A Statistical Framework for Performance Analysis of **Diversity Framed Slotted Aloha** With Interference Cancellation



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As the Internet of Things (IoT)/machine-to-machine market is rapidly growing, a special role is expected to be played by satellite communications in that they offer ubiquitous coverage and therefore enable typical monitoring, telemetry, and control services also in areas with a poor terrestrial infrastructure. In this respect, the case of massive IoT devices deployment calls for random access solutions, which have been long analyzed by the scientific satellite community in the last ten years. This article further elaborates on the relation between the normalized offered load and the achievable performance in terms of packet loss rate, which was not much addressed so far at high loads. The proposed theoretical framework has been validated through extensive simulation campaigns, which show an excellent match at different loads and number of interfering packets configurations, by significantly improving the results achievable through other existing theoretical frameworks.

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I. INTRODUCTION

The Internet of Things (IoT)/machine-to-machine (M2M) market has been tremendously growing over the last years, because of the compelling need to digitalize industry factories so as to develop the Industry 4.0 concept as well as to provide command and control means to general assets. As a matter of fact, this is not just limited to the case of industry "robotization" or energy plant monitoring but also addresses remote properties and assets, such a ship and cargo tracking, precision-agriculture, and overall comprehends small data sectors. Particularly interesting is the case of IoT services to be offered in remote areas, which is becoming an attractive market with an ever-increasing growth of the overall return-of-investment and with a consequent increase of the number of services and IoT devices being deployed [1].

In this context, particularly appealing is the use of satellite communications, thanks to the ubiquitous coverage and the recent advances in satellite antenna miniaturization so that IoT devices enabled to data communication through satellite are becoming a nonexpensive attractive option [2]. Moreover, the sectors, where the terrestrial infrastructure is not available or in any case the offered network capacity cannot sustain the traffic generated by massive IoT deployment certainly make the satellite the only viable solution. This trend is more importantly confirmed by the interest raised in the 5G standardization task carried out within 3GPP with respect to the use cases of massive machine-type communications [3]. In particular, the defined nonterrestrial network work item entails also the case of direct access from new-radio devices directly to satellite[4], which is well representative of IoT over satellite use cases. In this respect, different satellite systems are being considered as potential technology candidates to enable these services. On the one hand, geosynchronous (GEO) satellites are often considered as a proper solution, because of the large coverage with a single satellite and thanks to well-defined standards as well as consolidated experience in this field from many satellite vendors and operators. On the other hand, medium earth orbit constellations have been receiving more and and more consensus as they offer similar performance of GEO but a lower service delay and necessitate only few satellite platforms more. Finally, the renewed interest toward low earth orbit systems and in particular the plan and the launch of mega constellations (composed of hundreds/thousands satellite platforms) can further boost the market of IoT over satellite.

Independent of the specific satellite systems being considered, one of main design challenges to be faced is represented by the dense deployment of sensor nodes, which may need to access the satellite capacity simultaneously. As such, the problem of media access control is pivotal for achieving efficient data communication. In this respect, it is immediate to see that dedicated-access schemes cannot be considered a viable solution because of the bursty nature of the IoT traffic, which could cause a waste of the allocated satellite resources and ultimately lead satellite operators to reject the allocation requests from many of the IoT devices. On the contrary, the profitability of random access (RA) schemes is certainly much more appealing, also thanks to the evolution of the old ALOHA (in asynchronous and slotted mode) concept. To this regard, the scientific community has carried out important studies in the last ten years, aimed at finding out the best strategy to access the satellite capacity, by taking into account the available degrees of freedom to be exploited. In more details, some of the most important research lines have related to the use of multiple replicas so as to achieve time diversity transmission as well as interference cancellation (IC) implemented at the receiver side, in order to recover from slot collisions occurring especially in the case of moderate and high network traffic loads. Investigations have considered the cases of both slotted and unslotted access, in order to analyze the achievable performance also in relation to the induced implementation complexity in real devices.

More specifically, the scientific community has elaborated several schemes in the last years [5], [6], by taking the consolidated slotted ALOHA (SA) and framed slotted ALOHA (FSA) schemes as baseline and extending them with novel concepts such as time diversity (e.g., in diversity FSA (D-FSA) [7]) and IC, the latter being exploited in many proposals such as contention resolution diversity slotted ALOHA (CRDSA) [8], irregular repetition slotted ALOHA (IRSA)[9], and coded slotted ALOHA (CSA)[10]. In more detail, FSA and D-FSA schemes may introduce significant number of packet losses or even cause undesirable service unavailability. On the other hand, a great performance improvement, in terms of throughput, comes from applying IC to D-FSA. In this case, the replicas transmitted within a media access control (MAC) frame keep a pointer to the twin slots—whenever a clean burst is detected and successfully decoded, the potential interference contribution caused by the twin replica is canceled in the pointed slot, hence resulting in a remarkable throughput improvement with respect to SA. A further improvement may be achieved either by increasing the number of replicas to k, either letting k be a random variable sorted by a discrete distribution [9], or being k the coded fragments of a packet of m bits, with $m < k \cdot b$, and b being the size [bits] of a time slot [10].

