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Key Points:
- Geomorphic and stratigraphic scaling relations arise from noisy surface disturbance that is smoothed via sediment transport
- Power spectral density slope arises due to the presence and degree of nonlocal sediment transport
- Sadler slope arises from correlation in surface disturbance and presence and degree of nonlocal sediment transport

Correspondence to:
R. Schumer,
rina@dri.edu

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Theory connecting nonlocal sediment transport, earth surface roughness, and the Sadler effect

Rina Schumer[1], Alessandro Taloni[2,3], and David Jon Furbish[4]

[1]Division of Hydrologic Sciences, Desert Research Institute, Reno, Nevada, USA, [2]Center for Complexity and Biosystems, Department of Physics, University of Milan, Milan, Italy, [3]CNR-ISC Center for Complex Systems, Rome, Italy, [4]Department of Earth and Environmental Sciences, Vanderbilt University, Nashville, Tennessee, USA

Abstract Earth surface evolution, like many natural phenomena typified by fluctuations on a wide range of scales and deterministic smoothing, results in a statistically rough surface. We present theory demonstrating that scaling exponents of topographic and stratigraphic statistics arise from long-time averaging of noisy surface evolution rather than specific landscape evolution processes. This is demonstrated through use of "elastic" Langevin equations that generically describe disturbance from a flat earth surface using a noise term that is smoothed deterministically via sediment transport. When smoothing due to transport is a local process, the geologic record self organizes such that a specific Sadler effect and topographic power spectral density (PSD) emerge. Variations in PSD slope reflect the presence or absence and character of nonlocality of sediment transport. The range of observed stratigraphic Sadler slopes captures the same smoothing feature combined with the presence of long-range spatial correlation in topographic disturbance.

1. Introduction

One decade ago, a manuscript entitled "Self-organized complexity in geomorphology: Observations and models" [*Turcotte*, 2007] presented generic statistical physics models to explain the origin of scaling laws observed in topography and landforms. In self-organized complexity, simple dynamical rules give rise to macroscopic dynamics that exhibit fluctuations on all time scales [*Redner*, 2007]. For example, consider a model for 1-D surface evolution in which a particle falls on a random site on a line. If an adjacent site is lower, the particle falls downhill to that location. After a large number of particles are dropped, this simple "noisy diffusion" model produces topography and stratigraphy with specific statistical signatures [*Turcotte*, 2007; *Pelletier and Turcotte*, 1997]:

1. The power spectral density (PSD) of topography perpendicular to the dominant slope decays as $\sim k^{-2}$, where k is the wave number. This can be observed by calculation of the average power spectrum of the deviation from mean height across a surface, one metric for roughness. The PSD is linearly related to other roughness measures such as fractal dimension, the height-height correlation function, and spatial Hurst coefficient of topography.
2. The Sadler effect, a phenomenon in which measured rate of linear accumulation at a single point along a fluctuating surface decays as a power law function of temporal measurement interval, has a unique $\gamma = -0.75$ exponent. This is significant because long-term average linear sediment accumulation rates estimated from a finite number of dated points in one-dimensional cores tend decrease as a power law function of measurement interval Δt [*Sadler*, 1981] with slope close to -0.75. When the Sadler effect occurs, rates calculated over different time scales cannot be directly compared as longer measurement intervals tend to incorporate longer stratigraphic hiatuses, periods for which either no accumulation occurred or deposited sediment was subsequently eroded.

However, the local noisy diffusion model above only provides a "good approximation" of PSD and Sadler statistics [e.g., *Turcotte*, 2007; *Bell*, 1975]. The purpose of this manuscript is to use nonlocal generalizations of noisy diffusion models of surface evolution to explain processes that control a range of topographic PSD and Sadler slopes. We describe that the degree of horizontal spatial correlation in a fluctuating surface determines the PSD slope, while both horizontal spatial correlation and nonlocality in earth surface evolution determine the Sadler slope. We expect PSD slopes of topographic transects perpendicular to the direction transport

($-3<\beta\leq-2$) instead of exactly $\beta=-2$ as in *Pastor-Satorras and Rothman* [1998] and *Morris et al.* [2008], noting that most studies of topographic roughness find 1-D PSD slopes in this range, but have not specifically focuses on the direction normal to transport. We expect Sadler accumulation rate-time plot exponents between $-0.5<\gamma\leq-0.75$ instead of exactly $\gamma=-0.75$, as observed in studies spanning geologic settings [see *Jerolmack and Sadler* [2007] for a summary].

