Nonlinear water wave interactions with floating bodies using the δ⁺-SPH model

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HIGHLIGHT

1. A numerical wave tank (NWT) for violent wave-body interaction is built based on the recently developed δ^+-SPH scheme.
2. Adaptive-particle resolution is utilized to improve the local accuracy and reduce total computational costs.
3. Nonlinear interaction between a focusing wave and a floating body with green water overtopping is modeled.

1 INTRODUCTION

Due to its robustness in modeling free surface flows and violent fluid-structure interactions [1], SPH method has been widely applied for simulating hydrodynamic problems in different engineering fields, see the introduction in [2]. Among the different successful SPH variants, δ-SPH[1] is an enhanced weakly-compressible SPH model widely used in recent years thanks to its benefits to the standard SPH scheme, to its low CPU costs and to its ease of implementation. Recently, an enhanced scheme named as δ^+-SPH has been proposed in [2]. In the new model, adaptive particle resolution is also implemented. That means in some local regions, the coarse particle can be refined and de-refined. The particle-shifting procedure is introduced and generalized in the context of multi-resolutions in order to prevent numerical instabilities.

Wave-body interaction modeled with SPH method in multi-particle resolutions is rarely documented. In this work, δ^+-SPH scheme is adopted to build a numerical wave tank (NWT) which can be utilized to simulate violent wave-body interactions. In the NWT, a piston wave maker is adopted to generate a focusing wave. A damping zone with high viscous coefficient is adopted to prevent wave reflection. Floating bodies with arbitrary shapes moving under the wave excitation in any amplitude can be simulated. Green water overtopping can be modeled.

2 δ^+-SPH MODEL

A weakly-compressible hypothesis is considered to solve Navier–Stokes equations in the δ^+-SPH framework. The discretized governing equations are written as follows:

$$\begin{cases} \dfrac{D\rho_i}{Dt} = -\rho_i \sum_j (\boldsymbol{u}_j - \boldsymbol{u}_i)\cdot\nabla_i W_{ij} V_j + \delta h c_0 D_i \\ \dfrac{D\boldsymbol{u}_i}{Dt} = -\dfrac{1}{\rho_i}\sum_j \left(p_i + p_j\right)\nabla_i W_{ij} V_j + \boldsymbol{g} + \alpha h c_0 \dfrac{\rho_0}{\rho_i}\sum_j \pi_{ij}\nabla_i W_{ij} V_j \\ \dfrac{D\boldsymbol{r}_i}{Dt} = \boldsymbol{u}_i;\, p = c_0^2\left(\rho-\rho_0\right);\, D_i = 2\sum_j \psi_{ji}\left(\boldsymbol{r}_j-\boldsymbol{r}_i\right)\cdot\nabla_i W_{ij} V_j \Big/ \left|\boldsymbol{r}_j-\boldsymbol{r}_i\right|^2, \\ \pi_{ij} = 8\boldsymbol{u}_{ji}\cdot\left(\boldsymbol{r}_j-\boldsymbol{r}_i\right)\Big/\left|\boldsymbol{r}_j-\boldsymbol{r}_i\right|^2;\, \psi_{ij} = \left(\rho_j-\rho_i\right) - 0.5\left[\langle\nabla\rho\rangle_i^L + \langle\nabla\rho\rangle_j^L\right]\cdot\left(\boldsymbol{r}_j-\boldsymbol{r}_i\right) \end{cases}, \quad (1)$$

where m_i, ρ_i, $\boldsymbol{u}_i$ and $\boldsymbol{r}_i$ denote mass, density, velocity and position of the i-th fluid particle. V_j is the volume of the j-th particle which is one of all the neighboring particles within the compact support domain of i. $\boldsymbol{g}$ is the gravity acceleration. $\nabla_i W_{ij}$ is the gradient of the kernel function $W(|\boldsymbol{r}_i-\boldsymbol{r}_j|, h)$ with respect to $\boldsymbol{r}_i$. The smoothing length h is set as $\kappa\Delta x$ where Δx is the initial particle spacing and κ is a coefficient controlling the number of neighboring particles. Usually κ

is set to 2.0 when the C2 Wendland kernel is applied in order to obtain a neighboring particle number of 50. In the equation of state, ρ_0 is the particle density at rest. c_0 is usually chosen according to $c_0 \geq 10\max\left(U_{\max}, \sqrt{p_{\max}/\rho_0}\right)$ where $U_{\max}$ and $p_{\max}$ are the maximum expected velocity and pressure. The coefficient α is set as 0.01. Regarding the δ-term added in the continuum equation, δ is a problem independent parameter which is usually set to be 0.1 to avoid unphysical high frequency pressure fluctuations. $\langle \nabla \rho \rangle^L$ is the gradient of density evaluated using the renormalized gradient operator. A particle shifting is adopted to regularize the particle distributions (for more details see [2]).