The vast literature available so far was mostly focused on asymptotic limits or, in any case, the developed mathematical frameworks were not able to properly investigate the impact of the number of IoT devices on the overall performance [11] and, to the best of authors' knowledge, none of the existing studies is able to provide a complete statistical characterization of the system. In fact, when a RA system undergoes a variable traffic load (as for instance with sporadic and unpredictable M2M traffic), so that the system cannot be considered in steady state, then first- and second-order statistics are not sufficient to characterize the performance of the system. On the contrary, knowledge of the probability distribution of the number of collided users would help in carrying out a more thorough performance analysis, which is an important scientific problem not completely solved by the literature, to the best of the authors' knowledge.

To this aim, this article extends and improves the analytical framework presented in [12] to calculate both the distribution of the number colliding users and the average number of colliding packets per time slot in each frame of a D-FSA satellite-based system. More interestingly, the proposed framework shows important theory findings able to close the gap between the results obtained from realistic simulations and those coming from the existing theoretical frameworks. This aspect is particularly remarkable at moderate/high load, where the existing frameworks typically provide weak approximations, whereas the one proposed in this article is able to offer far better results.

The remainder of this article is structured as follows. The theoretical framework is described in Section II. Validation of the theoretical findings through a simulation campaign in a significant use case is in Section III. Finally, Section IV concludes this article.

II. THEORETICAL FRAMEWORK

A. Loop Model

As presented in Section I, IC is able to solve collisions throughout an iterative process if at least one replica of a packet can be correctly decoded. This means that the replica is alone in a time slot or takes advantage of the capture effect in case of unbalanced powers at the receiver. Otherwise, all the replicas of a user collide in a frame, incurring in a loop, by definition. Let us introduce the fundamental loop concept, which we use in what follows to characterize both D-FSA and the contention resolution in all the recent RA variants with IC. A loop occurs when two or more users send their replicas in the same time slot(s). For the sake of clarity, we provide just few examples. Two users u_1 and u_2 with two replicas each (k = 2) collide, in case of perfect power balancing, if they transmit the replicas in the same two time slots t_1 , t_2 out on *n* time slots in a frame. Let us say that u_1 and u_1 have v = 2 intersections. With k > 2, again, the two users share the same k-tuples in the case of loop, giving v = k intersections over the time slots $t_1 \cdots t_k$ out of *n*. Three users u_1, u_2 , and u_3 with two replicas each still share the same 2-tuple. However, with k = 3, the following two possible events may occur, given a loop:

- *u*₁, *u*₂, and *u*₃ share the same 3-tuples leading to 3 intersections over 3 time slots out of *n*;
- *u*₁, *u*₂, and *u*₃ sort different 3-tuples but leading to two intersections over four time slots out of *n*.

The second case is shown by means of an example in Table I: the *x* symbol represents a replica of the users u_1 , u_2 , u_3 belonging to the loop, and the pool of four time slots \mathcal{L}_l is $\{t_1, t_2, t_3, t_4\}$. Instead, the *o* symbol in Table I represents a replica transmitted by a user non in loop, i.e., with at least a replica in a time slot with no other packets, as in $(u_4 \rightarrow t_6, u_5 \rightarrow t_7)$. Note that, the users u_1 , u_2 , u_3 are in loop regardless of other users.

TABLE I Example of the Pool of Time Slots Involved in a Loop With l = 2 Users

	t_1	t_2	t_3	$t_4 \mid t_3$	$5 t_6$	t_7
u_1	х		х	x		
u_2	х	х	х			
<i>u</i> ₃	х	х		x		
<i>u</i> ₄	0		0		0	
u_5	0			o		0

More generally, an *l*-order loop is given by l + 1 users with $l \ge 1^1$, which have chosen exactly the same pool \mathcal{L} of time slots to send their replicas and all the time slots $t_i \in \mathcal{L}$ are affected by a collision generated at least² by the l + 1 users. Loops cause a contention between users that can be very hard to be decoded, unless other mechanisms are in place, i.e., when coding techniques are applied or when power imbalance between users occurs, thus giving rise to the *capture effect*. Not only, loops are one of the main reasons for packet loss rate (PLR) due to unresolved collisions via IC.

The loop probability in the first case, in which all the collided users share exactly the same time slots, has been investigated in [13]. Loops are one of the main reasons for packet losses, measured by means of PLR. In D-FSA, each user selects a set of *k* distinct time slots from a set of *n* elements in an MAC frame. Then, the user can choose any of the possible combinations with an equal probability $q = {n \choose k}^{-1}$. Consider now a generic packet of interest (PoI). The probability that a user, attempting a PoI, is in a loop with *l* other users, follows the binomial distribution, as in [13]

$$\mathcal{B}_L(u-1,q) = \binom{u-1}{l} \cdot q^l \cdot (1-q)^{u-1-l} \qquad (1)$$

with *u* as the number of users per frame.

