Decoding microwave modulation transfer: The impact of dissipation through stochastic processes

Cacciari Ilaria and Ranfagni Anedio

Istituto di Fisica Applicata "Nello Carrara", Consiglio Nazionale delle Ricerche, Via Madonna del Piano 10, Sesto Fiorentino (FI), 50019, Italy

E-mail: i.cacciari@ifac.cnr.it

An anomalous transfer of modulation from a modulated microwave beam (F_2) to an unmodulated one (F_1) has been experimentally demonstrated through accurate delay time measurements. These results are analyzed according to two possible interpretations: one based on an electromagnetic approach as reported elsewhere, and the other based on a purely stochastic model. This latter model, made up of random zig-zag paths, is the focus of the current paper. It is demonstrated that it is possible to obtain a plausible description of the experimental data.

Introduction: An unexpected modulation transfer from a modulated to a non-modulated (c.w.) microwave beam has been well-demonstrated in several papers, inaugurated by the pioneering one published 20 years ago [1]. In the years, numerous other works on this topic have been presented [2]. Recently, a work of particular importance dealt with studying the delay time observed during the modulation transfer [3]. In that work, the anomalous phenomenon is explained according to two possible interpretations. A first explanation is based on stochastic processes. In fact, the delay time is interpreted as resulting from random zig-zag paths experienced by the "particle": a type of motion equivalent to the telegrapher's equation [4, 5]. The second interpretation is based on a purely electromagnetic approach, which interprets the delay time data as due to competition (interference) between two waves. This second theoretical insight has recently been proposed in two other contributions [6, 7]. A question can be evidently posed searching for a possible link between these two, apparently so distant, kinds of approach.

The present work is devoted to deepening the investigation on the first interpretation, in light of more accurate experimental results provided. In particular, the theoretical model proposed here, in analogy to what was done in another work [8], describes also, the role played by dissipation.

In this work, we briefly summarize the key aspects of the proposed theoretical model. We present a new representation of experimental delay-time measurements, along with new results related to modulation transfer characteristics, and discuss the outcomes of testing the model.

Theory: As mentioned above, the stochastic modelling introduced to explain the experimentally observed anomalous modulation transfer is based on the equivalence of the telegrapher's equation with the zig-zag motion of particles [4]. This equivalence was first developed by DeWitt–Morette and Foong [9], who demonstrated that a solution to the telegrapher's equation can be expressed by a quadrature in which a density distribution $g(r, t)$ of a randomized time r (t being the normal time) enters. Namely,

$$F(x,t) = \int_{-\infty}^{+\infty} [\alpha\phi(r,t) + \beta\phi(x,-r)]g(r,t)dr, \tag{1}$$

where $\phi(x, r)$ is a solution of the wave equation without dissipation, α and β being arbitrary mixing coefficients so that $\alpha + \beta = 1$.

In our preliminary papers, dealing with a possible plausible theoretical interpretation of this anomalous phenomenon [10], an inversion of roles between r and t has been adopted. In that framework, it was assumed that r becomes an observable quantity, as generally manifests in classically forbidden processes [11].

According to reference [12] (see also references [5] and [10]), the average of $r(t)$ is given by

$$\langle r(t)\rangle = \int_{-\infty}^{+\infty} rg(r,t)dr = \frac{1}{2a}\left(1 - e^{-2at}\right), \tag{2}$$

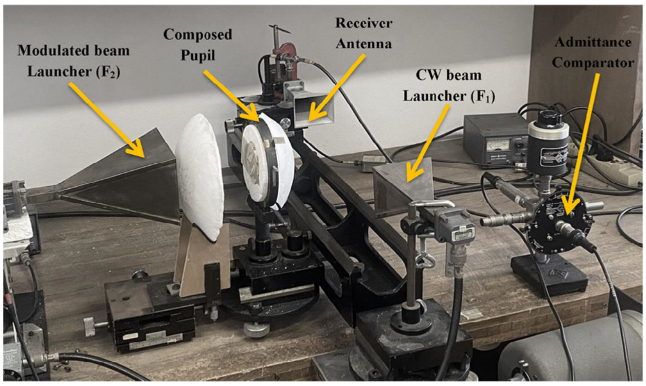

Fig. 1 *The experimental setup operated at 9.3 GHz. The typical geometry consisted of two horn antennas as launchers for the F_1 c.w. beam and the F_2 modulated beam, travelling through a composed pupil to reduce its width. The receiver horn antenna was placed at distance ρ in front of the F_1 launcher. The admittance comparator was also employed*