1.1. Background on Nonlocality

Also, in the last decade, the concept of "nonlocal transport laws" emerged in the earth surface processes literature. Nonlocality is a mathematical concept that allows linear generalization of diffusive transport models such that sediment flux may be a function of topography at a distance rather than just at the local point. Nonlocal flux is frequently implemented through use of a convolution integral, where flux, q, is proportional to the weighted sum of some function of slope with distance, as in [e.g., *Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010]

$$q(x,t)\propto\int_{-\inf}^{x}E\left(x'\right)R_r\left(x-x';x'\right)dx', \tag{1}$$

where $E(x')$ is the time averaged particle mobilization rate and $R_r(x-x';x')$ is probability that a particle travels further than $x-x'$ [*Furbish and Roering*, 2013]. Nonlocal models for sediment transport and landscape evolution have been invoked to explain the deviation from local predictions of long-time scaling behavior of topography, anomalous spread of gravel tracers, or hillslope curvature [*Ganti et al.*, 2010; *Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010; *Doane*, 2014].

1.2. Background on Long-Range Dependence and Fractional Brownian Motion

Long-range correlation or dependence (LRD) refers to power law decay of the autocorrelation function of a space or time series, implying that the future state of a process is a function of all prior states. While the idea that any physical property should have correlation through geologic time is unnerving, it has long been understood that LRD is an emergent property of systems that exhibit fluctuations across length or time scales. The stochastic model known as fractional Brownian motion (fBm) is the canonical model for LRD processes and plays a key role in the link between noisy surface evolution and both surface roughness and the Sadler effect. As we shall describe, noisy horizontal surface evolution leads to positive spatial LRD in surface height and negative temporal LRD in fluctuations at a point on the surface. Both are identified via autocorrelation functions identical to that of an fBM.

An fBm is a continuous time stochastic process with Gaussian increments that starts at zero and has zero mean. The increments of an fBm (fractional Gaussian noise) are stationary, but the fBm itself (the sum of the increments) is not a stationary stochastic process, as its (auto)covariance is not simply a function of the difference between two points in time:

$$\left\langle B_H(t)B_H\left(t'\right)\right\rangle=\frac{1}{2}\left(|t|^{2H}+|t'|^{2H}-|t-t'|^{2H}\right), \tag{2}$$

where the Hurst coefficient $0<H<1$ specifies the presence and type of correlation in increments of the fBm [*Mandelbrot and Van Ness*, 1968]. The autocorrelation structure in equation (2) is a defining property of an fBm. Classical Brownian motion is the uncorrelated increment subset of fBm ($H=1/2$) in which $\left\langle B_{1/2}(t)B_{1/2}(t')\right\rangle=t$, for $t<t'$. $H<0.5$ indicates negative LRD, while $H>0.5$ indicates positive LRD.

2. Equilibrium Surface Evolution Models

Earth surface roughness, defined by metrics such as the root-mean-square height difference between two points on a fluctuating surface, fractal dimension, or the power spectral density (PSD) of topography, is understood to be a measure of the topographic connectedness that arises from stochasticity in transport processes and the forcing mechanisms that drive them. Models for landscape evolution describe the change in surface elevation with time $\frac{\partial h(\vec{x},t)}{\partial t}$. Although landscapes are considered far from equilibrium, the fact that they exhibit self-affine scaling over a range of length scales has led to the use of surface evolution models that take the form of stochastic continuum elastic equations originally derived for thermodynamic systems [*Turcotte*, 2007; *Pastor-Satorras and Rothman*, 1998]. Elastic in the geomorphic context refers to the fact that gravity-driven forces pull the system toward a flat surface; mounds tend to smooth, and holes tend to fill, across all spatial scales. The analogy between thermodynamic systems and earth surface evolution stems from the fact that identical scaling statistics (universality classes) are observed in the long-time limit of both fluctuation-driven systems.

Studies of elastic systems have historically focused on the time to roughness saturation from an initially smooth surface and the resulting correlation length. These metrics of transient roughening and subsequent steady state surface evolution are driven by spatial correlation. The height-height correlation function $C(l)$, which describes the difference in height between points a distance l along the direction perpendicular to the dominant slope, is used to measure this correlation $C(l) = \left\langle \sqrt{[h(x+l,t) - h(x,t)]^2} \right\rangle$. The behavior of $C(l)$ in real space is uniquely determined by the PSD properties. If $C(l) \sim l^{H_x}$ for large l, where H_x is the *spatial* Hurst exponent in fBm, and the PSD $k^{-\beta}$, the following relation holds [*Taloni et al.*, 2012]:

$$H_x = \frac{\beta - 1}{2}. \tag{3}$$

Another feature that distinguishes behavior of one type of surface evolution from another is the h-autocorrelation function $\langle h(x,t)h(x,t')\rangle$ describing the correlation of height at two points in time at a single location on the fluctuating surface. The existence of such a metric implies a measurable relationship between spatial surface evolution and the stratigraphic record left at a point on that surface. Of course, only a partial record of positive fluctuations (deposition) can be observed in stratigraphic columns, and it is the character of this incompleteness that leads to the Sadler effect. The h-autocorrelation functions associated with the linear Langevin equations introduced below all have the form of an fBM with $H \leq 0.5$. These antipersistent surface fluctuations lead to a distribution of hiatus lengths with distribution that decays as $P(\text{hiatus} > t) \sim t^{-H}$ [*Schumer et al.*, 2011]. This in turn leads to a power law dependence of average linear rate on measurement interval in one-dimensional cores that decay as Δt^{H-1} [*Plotnick*, 1986; *Schumer and Jerolmack*, 2009]. We, thus, identify the emergence of specific PSD slopes and the Sadler effect [*Sadler*, 1981] as generic results of time-averaged noisy surface evolution.

2.1. Local, Linear Langevin Equation Driven by White Noise

There are two types of continuum partial differential equations (PDEs) commonly used to describe geophysical transport. Most common are Fokker-Planck equations, such as the advection-dispersion equation (ADE) or equations that combine geomorphic transport laws with conservation of mass, which describe the kinematics of transport under the assumption that the concentration of a cloud of particles is equivalent to the probability density (PDF) of single particle displacement. These deterministic equations describe the time evolution of particle plume growth statistically, with no explicit reference to the processes driving that growth. In the ADE, the PDF of displacement follows an average transport rate (advection) and a spread around that average (dispersion). Langevin equations, on the other hand, are stochastic PDEs that describe the time evolution of the random variable of interest itself—for example, location, rather than the PDF of location—as a function of a sum of forces acting on a particle. Some of these forces are deterministic, while unmeasurable forces are represented by a noise term. If transport is forced by noise, then the future location of a particle can only be characterized by a probability density. In the classic random walk example where displacement with time $\frac{\partial x(t)}{\partial t}$ is solely a function of a Gaussian (white) noise term (η): $\frac{\partial x(t)}{\partial t} = \eta(t)$, location with time is a Gaussian PDF. The corresponding Fokker-Planck equation describes the diffusive spreading of that density.

The evolution of the height $h(\vec{x},t)$ of a fluctuating surface is a function of noise-driven fluctuations $\eta(\vec{x},t)$ from a flat surface that are balanced by a so-called elastic (or relaxation, smoothing) term G that smooths the surface as a function of interface geometry, position, and time

$$\frac{\partial h\left(\vec{x},t\right)}{\partial t} = G\left(h\left(\vec{x},t\right)\right) + \eta\left(\vec{x},t\right).$$

For example, the Edwards-Wilkinson equation has been used to represent the evolution of a scale-invariant topographic profile perpendicular to the slope across a hillslope or fan ($x_\perp \equiv x$) [*Pelletier and Turcotte*, 1996; *Dodds and Rothman*, 2000; *Turcotte*, 2007]:

$$\frac{\partial h\left(\vec{x},t\right)}{\partial t} = \nu \frac{\partial^2 h\left(\vec{x},t\right)}{\partial \vec{x}^2} + \eta\left(\vec{x},t\right). \tag{4}$$

The white (Gaussian) noise is, by definition, mean zero and uncorrelated in space and time such that

$$\left\langle \eta\left(\vec{x},t\right)\eta\left(\vec{x}',t'\right)\right\rangle = 2D\delta^2\left(\vec{x}-\vec{x}'\right)\delta\left(t-t'\right), \tag{5}$$

where $\delta()$ is the Dirac delta function. Importantly, the prefactor D in the white noise autocorrelation term is a linear function of ν charactering the strength of the coupling between the disturbance and the smoothing term. ν represents material properties that control smoothing (filling of depressions and degradation of mounds caused by phenomena across scales, from rain splash to tree-throw to tectonics) rate, such as erodibility. The strength of noisy disturbance is presumably also a function of the erodibility; the more susceptible a surface is to a disturbance, the more easily it will resmooth.