However, [13, eq. (1)] does not accurately model the statistical occurrence of loops. In fact, (1) accounts for the probability that l users select the same tuple of replicas of the PoI and that the other u - l ones not. However, this condition does not guarantee the other u - l users to avoid creating a loop themselves. Yet, (1) accounts for a particular loop event, i.e., the loop takes only k time slots. This is highly probable only when l = 1.

Since (1) is not suitable to precisely characterize a loop event, let us introduce $P_{loop}(l)$ defined as

$$P_{\text{loop}}(l) = \Omega(l; u, n, k) \tag{2}$$

that accounts for the probability that l users are in a loop with a PoI with their k replicas, over a subset of time slots out of n, and u - 1 - l users must not belong to the loop. This condition guarantees that the u - 1 - l users will successfully take advantage of IC, actually being removed from the contention on the current frame. Accordingly, it is possible to derive a more precise analytical expression for $\Omega(l; u, n, k)$ which is discussed in the Appendix, where the novel mathematical framework applied in this article is illustrated.

B. Contention Resolution and IC

Let $\Gamma(E_b/N_0)$ be the function that describes the packet error rate for a given channel code and modulation scheme, as characterized in [13] and [14], as well. The loss probability ζ_l for a packet incurring in a loop with other *l* interfering packets is defined in [12] for an additive white Gaussian noise (AWGN) channel as

$$\zeta_{l} = \sum_{i=1}^{l} \Gamma^{k} \left(10 \log_{10} \frac{E_{b}/N_{0}}{1 + \sum_{j=1}^{i} (E_{b}/N_{0})_{j}} \cdot \frac{1}{r \log_{2} M} \right) f_{I|L}(i|l)$$
(3)

with coding rate r, M symbols, and $f_{I|L}$ being the probability of having i interfering packets of the set \mathcal{L}_l of the loop with a packet in a PoI of l-order. From [9], the probability that a user attempts a PoI is equal to $p_t = \frac{k}{n}$ and the probability that the PoI node has *degree i* out of l from the bipartite graph theory is given by the binomial distribution $\mathcal{B}_I(l, p_l)$. However, the condition of being in loop imposes excluding the probability of having less than two packets in a PoI. Therefore, $f_{I|L}(i|l)$ can be derived as the truncated distribution $\mathcal{B}_{I,\text{trunc}}(l, p_l)$ over the domain interval (1, l] according to

$$f_{X,\text{trunc}}(x) = \frac{f_X(x)}{F_X(x_{\text{max}}) - F_X(x_{\text{min}})}$$
(4)

with probability mass function (PMF) f_X and CDF F_X , respectively, over the domain (x_{\min}, x_{\max}) .

The IC process aims at reducing the degree l of a loop, by performing iterative cancellations of packets correctly decoded in other time slots. Yet, in the case of spread spectrum techniques, $(E_b/N_0)_j$ in (3) must be divided by the spreading factor Sf, as shown in [14].

Finally, the probability that a PoI remains in a loop of degree l with l = 1, 2, ..., u - 1 users (after IC) equals considering a generic user out of l + 1 to be in loop and it comes from (2) and (3) as

$$PLR = \frac{1}{u} \sum_{l=1}^{u-1} (l+1) P_{\text{loop}}(l) \zeta_l.$$
 (5)

C. Multiuser Interference Distribution Function

In order to account for more general conditions than an AWGN channel, we assume the the presence of possible multipath fading or shadowing effects. We assume that the spatial distribution of concurrently transmitting users follows a deterministic general model, i.e., a finite number of users are within a finite region and they transmit with the same level of E_b/N_0 [dB]. Under these assumptions,

¹The case l = 0 means that there is no loop.

²In fact, the other u-l-1 users can transmit at most k - 1 replica on \mathcal{L} in order to not belonging to the loop.

we provide a solution for the probability density function (PDF) of the interference generated by i colliding packets with a PoI, by exploiting the theories of stochastic geometry applied to the wireless networks [15], [16]. The interference generated by i packets in a time slot is

$$I = \sum_{j=1}^{I} \mathcal{B}_{I}\left(l, \frac{k}{n}\right) \left(\frac{E_{b}}{N_{0}}\right)_{j}$$
(6)