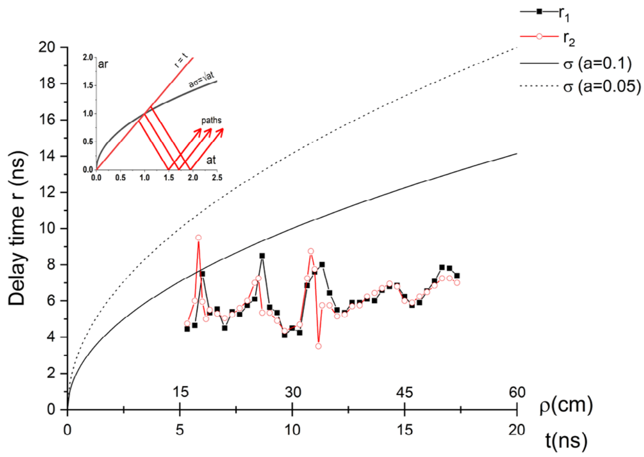

Fig. 2 *Two determinations of delay time, in the transfer of modulation, measured as a function of the distance ρ between the launcher of F_1 and the receiver horn. The continuous line represents $\sigma(t)$, with $t = \rho/v$, determined for $v = 3$ cm ns^{-1} and $a = 0.1 \times 10^9$ s^{-1}, while the dotted line is obtained for $a = 0.05 \times 10^9 s^{-1}$. Ideal paths with reversals are represented in the (ar, at) plane of the inset, after reference [3]*

where a is the dissipative parameter entering the telegrapher's equation, $t = \rho/v$ with ρ being the travelled distance and v the velocity. The density distribution of r can be approximated by a Gaussian as [10]

$$g(r,t) \simeq (a/2\pi t)^{1/2} \exp\left(-ar^2/2t\right). \tag{3}$$

Equation (3) is valid asymptotically, for $t \gg r$, and, in the same limit, its standard deviation is given by $\sigma \simeq \sqrt{t/a}$, with $t = \rho/v$.

Delay-time results: The experimental setup, similar to what has been described in various previous works [2, 3, 5–7], consists of two crossed microwave beams, one of which (F_1) is in continuous wave (c.w.) and the other (F_2) pulse-modulated. Both beams were derived from the same generator at $\nu = 9.3$ GHz. The schematic representation of the experimental setup is presented in Figure 1. The observed modulation transfer, between the two beams, consists of the fact that the originally unmodulated beam appears to be partially modulated, as revealed by the signal detected after the receiver horn, placed at a distance ρ, in front of the F_1 launcher.

The signals taken, one before the F_2 launcher, the other after the receiver horn, were sent to a high-temporal resolution oscilloscope. This allows measuring the delay time with an accuracy of a few tens of picoseconds.

The delay time data, measured in the modulation transfer between two microwave beams, is taken from reference [6] and are shown in Figure 2.

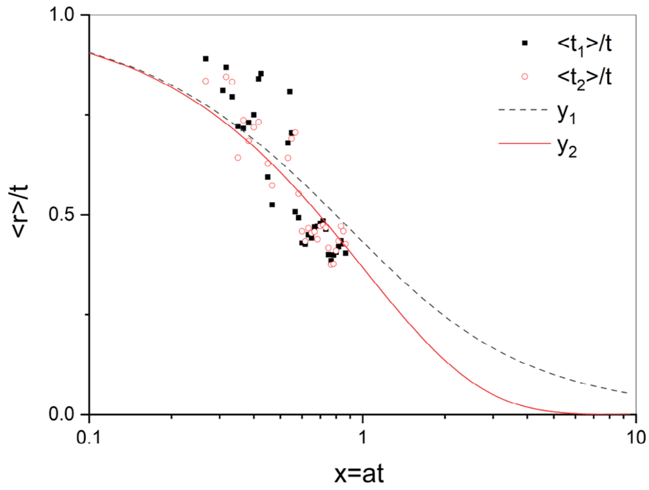

Fig. 3 *Average time $\langle r(t)\rangle$ normalized to t, versus the dissipative parameter a times t, x, evaluated using Equation (4), y_1, or $y_2 = e^{-x}$, for $a = 0.05 \times 10^9$ s^{-1}. Experimental results of $r(t)/t$ are taken from the two determinations in Figure 2*

They are obtained as an average between results corresponding to rise- and fall-time measurements of the pulse modulation.

In Figure 2, it is also disclosed that the shape of the data can be reasonably considered representative of the hypothesized zig-zag random path. As regards the estimate of the extension of the average steps of these paths (Δr), for $r^2 = \sigma^2 = t/a$ and $r \approx t$, we obtain $\Delta r \lesssim r = 1/a$ [3].