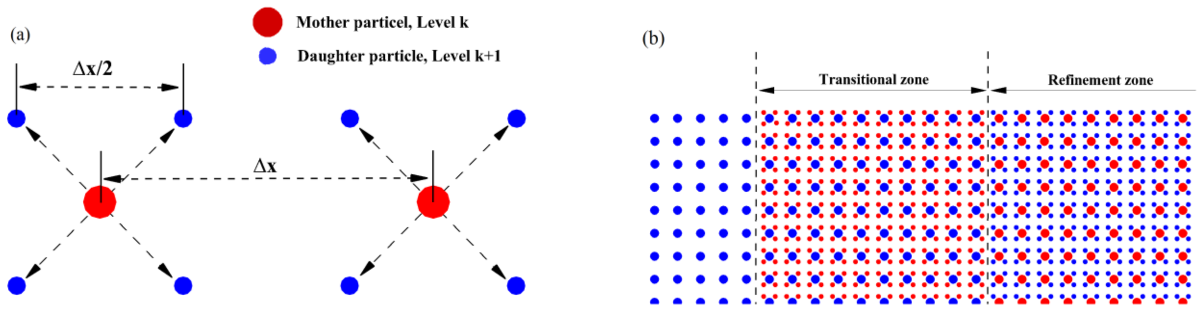


Fig. 1. Sketch of the particle splitting: (a) 1 mother particle (indicated by bigger circle in Level k) is split into 4 daughter particles (indicated by smaller circle in Level $k+1$) distributed in the 4 vertices of a square with a length of $\Delta x/2$ in 2D ; (b) Sketch of the transitional zone as an extension of the refinement zone: Blue particles indicate SPH particles evolving under the SPH equations while the variables of the red particles (named as projected particles) are interpolated from SPH particles.

In δ^+-SPH, a technique of adaptive-particle resolution with a particle refinement and de-refinement is applied according to [3]. We define a set of the particles with different particle resolutions by using L_k where k is the level of the particle resolution. The coarsest particles belongs to L_0. After the first refinement, the particles of L_0 (mother particles) are kept and particles of L_1 (daughter particles) are generated by a particle splitting technique (refinement), see Fig. 1 (a). However, the mother particle is kept but switch off during its trajectory in the refinement region. After the mother particle exits the refinement region, it is switched on again as a normal fluid particle. For the daughter particle exits the refinement region, it will be erased from the fluid domain (de-refinement). In order to avoid the kernel truncation, a transitional domain is extended from the boundary of the refinement domain with a length at least larger than the radius of the kernel function (see the sketch in Fig. 1 (b)). An interpolation is adopted to get the generic fluid variables ϕ from SPH particles to projected particles as

$$\phi_{i\in projected} = \sum_{j\in SPH} \phi_j W_{ij} \Big/ \sum_{j\in SPH} W_{ij} \tag{2}$$

In Eq. (2), ϕ can be density and acceleration. Note that the updating of the position and velocity of the projected particles is still based on the time integration. To maintain the distribution of projected particles regular, besides their interpolated accelerations, artificial viscous forces between projected particles are still calculated and the accelerations caused by the latter are added to its total accelerations. The SPH particles interact only with those particles belonging to the same level.

3 ALGORITHM FOR THE FLUID-FLOAT COUPLING

The hydrodynamic force on the floating body are derived based on the momentum balance between the fluid and body particles, see [4]. The governing equations for the motion of the body are

$$\begin{cases} M\dot{\boldsymbol{U}} = M\boldsymbol{g} + \boldsymbol{F}^{fluid-float} \\ I_R \dot{\Omega}_R = \boldsymbol{T}_R^{fluid-float} \end{cases} ; \begin{cases} \dot{\boldsymbol{r}}_R = \boldsymbol{U} \\ \dot{\theta} = \Omega_R \end{cases}, \tag{3}$$

where $\boldsymbol{U}$ and Ω_R denote the velocity of a pivotal point, *R*, and the angular velocity around it. $\boldsymbol{\theta}$ denotes the angle of rotation. M and I_R refer to the total mass and moment of inertia with respect to the point *R*. $\dot{f}$ means the time derivative of the general variable. The dynamic state of fluid particles and body particles can be expressed through the vectors $\boldsymbol{y}_f$ and $\boldsymbol{y}_b$ as follows:

$$\begin{cases} \boldsymbol{y}_f = \left(\ldots, \rho_i, \boldsymbol{a}_i, \boldsymbol{u}_i, \boldsymbol{r}_i, \ldots\right) & i \in \text{fluid} \\ \boldsymbol{y}_b = \left(\ldots, \rho_j, \boldsymbol{a}_j, \boldsymbol{u}_j, \boldsymbol{r}_j, \ldots\right) & j \in \text{body} \end{cases} \tag{4}$$