where j is the index relative to the j-interfering-packet with signal-to-noise ratio of $(\frac{E_b}{N_0})_j$ and $\mathcal{B}_J(l, \frac{k}{n})$ the distribution from (4). In [17], expressions for the PDF of the interference power in a network whose transmitters are arranged according to the Poisson point process are derived. Authors show that for a generic distribution $f_G(g)$ of the fading power, the resultant PDF $f_I(i)$ of the interference I can be obtained by numerical techniques. Precisely, they show some results of the inversion of the Laplace's transform of the interference power $\mathcal{L}_{I}(s) = \exp^{\{-\pi\lambda\mathbb{E}[g^{2/\eta}]\Gamma[1-\frac{2}{\eta}]s^{2/\eta}\}}$ where λ is the density of the transmitters, $\Gamma(\cdot)$ is the Gamma function, $\mathbb{E}[\cdot]$ represents the expectation of its argument, η is the path loss exponent, assuming that the fading value g is distributed according to distributions such as Rayleigh, Nakagami-m, and Rice. Note that, when $f_G(g)$ is Nakagami-*m* distributed, a direct expression of the signal-to-noise plus interference ratio (SNIR) distribution $f_{\Psi|I}(\psi \mid i)$ is calculated in [18] for both the cases, when the signal of interest (W) and the interfering signal (I) have different Nakagami-m distributions $(m_I \neq m_W)$ and when $m_I = m_W$, respectively. The SNIR is written as the ratio of two variables $U = \mathbb{E}\left[\frac{E_b}{N_0}\right]_W \cdot W$ and $V = 1 + \mathbb{E}\left[\frac{E_b}{N_0}\right]_I \cdot I$, respectively, whose distributions are

$$f_U(u) = \frac{f_W\left(\frac{u}{\mathbb{E}\left[\frac{E_b}{N_0}\right]_W}\right)}{\mathbb{E}\left[\frac{E_b}{N_0}\right]_W} \text{ and } f_V(v) = \frac{f_I\left(\frac{v}{\mathbb{E}\left[\frac{E_b}{N_0}\right]_I} - \frac{1}{\mathbb{E}\left[\frac{E_b}{N_0}\right]_I}\right)}{\mathbb{E}\left[\frac{E_b}{N_0}\right]_I}$$

respectively.

Calculating the quotient distribution of two random variables, the PDF for the SNIR $f_{\Psi|I}(\psi \mid i)$ at the receiver is given, numerically, in [18]. Rayleigh and Rician fading are particular cases of Nakagami-*m* fading with m = 1 and $m = \frac{(\kappa+1)^2}{(2\kappa+1)}$, respectively, being κ the Rice's factor, i.e., the ratio of the signal power in the dominant line of sight (LoS)-component to the power of other non-LOS components. Therefore, (3) can be generally written as

$$\zeta_l = \sum_{i=1}^l \Gamma^k(\mathbb{E}[\psi \mid i]) f_{I\mid L}(i \mid l)$$
(7)

where $\mathbb{E}[\psi \mid i] = \int \psi f_{\Psi \mid I}(\psi \mid i) d\psi$.

III. PERFORMANCE EVALUATION

For the performance evaluation, we selected an application scenario to be particularly attractive for IoT services over satellite systems. To this aim, a satellite system for data maritime communications among vessels operating far from harbors has been chosen as reference scenario. In fact, GEO systems for maritime communications are recently becoming more attractive than LEO systems, due to the larger data traffic injected into satellite systems, as a consequence of the data-traffic increment over oceans. Moreover, some of the main GEO satellite operators have extended their business toward satellite-on-the move systems, hence naturally embracing additional maritime data services (e.g., Inmarsat).

The reference scenario assume a satellite system operated in Ka frequency band, so as to enable highly reliable and timely M2M services, in terms of monitoring and control of vessel activities over vast coverage areas with users in mobility, while guaranteeing the needed data rate for real-time communications of diverse sensors and actuators displaced on vessels.

A. Channel Model

The characterization of the maritime satellite channel has been largely investigated [19]-[24] since the past decades. According to the conducted measurement campaigns and the corresponding channel models in [19], we assume in this work that the envelope of the signal received by the mobile terminal follows the Rice distribution, which describes the terminal-satellite communications in LoS, to which additional distorted signal replicas are summed up, to account for signal reflections caused by the water surface. In more detail, the Rice factor, characterizing the received power distribution of probability, depends on the elevation angle: by taking into account that the received power due to LoS communication increases with the elevation angle, the Rice factor increases as well. The delay, incurred between the reception of the LoS path signal and the multipath components, falls within hundreds of nanoseconds. On the other hand, the impulse channel response exhibits a number of echoes that can be assumed as Poisson-distributed with rate between one and two echoes within the rms delay spread τ , as in the case of open space communications, indicating a rapid decay of the impulse channel response. An extensive simulation campaign has been conducted on the Satellite Network Simulator (S-NS3) simulator [25]. In S-NS3, the system allocates 2 GHz to the feeder link from 17.7 to 19.7 GHz for the downlink and 500 MHz to the user link from 29.5 to 30 GHz for the uplink: the link budget for Ka-band system under investigation is provided, according to the Digital Video Broadcasting - Return Channel via Satellite (DVB-RCS2) specification, in [26]. Table II provides all the parameters³ used to set up the simulator while default values of the link budget are provided in Table III. A custom channel model has been developed in S-NS3, in order to account for a satellite maritime with fading. By using the measurements of the direct-to-multipath signal power ratio versus satellite elevation provided in [19] and [20] for the Inmarsat system, a Rice's factor $\kappa = 14.5$ dB can be obtained for an average elevation angle $\theta = 19^{\circ}$.