For a of the order of 10^9 s^{-1}, Δr turns out to be of the order of ns, and the corresponding $\Delta\rho = v\Delta r$, for $v = 3$ cm ns^{-1}, results to be of a few centimeters. These values are roughly comparable with the corresponding variation in Figure 2. Moreover, it should be noted that the velocity v, in this context, has to be comparable with $\Delta\rho/\Delta r$. Indeed, in Figure 2, the curves representing σ are evaluated for $a = 0.1 \times 10^9$ s^{-1} (continuous line) and $a = 0.05 \times 10^9$ s^{-1} (dotted line), maintaining $v = 3$ cm ns^{-1}. These curves plausibly represent the boundary line of the half area containing the random paths with a probability of $\sim 68\%$.

Model testing: As a test of the model, the data of Figure 2, once normalized to $t = \rho/v$, are reported in Figure 3 where are compared to Equation (2), also normalized to t, which become

$$\langle r(t)\rangle/t \equiv y_1 = (1/2x)\left(1 - e^{-2x}\right), \tag{4}$$

where $x = at$. The curve of $y_1(x)$, for small values of x is almost coincident with $y_2 \simeq e^{-x}$ [13][1]. By increasing x, the difference between them becomes more evident but the values supplied by y_2 are to be considered as more appropriate for the data fitting [8].

From the inspection of Figure 3, we evidence that the positions of the data, determined considering for the parameter a the value of 0.05×10^9 s^{-1}, turn out to be reasonably situated around the curves y_1 and y_2. The spreading of the data is evidently due to their undulation as shown in Figure 2.

As a further test of the model, we have investigated the modulation-transfer nature exploiting an experimental procedure based on an admittance comparator, as in reference [14]. This methodology allows to determine the propagation admittance components, by directly reading the conductance and susceptance scales of the comparator [15]. To do this type of measurement, the modulation at 30 MHz of the same carrier at 9.3 GHz, detected at the receiver horn antenna, was connected to the detector line of the admittance comparator, after a suitable attenuation. The attenuation value is selected to make the detected signal roughly comparable with the output of the comparator. The connection was realized through a coaxial cable of about one-half of the guided wavelength at 30 MHz (~ 3 m of cable), to reproduce, at the connection into the detector of the comparator, nearly the same situation detected at the receiver horn.

[1]This approximate relation, can be deducted from a transition-element analysis, as demonstrated in reference [8] in the limit of $a/2\omega \ll 1$, $\omega = 2\pi\nu$ being the circular frequency of the carrier.

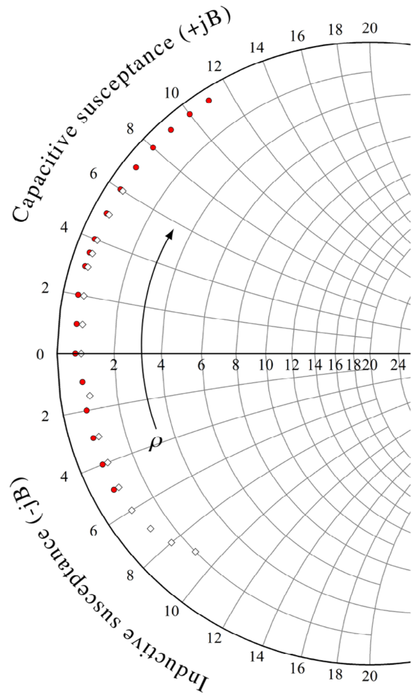

Fig. 4 *Portion of the Smith chart with two sets of admittance values, which have been obtained for two values of the lengths of the coaxial cable connecting the receiver to the admittance comparator. In one case ●, the length is about λ/2 at the modulation frequency of 30 MHz. In the other case (◇), with the length of the cable shortened by about* 10%*, the data are rightly translated in favour of the inductive zone. An opposite effect is observed when the length of the cable is more than λ/2*

The measure of the admittance critically depends on the length of the coaxial cable, as well as on the attenuation introduced to the received signal. Thus, the results of this measurement have only a semi-qualitative character. However, despite this criticality, they demonstrate an evident different nature when the distance ρ is varied from a few centimeters $(8 - 10)$ to a maximum value of ~ 42 cm.

The results are shown in the Smith chart with 20 mΩ^{-1} of characteristic admittance (see Figure 4) where two sets of data, relative to two lengths of the coaxial cable, are reported.

In particular, for $\rho \lesssim 14$ cm, the data result to be situated in the lower, inductive zone of the susceptance, while, for $\rho \gtrsim 14$ cm, they are positioned in the upper part of the diagram, denoting an evident capacitive behaviour of the susceptance.

