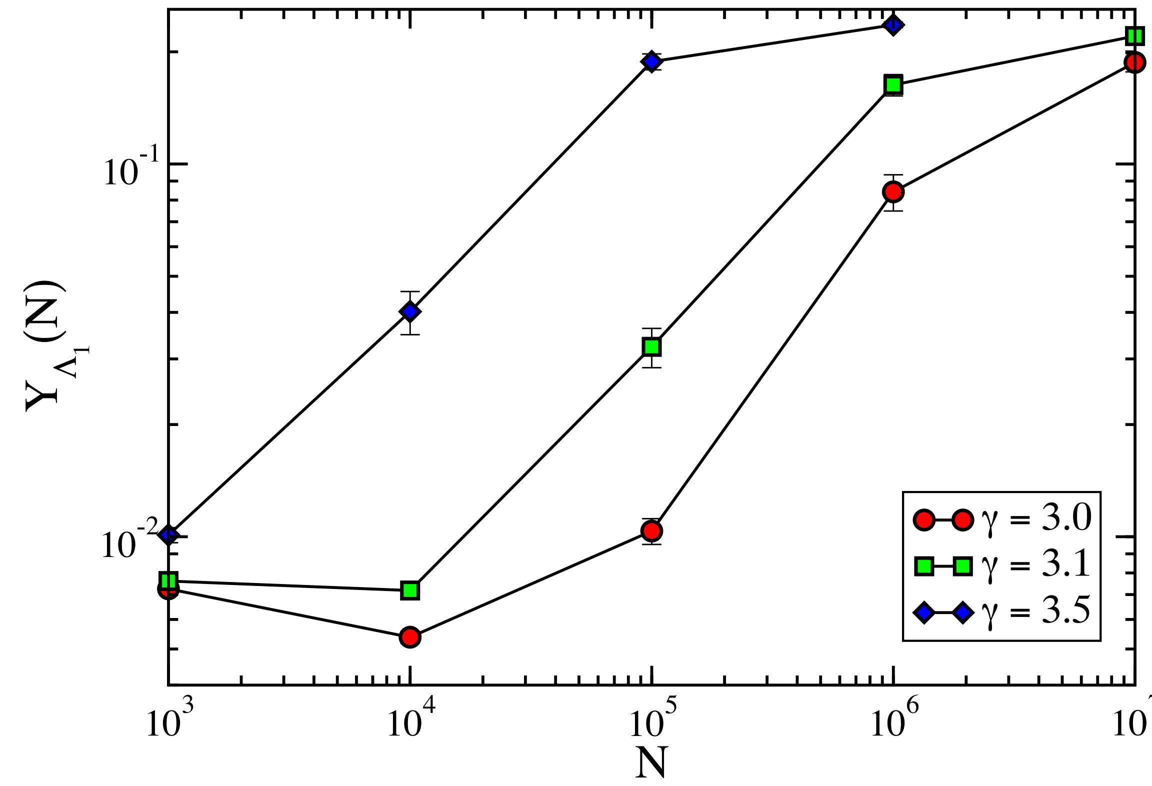

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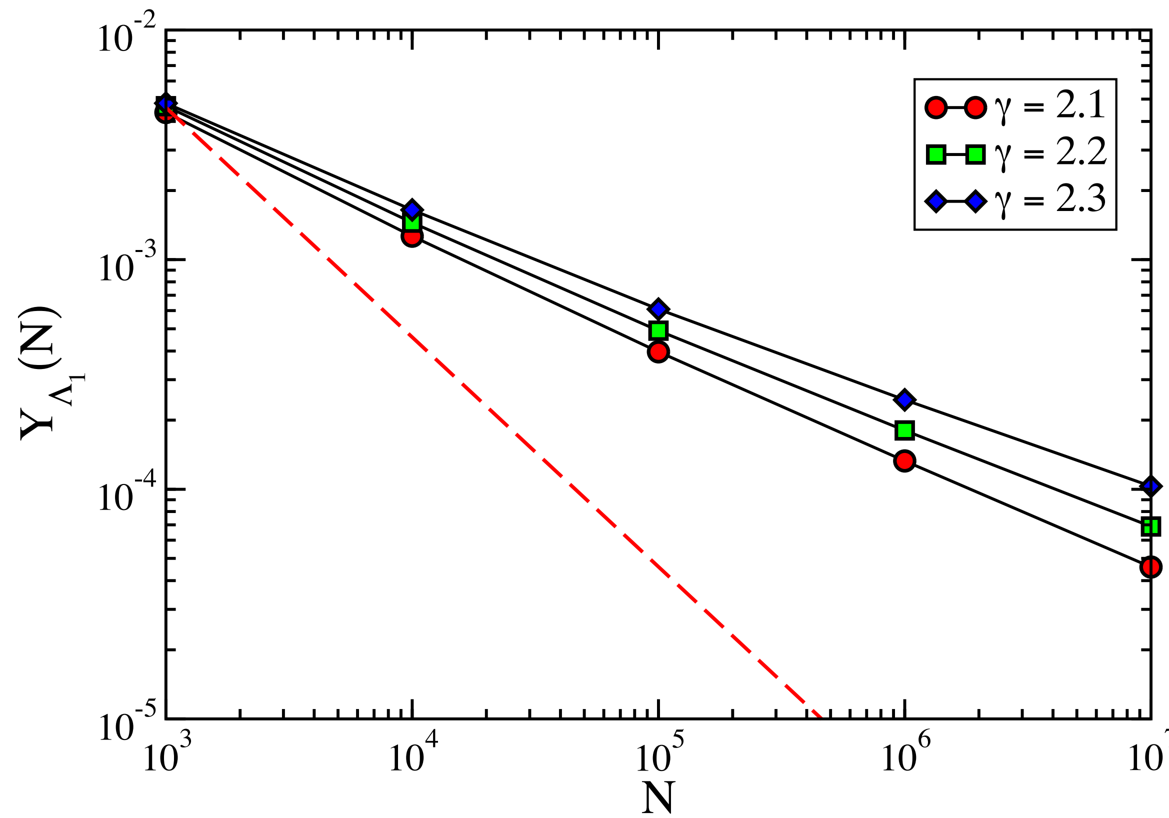
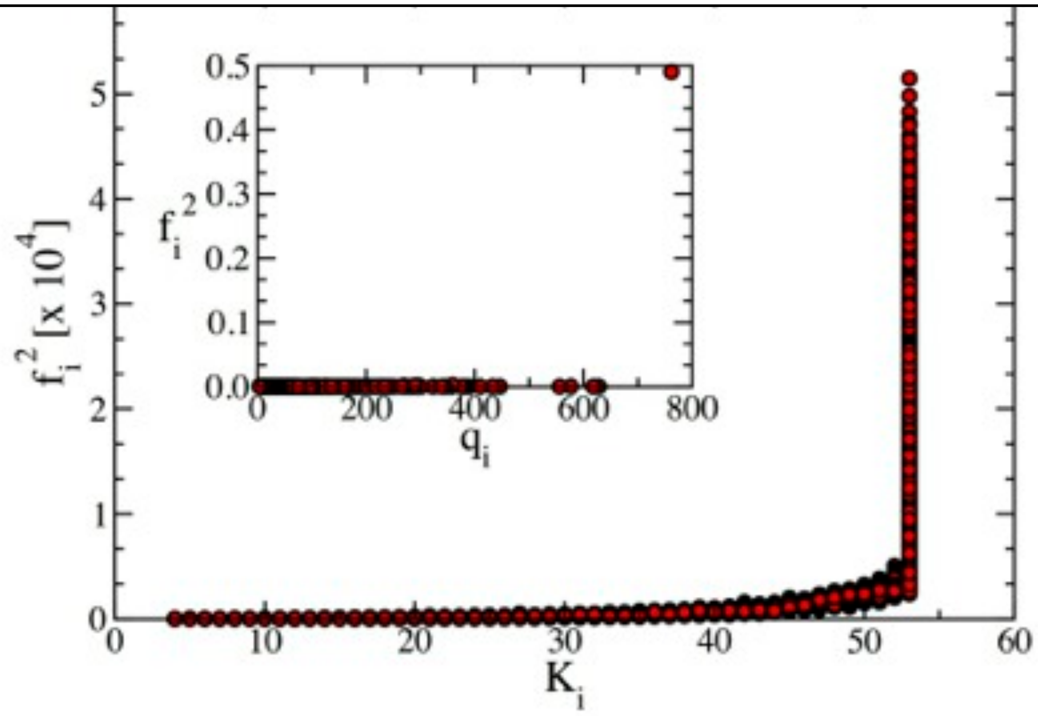