³The gross bandwidth of 12.5 MHz includes the guard band introduced through carrier spacing to limit the adjacent-channel interference.

		TABLE II			
	Sim	ulator Setup Para	ameters		
	Nar	ne	Value		
	Random Access	s (RA) scheme	2- 3-CRDSA		
	RA blocks pe	r superframe	1		
	RA block	duration	13 [ms]		
	Timeslots pe	r RA block	64		
	DVB-RCS2 V	Waveform Id	14		
	Mapping	Scheme	QPSK		
	Code	Rate	1/2		
	Burst L	ength	202 [B]		
	Payload	Length	188 [B]		
	Gross ba	ndwidth	12.5 [MHz]		
	Symbo	ol rate	8.01 [Msymbol/s]		
	Roll	off	0.2		
	Carrier s	spacing	0.3		
	One-way P	HY delay	0.13 [s]		
		TABLE III			
	Link B	udget Parameters	s in S-NS3		
S-NS	53 Parameter	User Terminal	GEO Satellite	Gatev	
txMa	xPower [dBW]	4	15	8.9	
txAn	ennaLoss [dB]	1	1	0	
rxAn	tennaLoss [dB]	0	1	0	
noiseTemperature [dBK]		24.6	28.4	24.0	
txMaxAntennaGain [dB]		45.2	54	65.	
rxMaxA	ntennaGain [dB]	44.6	54	61.:	
0	105				
0.	125				
	\$		Empirical	I=0	
	7	Empirical	=2		



Fig. 1. Example of empirical versus analytical Probability Density Function (PDF) $f_{\Psi|I}$ of the Signal to Noise plus Interference Ratio $(SNIR) (G = 0.5, i = \{0, 1, 2\}).$

The corresponding delay power profile of the simulated channel exhibits an rms delay spread $\tau = 49$ ns and a coherence bandwidth of $B_c = 3.2$ MHz, the latter computed according to $B_c = \frac{1}{2\pi\tau}$. The channel coherence time can be approximated as $T_c \approx 1/(f_c \frac{v \cos(\theta)}{c})$, yielding $T_c = 2.6$ ms for a carrier frequency $f_c = 29.5$ GHz and an average speed of the mobile terminal v = 15 Km/h, as then used in our simulations.

Fig. 1 shows $f_{\Psi|I}(\psi \mid i)$ at the receiver for $i = \{0, 1, 2\}$ interfering packets with the PoI, at load G = 0.5 (i.e., 32) users experiencing Rician fading and transmitting in a frame composed of 64 time slots). The plotted areas represent the empirical distributions of $f_{\Psi|I}(\psi|i)$ obtained by simulations, while the dotted lines with markers are the relative analytical curves. Finally, the values of the probabilities of having $i = 0 \dots 2$ interfering packets, derived from simulations, are reported in the legend of the plot. In fact, if i > 2, a packet could be decoded either thanks to a spreading factor $S_f > 1$ (this is not the case of this article), or through IC, as addressed in this article and further discussed in Section II-B.

B. Numerical Results

The modulation and coding scheme, used in the simulations, account for a QPSK mapping scheme with forward error correction code rate 1/2. Yet, each user can transmit a number of replicas k = 2, 3 over a frame of n = 64 time slots.

We focused our analysis in a significant range of normalized traffic load G between 0.5 and 1.0, being G = U/Nthe number of transmitting user per slot in each frame. Fig. 2 shows the empirical probability mass function $P_{\text{loop}}(l)$ of the number of users interfering with a PoI for values of G simulated on S-NS3, in the range mentioned before, for both 2- and 3-CRDSA. With G < 0.5, loop events become rare and the analysis is not significant. However, it has been well demonstrated in the literature that a RA scheme such as CRDSA performs almost ideally with $PLR \approx 0$. In fact, Packet Loss Rate (PLR) models in [13] and [14] fit accurately the simulations up to G = 0.6.

However, if G > 0.6 and, more generally, using more than two replicas, the model in [13] provides a not negligible error as discussed in [12].

Figs. 3 and 4 show the CDFs $F_{I|L}$ for 2- and 3-CRDSA, by comparing (4) with the empiric distribution derived from the simulation campaign with S-NS3.