And the dynamic state of the floating body is expressed through $\boldsymbol{y}_B$ as:

$$\boldsymbol{y}_B = \left(\dot{\boldsymbol{U}}, \boldsymbol{U}, \boldsymbol{r}_G, \dot{\boldsymbol{\Omega}}, \boldsymbol{\Omega}, \boldsymbol{\theta}\right). \tag{5}$$

In Eqs. (4) and (5), $\boldsymbol{y}_f$ and $\boldsymbol{y}_B$ are updated using a 4$^{\text{th}}$ order Runge-Kutta integration method. $\boldsymbol{y}_b$ is updated based on the components in $\boldsymbol{y}_B$ in each time step, see more in [4]. Fixed ghost particle boundary is adopted to model the solid wall boundary, see [2].

4 NUMERICAL RESULTS

The case of a focusing wave-body interaction in [5] is employed to test the performance of the NWT. The initial set-up for the case is shown in Fig. 2 (for more details see [5]). On the left of Fig. 3 the evolution of the wave maker position to generate the focusing wave is shown. On the right-hand side of the same figure, a snapshot with three different particle resolutions is shown. L_0 with an intial particle spacing of $H/\Delta x = 20$ is used to descretize the whole fluid. L_1 with a length of 2.4 m and L_2 with a length of 1.6 m indicate the refinement zones. Close to the floating body, particle spacing of $H/\Delta x = 80$ is obtained after 2 times of splitting. Snapshot comparisons between δ$^+$-SPH [2] and experiment [5] are depicted in Fig. 4. Time histories for the body motions, wave elevation and pressure evolution on the gauge are shown in Fig. 5.

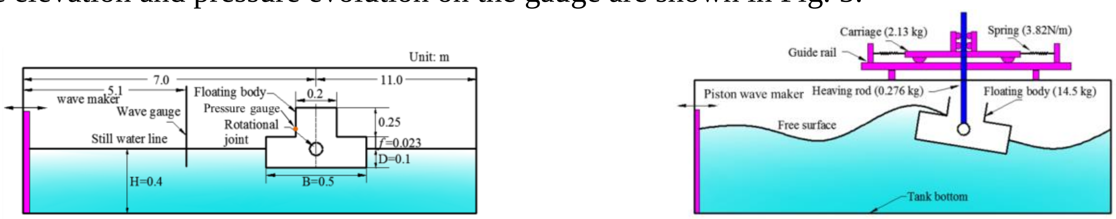


Fig. 2. The set-up for the focusing wave-body interaction.

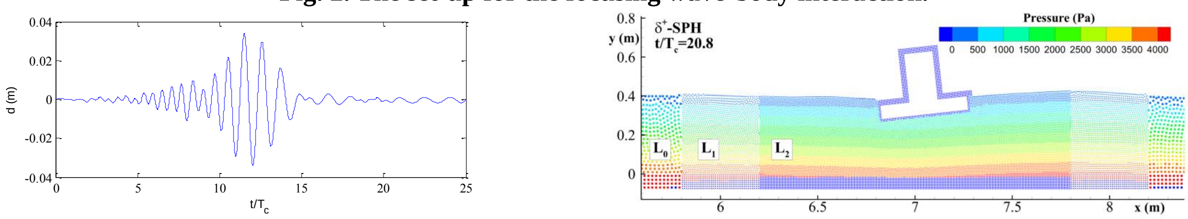
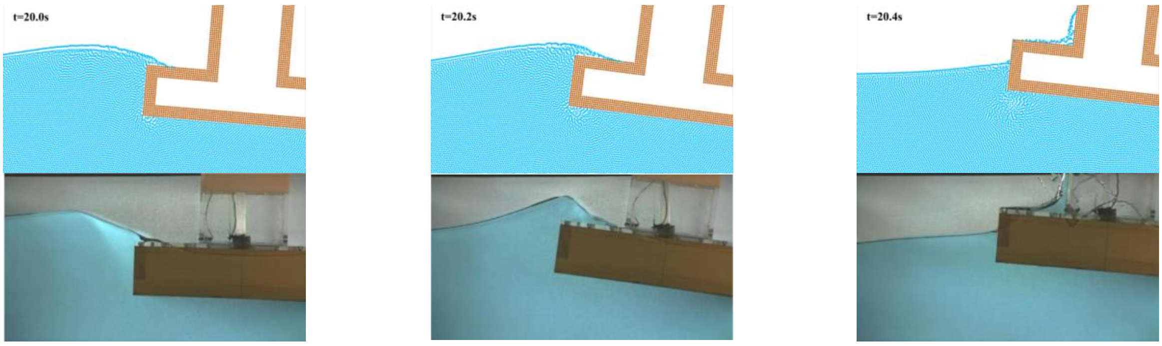