A different type of localization

Nodes in max K-core have large centrality



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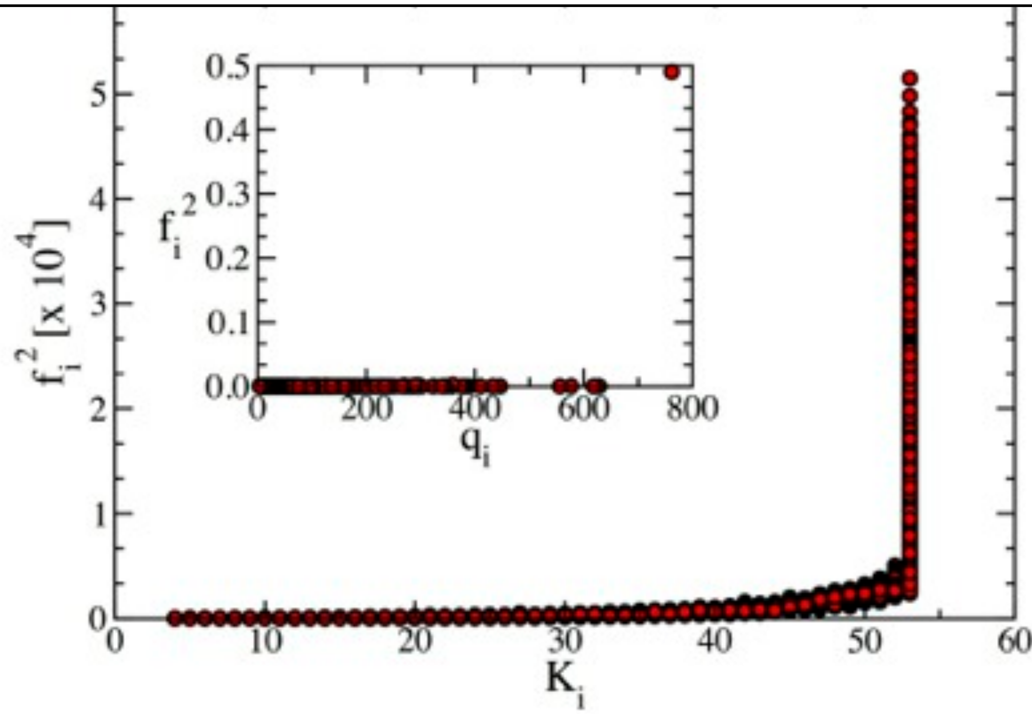