The presence of D in the second moment of the noise fluctuations is a consequence of a fluctuation-dissipation theorem that ensures that the variance of the noise is balanced by the strength of the dissipating force so that the macroscopic solutions are stable [*Sornette*, 2006; *vanKampen*, 1981]. From a physical perspective, the surface continues to fluctuate, without growing unbounded or becoming flat. In the Edwards-Wilkinson equation, Gaussian noise is smoothed by its relative diffusion. Regardless of the form of the noise or smoothing term, all Langevin equations describing fluctuating systems in the absence of external forcing must satisfy a fluctuation-dissipation theorem. We make this point now as a preface to the explanation of equations presented later in the manuscript.

The Edwards-Wilkinson equation (equation (4)) is the continuous version of the discrete particle dropping model described in section 1 [*Barabasi and Stanley*, 1995]. There are a number of discrete particle models whose long-term behavior is described by a single PDE that implies specific scaling regardless of microscopic details. This is self-organization. Further, the Edwards-Wilkinson equation is local in space, meaning that the flux of sediments depends solely on the local slope. The Edwards-Wilkinson equation produces a rough surface with power spectral density slope equal to -2 [*Krug et al.*, 1997]. A fluctuating Edwards-Wilkinson surface also produces fractional Gaussian noise fluctuations at a point with $H=0.25$ [*Pelletier and Turcotte*, 1996]. That is, the h-autocorrelation function of the Edwards-Wilkinson equation is that of an fBm (equation (2)) with $H = 0.25$.The slope of a corresponding Sadler plot is $H - 1 = -0.75$ [*Pelletier and Turcotte*, 1996; *Schumer et al.*, 2011].

In previous geomorphic applications of Langevin equations, the noise term represented deposition and the smoothing term erosion [e.g., *Dodds and Rothman*, 2000]. However, elastic equations more generally specify that all deviations from a flat surface, whether creation of a mound or depression, are driven by noisy processes, while transport-driven smoothing is a deterministic function of slope. The Edwards-Wilkinson equation has been likened to the Culling erosion model [*Culling*, 1963] with the addition of a white noise driver [*Turcotte*, 2007]. This analogy can be misleading, as the Culling model is a Fokker-Planck equation that can be derived from conservation of mass (Exner equation) and Fick's law, while the Edwards-Wilkinson equation is a Langevin equation that is not derived, per se, but generically states that noisy disturbances from an equilibrium surface are smoothed deterministically. There are no closed-form Fokker-Planck equations that correspond with the Langevin equations described here.

2.2. Nonlocal, Linear Langevin Equation Driven by White Noise

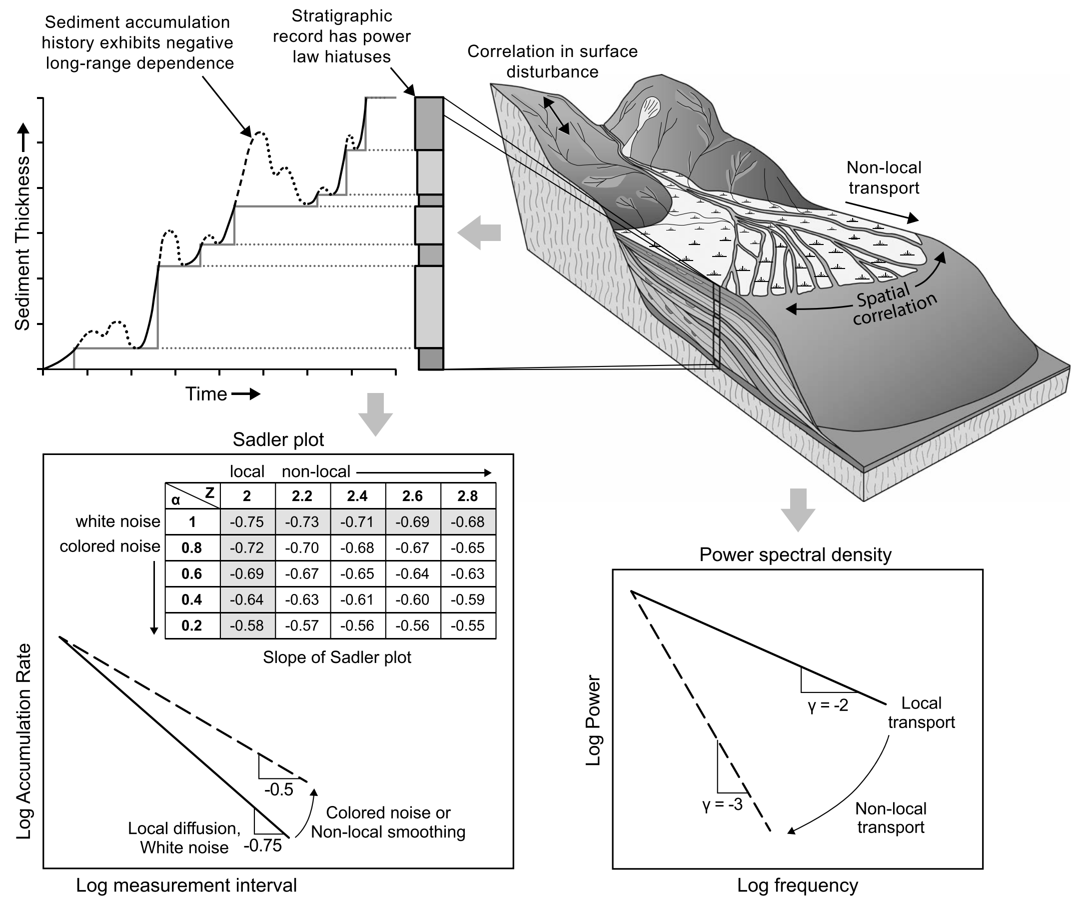
A more general class of linear Langevin equations takes the form [e.g., *Krug and Dobbs*, 1996]