Fig. 3. Wave maker position versus time for the focusing wave generation [5]. The SPH particle distribution at $t/T_c = 20.8$ **using three particle levels. The pressure field is also contoured.**



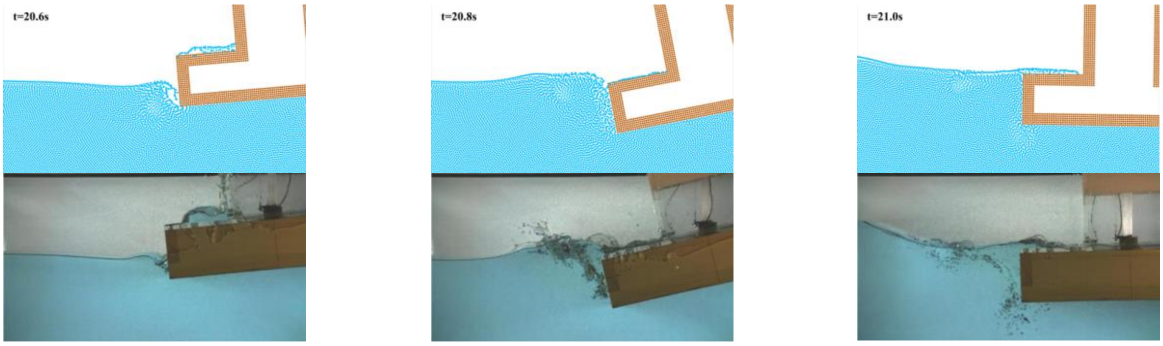


Fig. 4. Snapshots comparison between δ⁺-SPH particle positions [2] and experimental pictures [5].

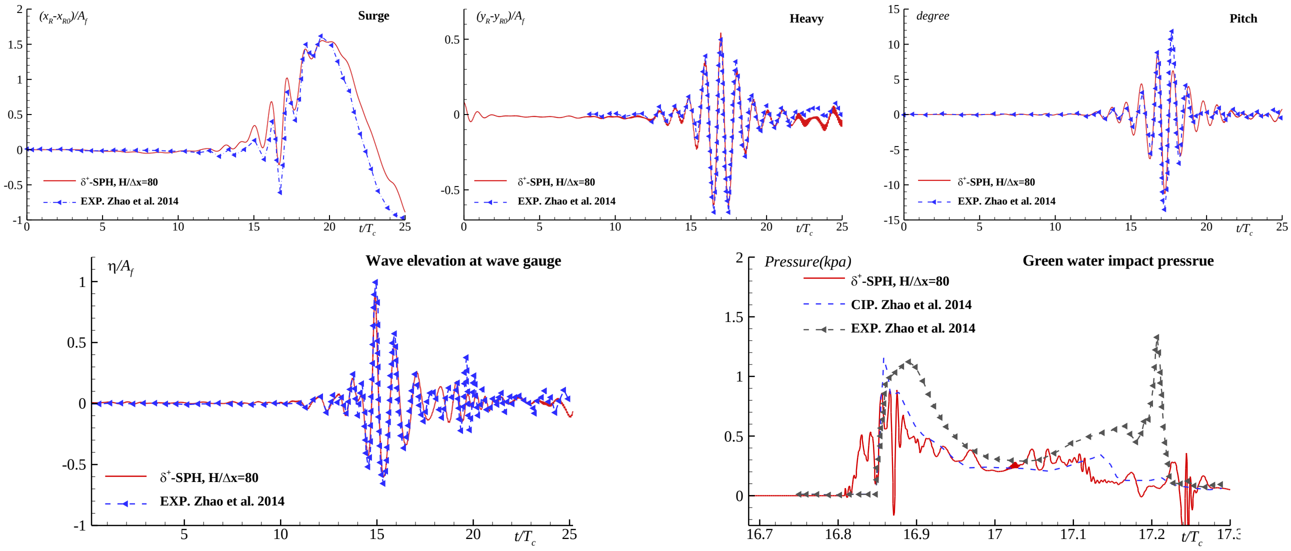


Fig. 5 Comparisons of the body's surge, heavy and pitch motions and the wave elevation at the wave gauge between δ⁺-SPH results [2] and experimental data [5]. Pressure time history on the gauge is also compared with CIP [5].

5 CONCLUSIONS

The δ⁺-SPH model is employed to build a 2D numerical water wave tank. Simulation with adaptive-particle resolution is carried out with a significant computational costs reduction. Validations using experimental data demonstrate the robustness of the δ⁺-SPH scheme in modeling interactions between focusing waves and floating body with green water overtopping.

ACKNOWLEDGEMENTS

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