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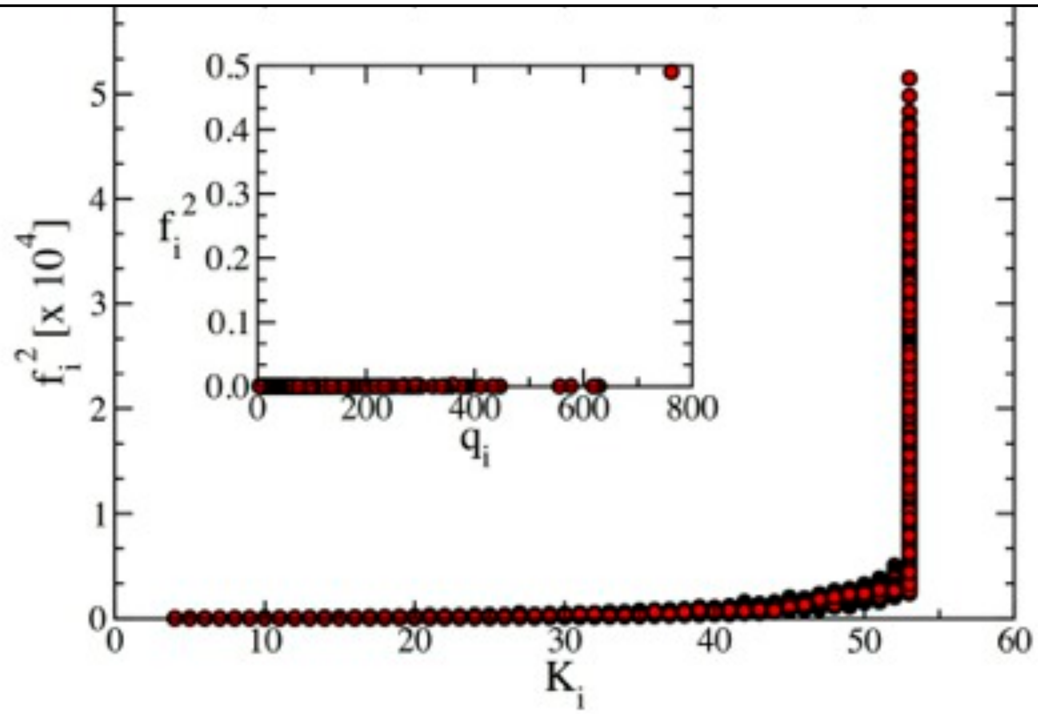

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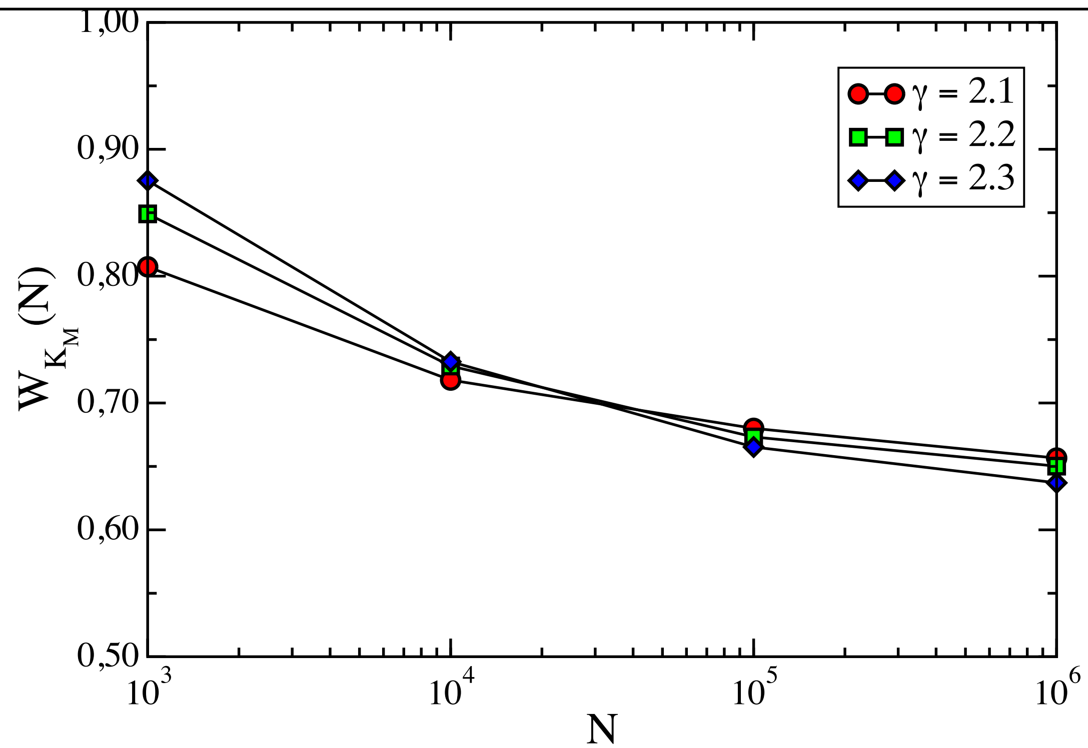

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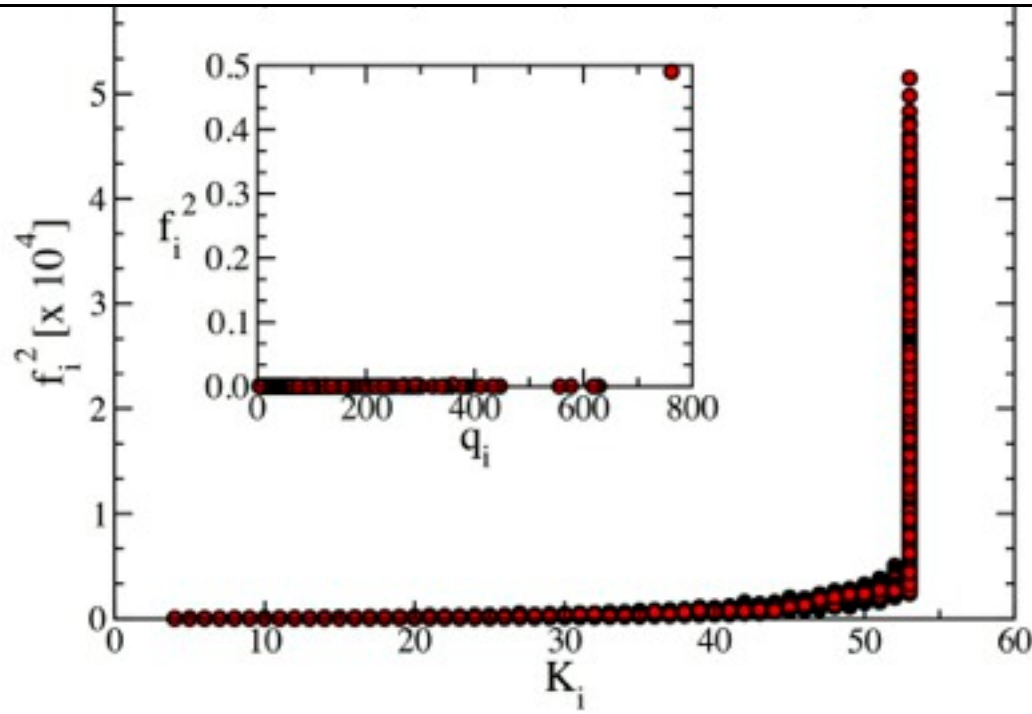