$$\frac{\partial h\left(\vec{x},t\right)}{\partial t} = \nu \nabla^z h\left(\vec{x},t\right) + \eta\left(\vec{x},t\right) \tag{6a}$$

$$\left\langle \eta\left(\vec{x},t\right)\eta\left(\vec{x}',t'\right)\right\rangle = 2D\delta^2\left(\vec{x}-\vec{x}'\right)\delta\left(t-t'\right), \tag{6b}$$

where the *fractional* Laplacian is defined as $-\left(\nabla^2\right)^{z/2}$ [*Samko et al.*, 1993]. The Langevin equation terms are macroscopic quantities resulting from time-averaged disturbance and smoothing processes. Nonlocality in the smoothing term of equation (6a) arises from anomalous transport. For example, the likelihood that a particle will be entrained is a function of how well it is protected by upslope topography (a shadowing mechanism [*Hergarten et al.*, 2008]). Alternately, the travel distance of entrained sediment depends upon downslope topography and connectivity of flow paths. The key here is that long-time averaging leads to the dominance of a few important factors that determine scaling exponents of topography. The temporal autocorrelation structure in landscape fluctuations at a point comes from horizontal dynamics of the fluctuating surface. For local or nonlocal linear Langevin equations with Gaussian noise (equation (6a)), the PSD of the surface decays as k^{-z} [*Krug et al.*, 1997]. The solution $h(\vec{x},t)$ is Gaussian, and its temporal autocorrelation function is well known to be that of an fBm (equation (2)) [*Krug et al.*, 1997]

$$\left\langle \left[h(x,t)-h(x,0)\right]\left[h(x,t')-h(x,0)\right]\right\rangle = K\left[(t')^{2H}+(t)^{2H}-|t'-t|^{2H}\right],$$



α \ z	2	2.2	2.4	2.6	2.8
1	-0.75	-0.73	-0.71	-0.69	-0.68
0.8	-0.72	-0.70	-0.68	-0.67	-0.65
0.6	-0.69	-0.67	-0.65	-0.64	-0.63
0.4	-0.64	-0.63	-0.61	-0.60	-0.59
0.2	-0.58	-0.57	-0.56	-0.56	-0.55

Figure 1. Nonlocal transport ($2 < z < 3$ in the GEM) affects the power law slope of the topographic PSD and the Sadler plot. The presence of long-range correlation in surface disturbance (α in the power law convolution kernel of the GEM) also flattens the slope of the Sadler plot.

where K is a positive constant and the Hurst coefficient is a function of the order of the fractional derivative z:

$$H = \frac{1}{2}\left[1 - \frac{1}{z}\right]. \tag{7}$$

This generalizes the local subset, the Edwards-Wilkinson equation; when $z = 2$, $H = 0.25$. Equation (7) states that as smoothing is driven by increasing nonlocality in transport such that each point in space has increasing probability of fluctuations driven from afar (z increases from $2 \rightarrow 3$), vertical fluctuations become less strongly anticorrelated (H increases from $0.25 \rightarrow 0.33$). Thus, the Sadler slope $\gamma = (H - 1)$ flattens (from 0.75 toward 0.66) when noise is Gaussian and smoothing is increasingly nonlocal (Figure 1). To summarize, nonlocal smoothing can balance Gaussian noise disturbances but modifies the scaling exponents describing incompleteness of the resulting stratigraphy and roughness of the surface.