Tables IV and V show the average number of packets E[I | L] interfering with PoI in a loop of order l over a pool $\mathcal{L}_{l+1} = \{t_1 \cdots t_i\}$ of time slots for 2- and 3-CRDSA, respectively. The number of interfering user l can assume the values in the first column of the two tables with probabilities that can be read in the PMF in Fig. 2. When l = 1, only a single packet can interfere with a PoI. Such an event occurs with a probability that decreases with G. For G > 0.8, the probability of having l = 1 is almost zero [27]. Again, (1) assumes that l users in a loop draw the same k-tuple of the PoI in a frame. When this occurs, the number of mutually interfering packets in one of the k time slots is exactly l + 1. For example, let us consider the event with 61 users in a loop at G = 1 and k = 3: it would occur with a probability in the order of 10^{-199} according to (1). Contrarily, Table V shows that only 1.26 packets on average interfere with a PoI and the event has a probability $\approx 10^{-4}$ as in Fig. 2: the pool of slots in the loop is $(61 \cdot 3)/(1 + 2.26) \approx 56$. In fact, only eight slots (64-56) are not in the pool of those in loop. It is worth noting that having 3 users (out of 64, i.e., G = 1) not in the loop requires 5 to 9 time slots not in the loop, as exemplified in Table VI.

Finally, Fig. 5 shows the resulting PLR for 2- and 3-CRDSA by comparing: 1) the simulation results; 2) PLR in [13]; and 3) PLR calculated by exploiting (5). Fig. 5 takes into account a range of normalized offered loads around realistic system working points identified in [28] and [29].



Fig. 2. Empirical distributions of the number of interfering users in a PoI $l = 1 \cdots 63$ for $G = 0.5 \cdots 1.0$ in 2- and 3-CRDSA with Rice fading.



Fig. 3. Analytical versus empirical Cumulative Density Function (CDF) $F_{I|L}$ of the number of interfering packets *I* with a packet in a PoI in *l*-order loop for 2-CRDSA at different normalized load G = 0.6, 0.8, 1.0.



Fig. 4. Analytical versus empirical CDF $F_{I|L}$ of the number of interfering packets *I* with a packet in a PoI in *l*-order loop for 3-CRDSA at different normalized load G = 0.6, 0.8, 1.0.

 TABLE IV

 Average Value of the Number of Interfering Packets E[I | L]

 With a Packet in a PoI for a *l*-Order Loop for 2-CRDSA

	$E[I \mid L]$							
l	G=0.5	G=0.6	G=0.7	G=0.8	G=0.9	G=1.0		
1	1	1	1	1	-	-		
3	1.02	1.02	1.03	1.04	-	-		
5	1.08	1.08	1.10	1.12	-	-		
7	1.13	1.14	1.16	1.20	1.24	-		
10	1.18	1.20	1.22	1.25	1.35	-		
20	-	1.33	1.33	1.37	1.44	1.54		
30	-	-	1.40	1.45	1.52	1.63		
40	-	-	-	1.56	1.57	1.66		
50	-	-	-	-	1.68	1.69		
60	-	-	-	-	-	-		

We remark that 2-CRDSA provides a slightly better performance at G = 1 than 3-CRDSA as confirmed both by the proposed analytical framework and by the simulation results. This is also evident looking at both the PMF of the number of interfering users in Fig. 2, and at the average number of interfering packets in Tables IV and V, with always more than two interfering packets in a PoI with G = 1 at any loop order *l*. Anyway, 3-CRDSA provides better performance in terms of throughput w.r.t. 2-CRDSA or Irregular Repetition Slotted ALOHA (IRSA) [30] with frame sizes between 64 and 128 time slots. Moreover, if congestion control algorithms are operated by the upper layer protocols (i.e, transport layer), it turns out that the

 TABLE V

 Average Value of the Number of Interfering Packets E[I | L]

 With a Packet in a PoI for a *l*-Order Loop for 3-CRDSA

	$E[I \mid L]$						
l	G=0.5	G=0.6	G=0.7	G=0.8	G=0.9	G=1.0	
1	1	1	1	-	-	-	
3	1.02	1.03	1.08	-	-	-	
5	1.10	1.10	1.14	1.15	1.25	-	
7	1.16	1.19	1.19	1.22	1.29	-	
10	1.16	1.22	1.23	1.28	1.48	-	
20	-	1.39	1.42	1.47	1.64	-	
30	-	-	1.58	1.65	1.83	2.15	
40	-	-	-	1.78	1.94	2.18	
50	-	-	-	-	2.00	2.22	
60	-	-	-	-	-	2.26	

TABLE VIExample of the MinimumNumber of Time Slots $(t_i = 5)$ inOrder to Allocate $u_j = 3$ UsersWithout Loops

	t_1	t_2	t_3	t_4	t_5
u_1	0			0	0
u_2		0		0	0
u_3			0	0	0

working points are subject to PLR close to 10^{-3} and to 10^{-2} , for transmission control protocol (TCP) [28] and TCP



Fig. 5. PLR of 2- and 3-CRDSA with Rice fading: S-NS3 simulations versus PLR in [5] versus novel statistical analysis PLR.

friendly rate control-like [29], respectively, when 3-CRDSA is in use, i.e., 3-CRDSA will never pull *G* over 0.8 with a congestion control in use.