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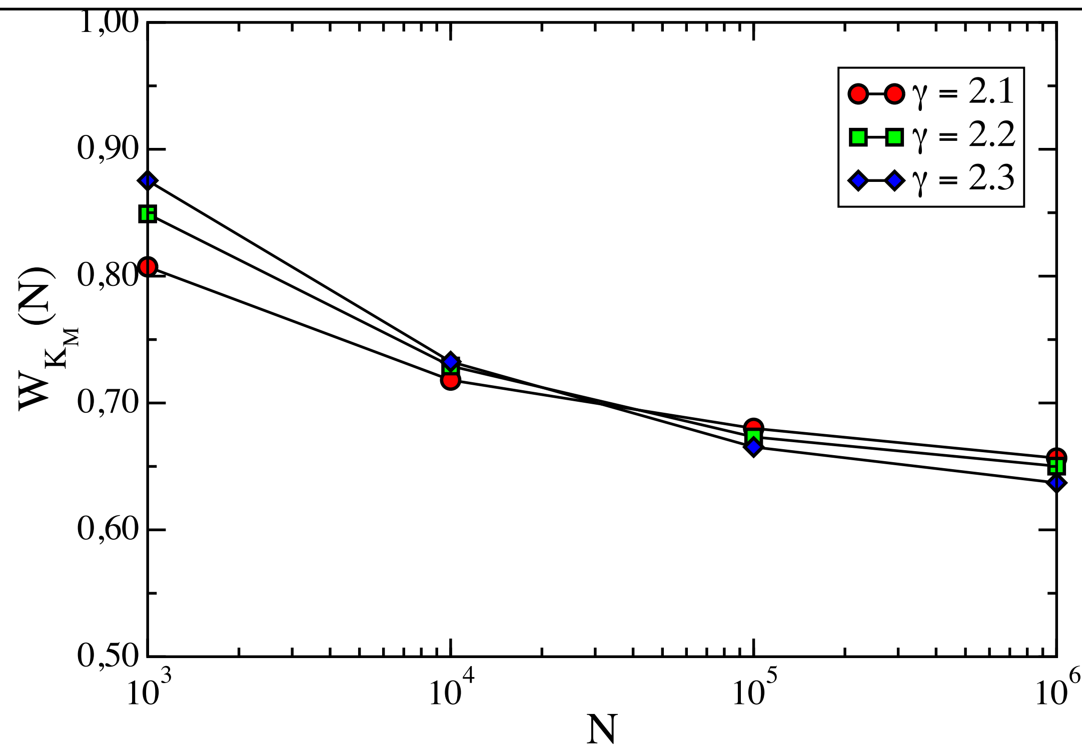