This fact, in analogy with the case of surface waves [14, 16], represents a situation of fast waves for the capacitive character, while it is of slow waves for the inductive one. There is, however, a significative difference with respect to the case of the complex waves, where the resulting admittance values resulted to be typically complex (see fig. 4 in reference [14]), while in the present one, they turn out to be nearly purely imaginary. This fact seems to agree with the small value of the dissipative parameter ($a = 0.05 \times 10^9$ s^{-1}) required for fitting the data in Figure 3.

Another important difference consists in the fact that, still according to the results of delay as represented in Figure 3, where the ratio $\langle r\rangle/t$ tends to decrease with increasing $x = at$, the time t, as shown in Figure 2, being obtained by ρ/v, with $v = 3$ cm ns^{-1}, which represents a situation of very slow velocity. Hence, when it is said, as before, fast and slow waves, they are to be considered with respect to this very small value of velocity.

Discussion and concluding remarks: The set of results reported above confirms the validity of the stochastic model expressed by Equation (2), hence the curve $y_1(x)$, but even too $y_2(x)$, as capable of interpreting the experimental results in an alternative way.

Table 1. Comparison of the quantities (Δr, $\Delta\rho$) for the two different theoretical models with $v = 3$ cm ns^{-1}

	a (10^9 s^{-1})	Δr (ns)	$\Delta\rho = v\Delta\tau$ (cm)	Ref.
Experiment	–	2−4	∼9	[6]
Electromagnetic model	1.17	2.66	8	[6]
Stochastic model	0.1	<10	<30	This work
Stochastic model	0.1	3	9	This work

A reasonable agreement is achieved for the values in the second line of the Stochastic model.

It is particularly interesting that the observed phenomenology, specifically the transfer of modulation between the two microwaves beams, has received two different interpretations. As mentioned since the Introduction, one way of interpreting this phenomenon is based on the competition between two waves: one due to a saddle-point contribution and the other to pole singularities situated near the saddle point [6, 7].

The interpretation adopted in the present work is quite different from the other one, being based on a stochastic process consisting of random zig-zag paths experienced by the 'particle', in analogy with the telegrapher's equation [4, 5]. What the two approaches have indeed in common is the role played by the dissipative parameter a entering the telegrapher's equation. In the cases of reference [6], this parameter determines the propagation constant $2a\rho/v$. In the present case, its role is more explicit, as it directly determines the average of the randomized time, identified with the observed delay-time, and its density distribution.

However, there is a substantial difference in the value of the parameter a required in the two different interpretations. In the case of the more conventional electromagnetic approach [6], the required value for a is such that $a/v \simeq 0.39$; hence, for $v = 3$ cm ns^{-1}, a turns out to be $\sim 1.17 \times 10^9$ s^{-1}. The variation Δr, if evaluated as $\Delta\rho/v$ for $\Delta\rho \simeq 8$ cm [6], is approximately 2.66 ns, which is more compatible with the experimental data than the value estimated as $a^{-1} \simeq 0.85$ ns.

On the contrary, in the case of the stochastic approach that is described in this work and in reference [5], the required value for a turns out to be decidedly smaller: $0.05 - 0.1 \times 10^9$ s^{-1}.

This last aspect deserves to be re-examined in more detail in an attempt to establish a closer relationship between the two approaches considered.

According to the same approximate relation $\Delta r \lesssim r \simeq a^{-1}$, we should obtain, even for $a = 0.1 \times 10^9$ s^{-1}, $\Delta r \lesssim r = 10$ ns; hence, $\Delta\rho = v\Delta r \lesssim 30$ cm, a value that is excessively large. However, considering that for average values of $at \approx 0.5$ (see Figure 3), the exact value of the distribution $g(r, t)$ is reduced to about $1/3$ of the approximated value $\sigma = \sqrt{t/a}$ ($\simeq a^{-1}$) (see fig. 1 in reference [17], we have $\Delta r \lesssim 1/3a \simeq 3$ ns; hence, $\Delta\rho = v\Delta r \simeq 9$ cm. These latter values can be considered compatible with those previously determined for the electromagnetic model, as summarised in Table 1.

What has been discussed so far only shows that the quantities (Δr, $\Delta\rho$) are indeed comparable in the two different approaches. More important analogies should be sought in the models. One model, based on the telegrapher's equation, involves zig-zag particle motion. The other, based on the competition (interference) of two components characterized by the presence of partially standing waves, can also be interpreted as forward and backward motion, similar to the aforementioned zig-zag motion. In other words, by recalling Equation (1), which, as said before, represents a solution of the telegrapher's equation, when $g(r, t)$ can be considered as mainly constituted by the "undistorted" part $e^{-at}\delta(t - r)$ [9, 12], we have that $F(x, t)$ is given by an attenuated integrand for $r = t$, which represents a situation of partially standing wave, which is peculiar of the electromagnetic model [6].

These insights reveal a compelling connection between the two approaches under consideration.

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