Fractional derivatives are linear operators that correspond, in the limit, to a weighted average of values across the operand. The weights decay as a power law function of the order of the fractional derivative [*Schumer et al.*, 2001]. The fractional operator is mathematically well defined for any value of z; varying z changes the shape of the weight function. In this application, the weight function describes the character of nonlocal interactions during transport, and there are no specific limits on the order of the derivative. In linear Langevin equations applied to the earth surface, we expect $2 \leq z < 3$ by equation (7). It may be surprising to those familiar with the fractional ADE literature to see a fractional operator of order greater than 2 or an increase in the order of a fractional derivative when the description of transport changes from local to nonlocal. However, these requirements are specific to fractional Fokker-Planck equations, not to fractional PDEs in general. Specifically, space-fractional ADEs describe the time evolution of the probability of particle location, and their solutions are either Gaussian or alpha-stable densities. The order of the fractional derivative in these

Table 1. The 1-D Generalized Elastic Model (GEM) Is a Linear, Nonlocal Langevin Equation That Generalizes Previous Linear Fluctuation/Erosion Models. The Hurst Coefficient Characterizing the Time Series of Height at a Point on a Fluctuating Surface is a Function of GEM Parameters and Drives the Slope of the Sadler Plot

Equation Characteristics	Langevin Equation for Surface Evolution	PSD Slope	Fluctuation Hurst Coefficient (H)	Sadler Slope ($H-1$)
$\eta(\vec{x},t) \equiv$ white or colored noise	$\frac{\partial h(\vec{x},t)}{\partial t} = \int \Lambda(x-x') \frac{\partial^z h(\vec{x},t)}{\partial \lvert\vec{x}'\rvert^z} dx' + \eta(\vec{x},t)$, $\langle \eta_j(\vec{x},t)\,\eta_k(\vec{x}',t')\rangle = 2D\Lambda(\vec{x}-\vec{x}')\,\delta_{j,k}(t-t')$ $\Lambda(\vec{r}) = \begin{cases} = \delta(\vec{r}), & \text{local} \\ \propto \frac{1}{\lvert\vec{r}\rvert^\alpha}, & \text{nonlocal} \end{cases}$ Generalized Elastic Model	$-z$	$H = \frac{1}{2}\frac{z-1}{\alpha-1+z}$ [a]	$-\frac{1}{2}\frac{2\alpha-1+z}{\alpha-1+z}$
$\eta(\vec{x},t) \equiv$ white noise	$\frac{\partial h(\vec{x},t)}{\partial t} = v\frac{\partial^z h(\vec{x},t)}{\partial \lvert\vec{x}'\rvert^z} dx' + \eta(\vec{x},t)$, $\langle \eta_j(\vec{x},t)\,\eta_k(\vec{x}',t')\rangle = 2D\delta_{j,k}(x-x')\,\delta_{j,k}(t-t')$ Linear Langevin equation	$-z$	$H = \frac{1}{2}\frac{z-1}{z}$	$-\frac{1}{2}\frac{1+z}{z}$
$\eta(\vec{x},t) \equiv$ white noise $z=2$	$\frac{\partial h(\vec{x},t)}{\partial t} = v\frac{\partial^2 h(\vec{x},t)}{\partial \lvert\vec{x}'\rvert^2} dx' + \eta(\vec{x},t)$, $\langle \eta_j(\vec{x},t)\,\eta_k(\vec{x}',t')\rangle = 2D\delta_{j,k}(x-x')\,\delta_{j,k}(t-t')$ Edwards-Wilkinson equation	-2	$H = \frac{1}{2}\frac{2-1}{2} = \frac{1}{4}$	$-\frac{3}{4}$

[a] In multiple dimensions $H = \frac{1}{2}\frac{z-d}{\alpha+z-d}$.
The local formulation occurs when $\alpha \to d$.

equations corresponds with the largest finite moment of these densities, which happen to be 2 in the local case and $1 < \alpha < 2$ in the nonlocal case [*Metzler and Klafter*, 2000]. Since Langevin equations are about the evolution of surface location itself, rather than the probability of location, these bounds are not applicable to order of their fractional operators.