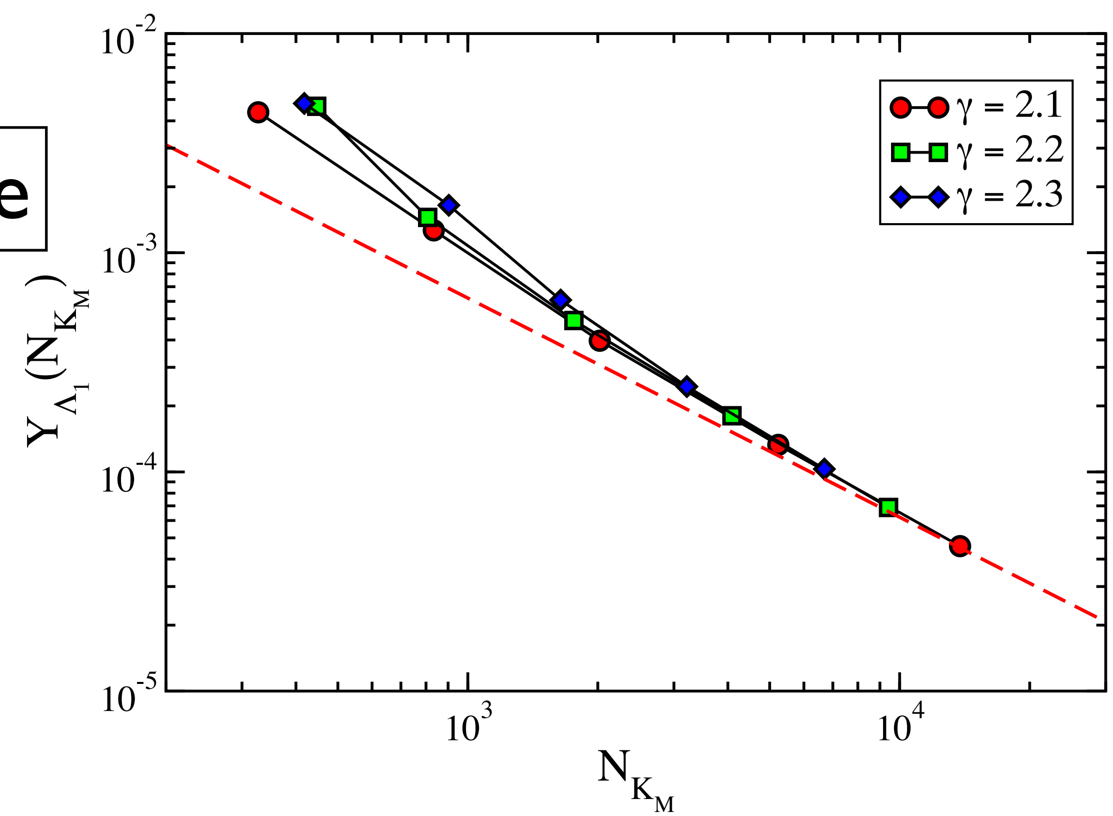

Localization on the max K-core

A new type of eigenvector centrality localization

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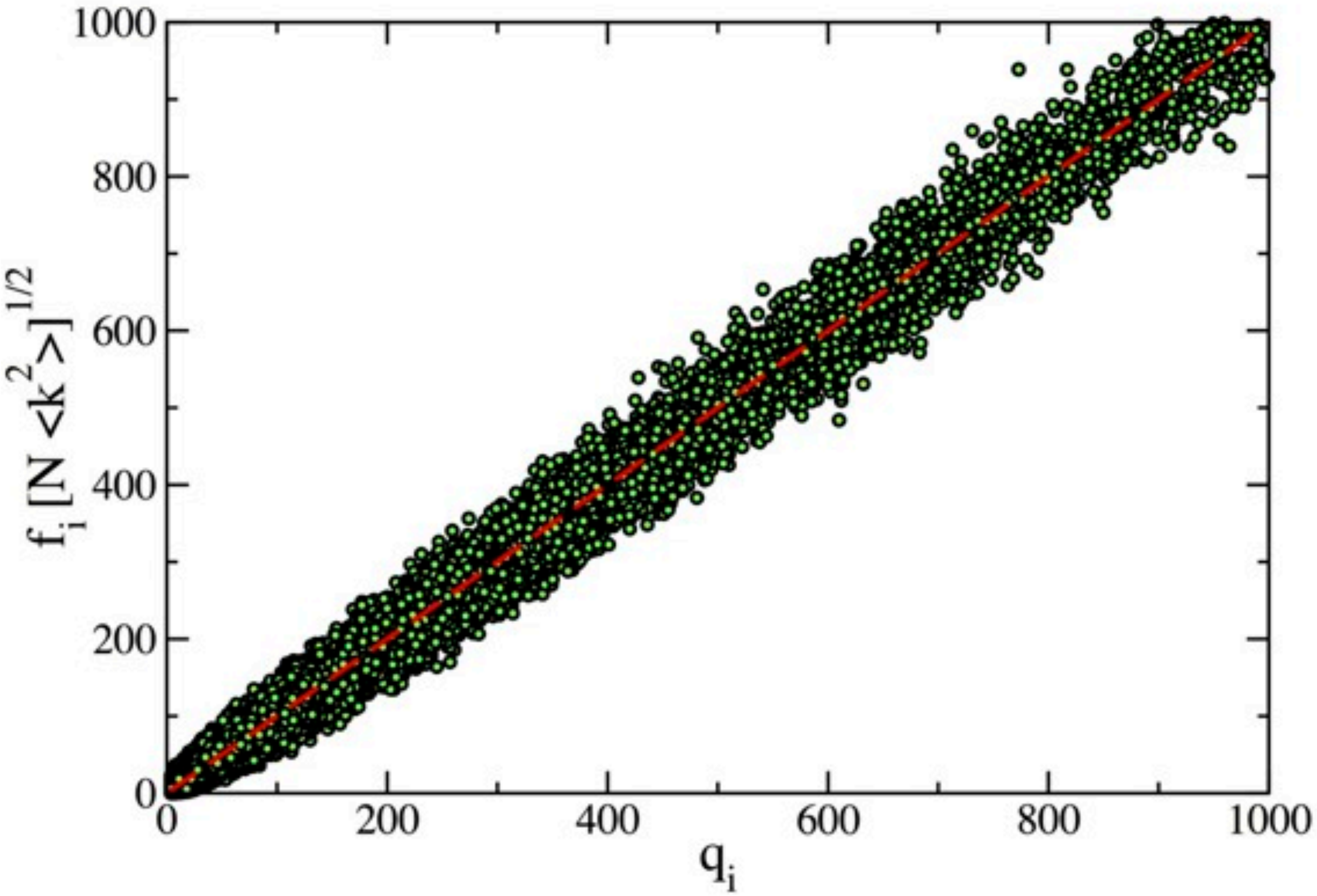

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- For $\gamma > 5/2$, EC is localized around the hub: the hub is central because of its neighbors, which are central only because of the hub.

- For $\gamma < 5/2$, EC is localized around a mesoscopic subgraph: the max K-core.
  It is just proportional to degree centrality: $f_i \sim k_i$

An alternative centrality

An alternative centrality

- Martin, Zhang and Newman propose to use an alternative centrality, computed starting from the components of the leading eigenvector of the Non-Backtracking matrix