It must be noted that the proposed framework could be adopted also with irregular repetitions of replicas, such as IRSA, by exploiting the proper column margins and the relative probabilities, which would occur with.

IV. CONCLUSION

This article propose a novel and accurate framework of analysis of Diversity - Framed Slotted ALOHA (D-FSA) schemes with IC for RA satellite platforms, such as CRDSA. To this end, advanced analytical frameworks and suitable simulation tools have been adopted for carrying out the performance evaluation, by taking, as reference, some recent scientific works on this subject. Particular attention has been devoted to the existing analytical models and simulation results of the literature, deriving the difficulty to correctly estimate the loop probability and the resulting PLR in a D-FSA system. To take these important aspects into consideration, this article has provided a deep and complete statistical analysis on loop events, the distribution of packets, and of users in a loop, and how they can be combined in frames with the presence of loops, eventually proposing a theoretical framework. Closed forms of the probability distribution functions are obtained fostering on a mathematical framework to count the number of all the binary matrices with prescribed columns and rows, which account for all the possible manifestations of a given number of user u over n time slots in a frame with k attempts each. The performance evaluation shows the accuracy of the model in many significant cases with two different configurations: 2- and 3-CRDSA.

APPENDIX

This appendix describes the theoretical framework for deriving the analytical expression for $p_{loop}(l)$, i.e., the probability that only l users are in a loop with a PoI, after performing IC operations, given the relevant configuration parameters of CRDSA, i.e., n, u, and k.

A. Binary Matrix With Prescribed Row and Column Margin

Let the *column margin* $\overline{c} = (c_1, c_2, \dots, c_n) \in \mathbb{N}^n$ be an ordered vector and $\langle \overline{c}(z) \rangle := \{\overline{c} \in \mathbb{N}^n : \sum_{i=1}^n c_i = z\}$ the set of the whole vectors, whose elements sum to *z*, i.e, which is given by all the possible unique permutations ϵ_c of the c_i parts of each ordered vector *c*, such that

$$\epsilon_c = \binom{n}{m_1, m_2, \dots, m_s}$$

with m_1, m_2, \ldots, m_s the multiplicities of the c_i parts. Analogously, let the row margin $\overline{r} = (r_1, r_2, \ldots, r_u) \in \mathbb{N}^u$ be the vector with $r_j = k$, which accounts for the *k* replicas transmitted by each user u_j . Note that $\langle \overline{r}(z) \rangle \equiv \overline{r}$ for CRDSA with *k* constant replicas and ϵ_r is trivially 1. Then, let us consider the set $\mathcal{M}_{\langle \overline{c} \rangle \times \overline{r}}(z)$ of binary matrices with $\langle \overline{c} \rangle$ columns and \overline{r} row such that the total number of elements (ones) of each binary matrix $\in \mathcal{M}_{\langle \overline{c} \rangle \times \overline{r}}$ is equal to *z*. Therefore, for *u* users (rows) and *k* replicas, it stands that

$$\sum_{i=1}^{n} c_i = \sum_{j=1}^{u} r_j = k \cdot u.$$

The set $\mathcal{M}_{\langle \overline{c} \rangle \times \overline{r}}(k \cdot u)$ accounts for all the possible manifestations of *u* users transmitting *k* replicas over a frame of *n* time slots and its cardinality $|\cdot|$ can be calculated



Fig. 6. Analytical versus empirical Probability Mass Function (PMF) $p_{loop}(l)$ of the number of interfering users *l* with a packet in a PoI in *l*-order loop for 3-CRDSA.

by

$$|\mathcal{M}_{\langle \vec{c} \rangle \times \vec{r}}(k \cdot u)| = \binom{n}{k}^{u}.$$
(8)

B. Counting Binary Matrices

Let $\mathcal{M}_{\overline{c} \times \overline{r}}$ be the set of binary matrices with prescribed column and row vectors \overline{c} and \overline{r} , respectively. The cardinality $N(\overline{r}, \overline{c}) = |\mathcal{M}_{\overline{c} \times \overline{r}}|$ is recursively calculated in [31] as

$$N(\overline{r}, \overline{c}) = \sum_{\overline{s} \in \langle \overline{c}(z) \rangle} \left(\frac{\overline{c}}{\overline{s}} \right) N(L\overline{r}, \overline{c} \setminus \overline{s})$$
(9)

where *L* denotes the left-shift map $L\overline{r} = (r_2, r_3, ...),$ $\overline{c} \setminus \overline{s} := \overline{c} - \overline{s} + L\overline{s}$, which can be read as \overline{c} reduce \overline{s} , and $\binom{\overline{c}}{\overline{s}} := \binom{c_1}{s_1} \binom{c_1}{s_1} \cdots \binom{c_n}{s_n}$.