2.3. Linear Langevin Equations With Colored Noise: Generalized Elastic Model

The linear Langevin equations above are valid for Gaussian noise disturbances that are uncorrelated in both space and time. But what if disturbance from a flat surface has spatial correlation, such that uplift or subsidence affect more than a local area? This case can be represented by use of a fractional Gaussian noise instead of a white noise. The generalized elastic model (GEM) is the equilibrium equation for a fluctuating surface (in 1-D) [*Taloni et al.*, 2010a, 2010b; *Nezhadhaghighi et al.*, 2014]:

$$\frac{\partial h(\vec{x},t)}{\partial t} = v\int \Lambda(\vec{x}-\vec{x}') \frac{\partial^z h(\vec{x},t)}{\partial \lvert\vec{x}'\rvert^z} d\vec{x}' + \eta(\vec{x},t), \tag{8a}$$

$$\langle \eta_j(\vec{x},t)\,\eta_k(\vec{x}',t')\rangle = 2D\Lambda(\vec{x}-\vec{x}')\,\delta_{j,k}(t-t'), \tag{8b}$$

$$\Lambda(\vec{r}) \begin{cases} = \delta(\vec{r}), & \text{local (white noise)} \\ \propto \frac{1}{\lvert\vec{r}\rvert^\alpha}, & \text{nonlocal (colored noise),} \end{cases} \tag{8c}$$

with $[0 < \alpha < 1]$. As in the previous linear Langevin equations, change in height with time on a fluctuating surface is described by both deterministic and random drivers (Table 1). The noise is, as usual, mean zero and uncorrelated in time but can be either uncorrelated in space (white noise in space and time) or have long-range spatial correlation (colored noise in space, white noise in time). If the noise η is white, then the fluctuations are uncorrelated in space and the convolution kernel on the left-hand side of equation (8a) is a Dirac delta function so that the GEM is equivalent to the nonlocal linear Langevin equation in equation (6a). If, instead, the noise has long-range spatial correlation, then a fluctuation dissipation theorem specifies that the relaxation term includes convolution with a power law kernel [*Taloni et al.*, 2010a]. In geomorphic terms, disturbances to the surface may affect a significant area, not simply a localized site. The (asymptotically) nonlocal formulation of the kernel only applies for $|\vec{r}|$ between the smallest characteristic scale described by a continuum and the size of the system. A fluctuation-dissipation theorem ensures that the variance of noise fluctuations and relaxation must occur such that neither dominates. This highlights the way in which nonlocality and long-range correlation in space are linked.

The PSD of topography is a function of smoothing properties and not disturbance. Thus, PSD decay remains $|k|^{-z}$ as in equation (6a). However, the time series of fluctuations at a point are affected by the character of both disturbance and smoothing. The h-autocorrelation function for fluctuations generated by the GEM are, as expected, that of an fBm (equations (2) and (6b)), where the general relationship between the (temporal) Hurst coefficient of fluctuation increments and surface transport parameters is [*Taloni et al.*, 2010a]

$$H = \frac{1}{2}\frac{z-1}{(\alpha - 1 + z)}. \tag{9}$$

Thus, noisy surface evolution (local or nonlocal) produces temporal structure in surface fluctuations at a point (Brownian motion or fractional Brownian motion) that, in turn, results in power law distributed hiatuses in the stratigraphic record (with power law tail slope of −0.25 or $-0.5 < -H \leq -0.25$) which cause the Sadler effect (Sadler slope of −0.75 or $-0.75 \leq \gamma < -0.5$). The exact value arises from both the order of the convolution kernel and the order of the fractional derivative (Figure 1). Both quantities are contained in the smoothing term. The Sadler effect arises from the combined contribution of the noise and smoothing statistics, while the spatial spectral density arises from the character of the smoothing only. Thus, combined measurements of both the Sadler effect and the spectral density allow parametrization of the GEM that surface fluctuations obey.

3. Discussion

The ubiquity of power laws and $\frac{1}{f^\alpha}$ (colored) noise in natural systems continues to be explored. The "generic occurrence of power law spatiotemporal correlations in noisy nonequilibrium systems" [*Bak et al.*, 1988] extends to geomorphic systems and the record they leave. The presence of the noise is an essential ingredient in the production of self-affine surfaces [*Pastor-Satorras and Rothman*, 1998; *Rodriguez-Iturbe and Rinaldo*, 1997] and in explaining the spatial and temporal heterogeneities observed in landscape formation [*Sornette and Zhang*, 1993]. Stochastic PDEs demonstrate the statistical patterns that emerge from the time evolution of spatially averaged noisy transport. Coherent disturbance and transport at the earth surface leads to horizontal correlation across a landscape. This in turn translates into vertical time correlations. The mechanism is encapsulated in equation (9) and depicted in Figure 1. We do not intend that the many-parameter GEM is a replacement for physically based earth surface evolution models. These generic models explain emergent patterns that arise in disordered systems that need to be taken into account when trying to separate autogenic effects from external forcing. Further, calculation of both the spatial PSD of a surface and the Sadler slope of stratigraphy points toward local versus nonlocal transport in surface smoothing and white or colored noise in disturbances to the surface.