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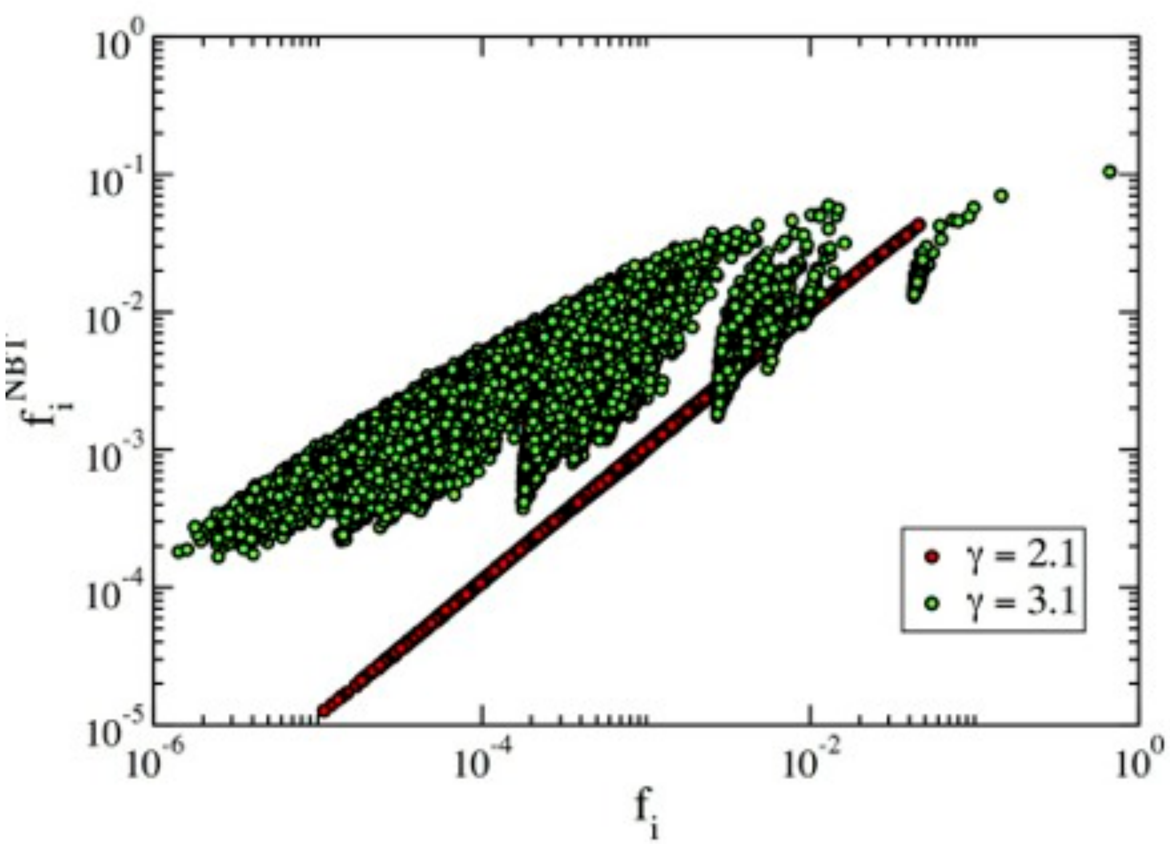

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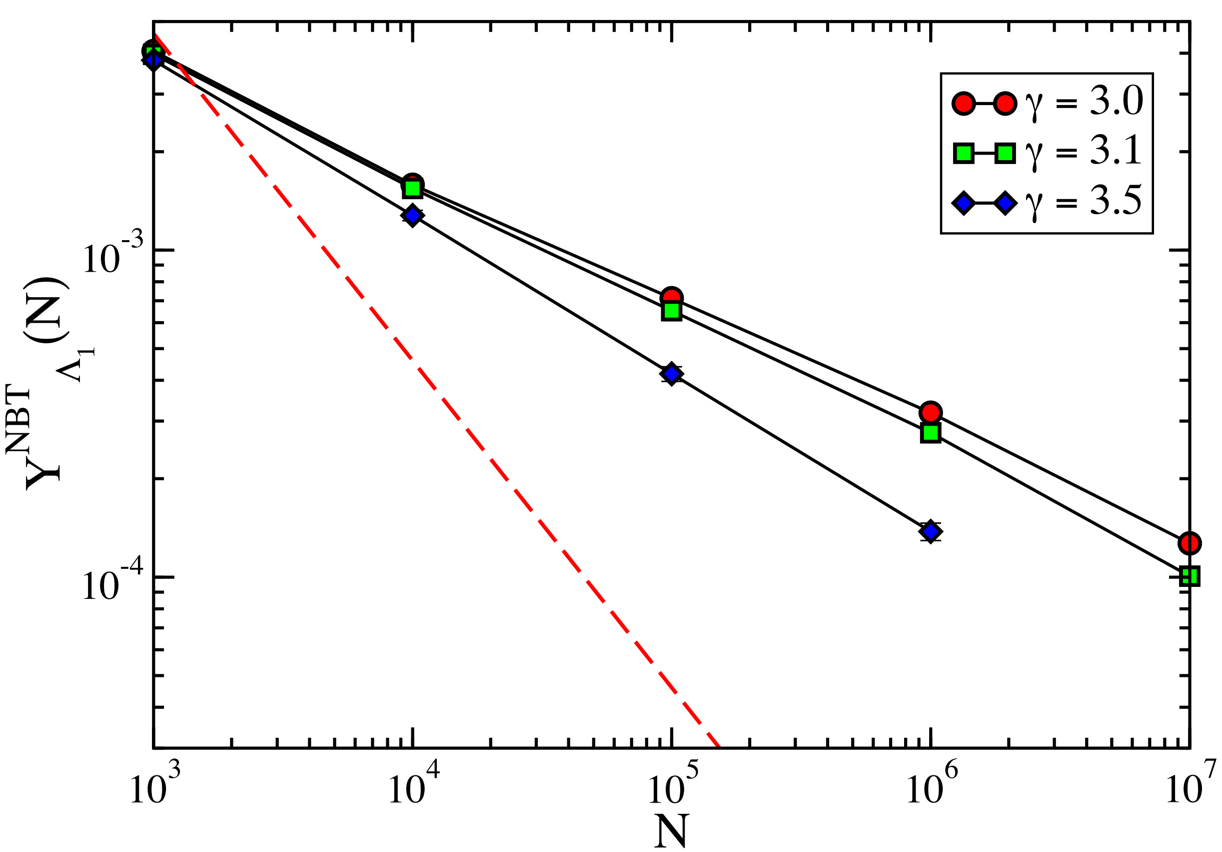