Therefore, the probability $\mathbb{P}_{\overline{c} \times \overline{r}}$ is given by (8) and (9) as

$$\mathbb{P}_{\overline{c}\times\overline{r}}(u,n,k) = \frac{N(\overline{r},\overline{c})}{\binom{n}{k}^{u}}$$
(10)

C. Column Margins for Users in Loop and Not

Let us consider l users in loop with a PoI such that l + 1users shares the same set of time slots with their replicas, then set of column margins $\langle \overline{c} \rangle^{(l+1)}$, allowed by our counting problem, contains the column margins that generate exactly l + 1 users in loops and that use $k \le n$ time slots, once the IC process is completed. Hence, we are looking for the set of column margins

$$\langle \overline{c} \rangle^{(l+1)}((l+1)k) := \{ \overline{c}^{(l+1)} \in \mathbb{N}^n : \sum_{i=1}^n c_i^{(l+1)} = (l+1)k \}$$
(11)

which generates the set of binary matrices $\mathcal{M}_{\langle \overline{c} \rangle^{(l+1)} \times \overline{r}}$.

The elements in $\langle \overline{c} \rangle^{(l+1)}$ can be evaluated, by assuming that each margin is composed by two submargins $\langle \overline{c} \rangle_{L}^{(l+1)}$ and $\langle \overline{c} \rangle_{\mathcal{NL}}^{(l+1)}$, respectively. The first one counts the number of replicas per time slot of the users in loop and the second one counts the number of replicas per time slot of the users not in loop, i.e, those resolved by IC. The set $\langle \overline{c} \rangle_{\mathcal{L}}^{(l+1)}$ is obtained by the integer partition of the number of replicas (l + 1)kin $[k, \lfloor (l+1)k/2 \rfloor]$ integers, where the integers range in [2, l+1]. The set $\langle \bar{c} \rangle_{\mathcal{NL}}^{(l+1)}$ is obtained by the integer partition of the number of replicas (u - l - 1)k in [k, n] integers, where the integers range in [1, u - l - 1], and then by filtering this set, as explained in the following. We assume that are the sets of elements in the column margin greater than 1, equal to 1, and equal to 0, respectively. The column margin can be selected, whether the number of users in loop at the end of IC steps is l + 1: this can occur only by removing the replicas of the users that have at least a replica alone in a time slot. This means that the number of time slots must be large enough to contain at each step of IC, the number of integers needed to partition the number of replicas of the users not in loop minus the replicas of users not in loop with at least one replica alone in a time slot, assuming the integers of the partition strictly equal to two. Formally, such a condition can be stated as follows: the number of available time slots $n' = |\langle a_2 \rangle| + \max[0, |\langle a_2 \rangle| - (u - l - 1)]$ must be greater than or equal to the number of needed time slots

$$n'' = \left\lfloor \frac{\sum_{i=1}^{|\langle a_2 \rangle|} a_{2,i} - |\langle a_1 \rangle|(k-1)}{2} \right\rfloor + 1.$$

The elements in $\langle \overline{c} \rangle_{\mathcal{L}}^{(l+1)}$ and $\langle \overline{c} \rangle_{\mathcal{NL}}^{(l+1)}$ have to combined so that the resulting column margins $\langle \overline{c} \rangle^{(l+1)} \ni \overline{c}^{(l+1)} =$ $(\langle b_2 \rangle, \langle b_1 \rangle, \langle b_0 \rangle)$ satisfy the conditions stated below, assuming that $\langle b_2 \rangle, \langle b_1 \rangle, \langle b_0 \rangle$ are the sets of elements in the column margin greater than 1, equal to 1, and equal to 0, respectively. The following are filtering rules for the column margins $\overline{c}^{(l+1)}$.

- 1) If $|\langle b_1 \rangle| \ge u l 1$, the integer partition of $|\langle b_1 \rangle|$ in u - l - 1 integers can be calculated with integers less or equal to k, and, for the column margins that satisfy this constraint, only the number of matrices, which allow exactly u - l - 1 transmitting users, have to be taken in to account (further details are provided in [32]).
- 2) If $|\langle b_1 \rangle| < u l 1$, given $n''' = |\langle b_2 \rangle| + |\langle b_1 \rangle|$ the number of elements in the column margin greater than zero, the difference $c_{i+1}^{(l+1)} c_i^{(l+1)} \quad \forall i \in n'''$ has to be less or equal to 1.

D. Loop Probability

1

The loop probability function $P_{\text{loop}}(l) = \Omega(l; u, n, k)$ can be obtained by combining (10) with (11) as

$$\Omega(l; u, n, k) = \binom{u}{l+1} \sum_{\overline{j} \in \langle \overline{c}^{(l+1)}((l+1)k) \rangle} \mathbb{P}_{\overline{j} \times \overline{r}}(u, n, k).$$
(12)

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Fig. 6 shows the perfect match between analysis and simulation in a simple test case with u = 16, n = 16, k = 3, equivalent to a system load G = 1.

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