3.1. Implications for Simulation of Synthetic Stratigraphy

There is now a long history [*Kolmogorov*, 1951; *Tipper*, 1983; *Strauss and Sadler*, 1989; *Ganti et al.*, 2011; *Dacey*, 1979; *Straub et al.*, 2011; *Molchan and Turcotte*, 2002] of numerical generation of synthetic stratigraphy using random walk models to probe the statistical characteristics of bed thickness and stratigraphic hiatuses as well as signals of allogenic cyclicity and trends. The form of the h-autocorrelation function associated with the GEM implies that surface fluctuations at a point on an evolving surface follow an fBm with negative LRD. This fBm is itself the solution to a PDE describing vertical fluctuations of the surface. An fBm with $0.25 \leq H < 0.5$ is the appropriate theoretical model for synthetic stratigraphy surface fluctuations. Use of the fBm model was suggested by *Pelletier and Turcotte* [1997] and *Molchan and Turcotte* [2002], but not incorporated into subsequent synthetic stratigraphy studies to our knowledge. Specifically, if landscape evolution is governed by the GEM (8a), then the fluctuation time series at a point follows a fractional Langevin equation where the Caputo fractional time derivative of surface height is proportional to a fractional Gaussian noise [*Taloni et al.*, 2010b]

$$K\frac{\partial^{2H} h\left(\vec{x},t\right)}{\partial t^{2H}} = \zeta\left(\vec{x},t\right), \tag{10}$$

where H is defined as in equation (9); the order of the time fractional derivative arises as a result of nonlocality of the noise in the GEM and nonlocality of the smoothing. $\zeta(\vec{x}, t)$ is a fractional Gaussian noise with $H_\zeta = 1 - H$ [*Taloni et al.*, 2010b]. The Caputo fractional time derivative is defined

$$\frac{\partial^\alpha f(x,t)}{\partial t^\alpha} = \frac{\partial f(x,t)}{\partial t} * \frac{t^{-\alpha}}{\Gamma(1-\alpha)}, \tag{11}$$

highlighting that the equation for vertical fluctuations incorporates the time history of fluctuations as a power law decreasing function of time.

The Hurst coefficient is a function of the exponent z in the smoothing term and the exponent α that characterizes the degree of correlation in roughening in the convolution kernel in the GEM (equation (8)). The value of the Hurst coefficient leads directly to both the power law decay in the distribution of stratigraphic hiatuses and a power law dependence of average linear deposition rate on measurement interval.

4. Conclusions

The disordered nature of earth surface evolution results in a statistically rough surface characterized by scaling relations seen in a variety of phenomena typified by random roughening on a wide range of scales and deterministic smoothing driven by gravity. When resmoothing due to sediment transport is a purely local processes, the geologic record self organizes such that a specific Sadler effect and topographic power spectrum emerge. The GEM, an "elastic" linear Langevin equation, generalizes previous equilibrium surface evolution models by permitting long-range correlation in surface disturbance and nonlocality in smoothing by sediment transport. The model demonstrates that noisy surface evolution (local or nonlocal) produces spatial statistical structure in topography such that the slope of topographic PSD corresponds with the order of the fractional derivative ($2 \le z < 3$) in the GEM describing the character of local or nonlocal transport. Further, we find temporal structure in surface fluctuations at a point (fractional Brownian motion with Hurst coefficient H) that, in turn, results in power law distributed hiatuses in the stratigraphic record (with power law tail corresponding with the Hurst coefficient $-0.5 < -H \le -0.25$) which cause the Sadler effect (slope $-0.75 \le \gamma < -0.5$). The exact value arises from both the order of the convolution kernel and the order of the fractional derivative in the GEM (Figure 1). Combined measurement of both the Sadler effect and the topographic PSD theoretically allow parametrization of the GEM that surface fluctuations obey. Generic statistical equations like the GEM are not intended to replace phenomenological models of surface evolution. However, they explain the emergence of measurable scaling statistics observed at the earth surface across geologic settings. The noisiness of surface evolution across space and time scales produces patterns that can be used as null hypotheses from which change due to external forcing can be tested. Further, this analysis opens the door for comparisons and interpretation of PSD and Sadler slopes across depositional and tectonic settings.

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