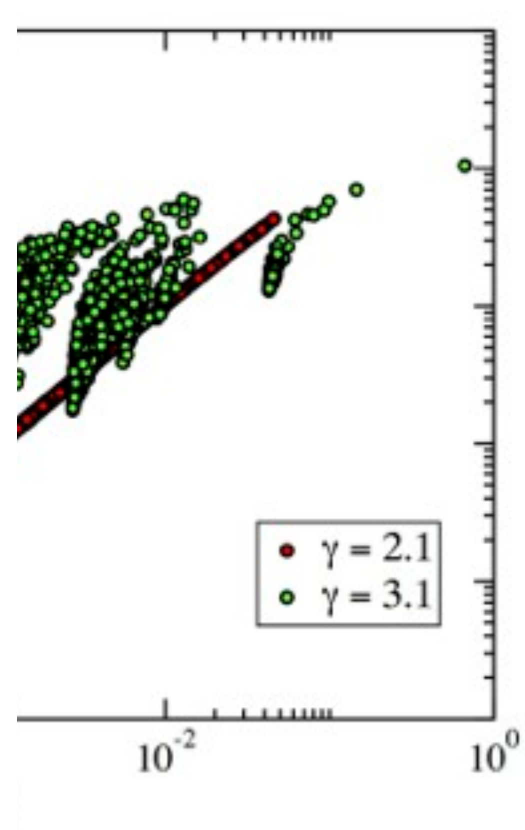

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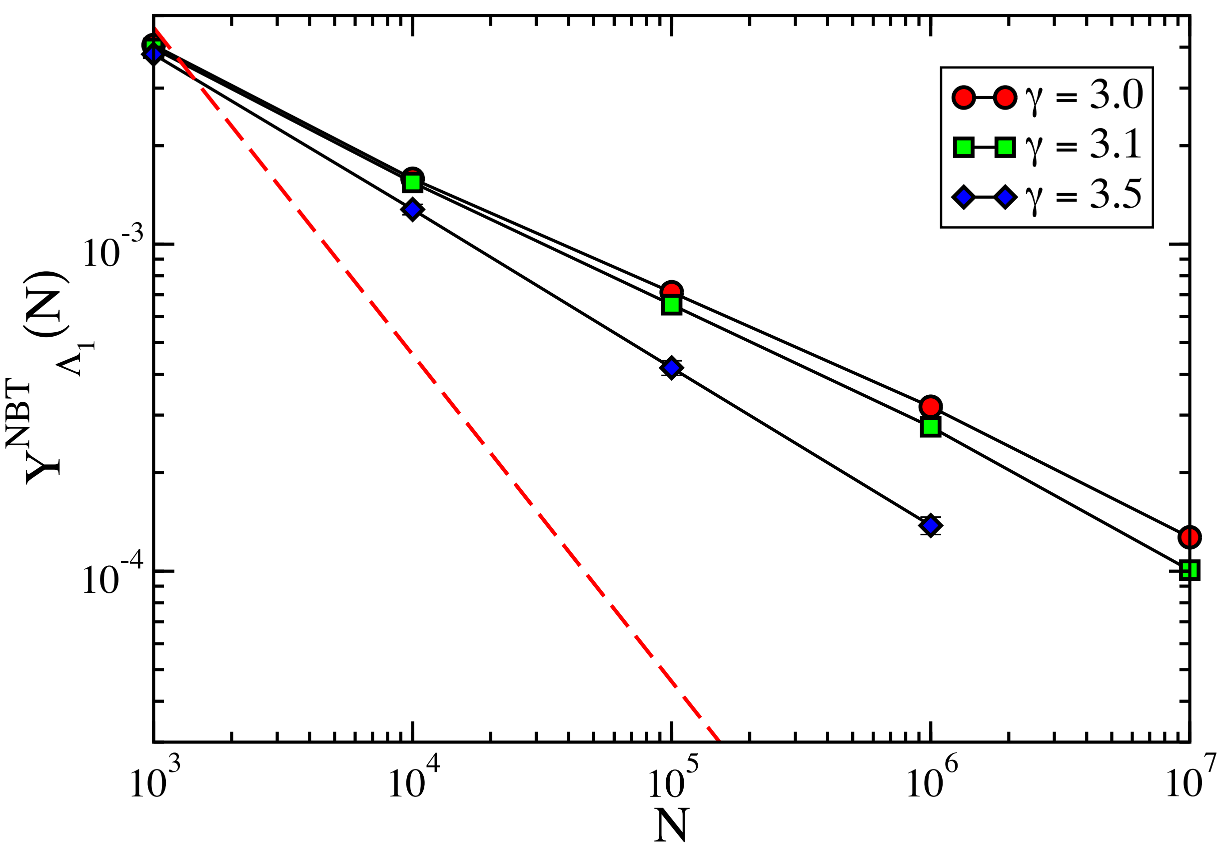
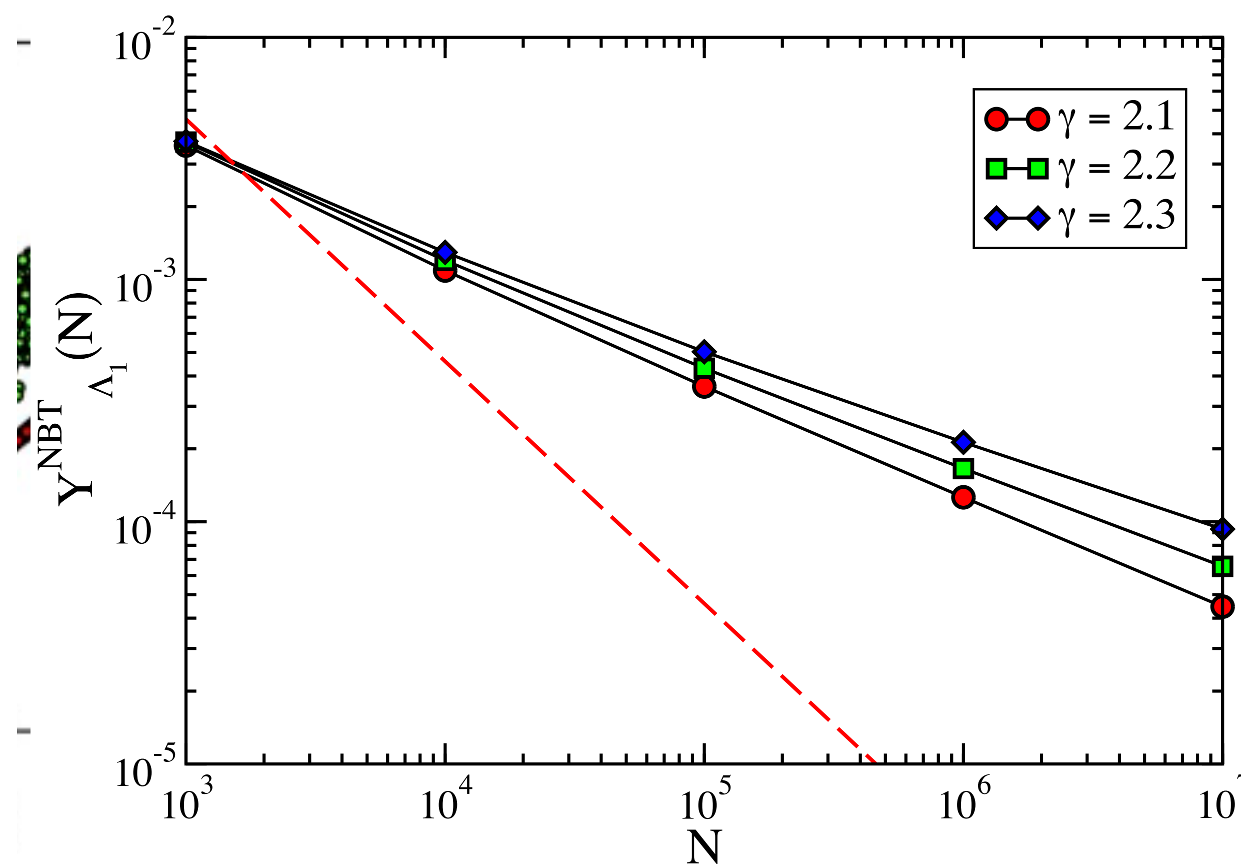
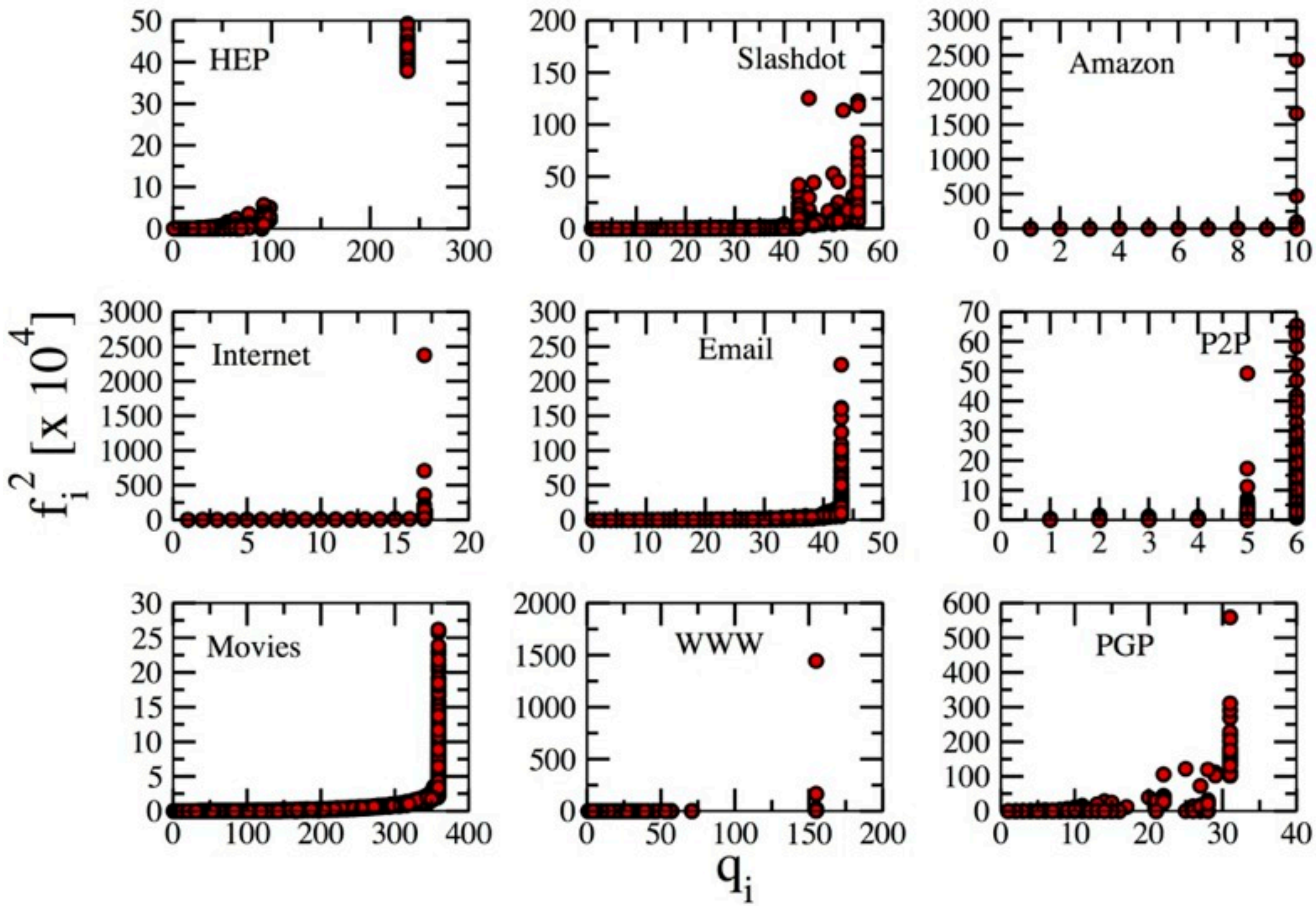




Real Networks



Take home message

- SIS epidemic threshold always vanishes in the large size limit
- Mean-field approaches capture only part of the picture
- Depending on heterogeneity
  - Different mechanisms trigger the epidemic transition
  - Different types of eigenvector centrality localization may occur
- Networks with $\gamma < 5/2$ are much different from those with $\gamma > 5/2$

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