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Quantum computing for swarm robotics: a local-to-global approach

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Quantum computing is a branch of computer science derived from the fundamental laws of quantum mechanics, such as state superposition, multi-value logic and destructive measure. An open challenge in itself is to re-think in quantum terms classic problems and solving techniques. Another nature-inspired field is the development of swarm-based robotic applications, where the challenge is catching the fundamental laws governing swarm dynamics, such as pattern formation and target reaching. Here, we review some recent approaches on swarm dynamics, the organizational rules of which are formalized according to quantum computing. In this way, the shades of probability in decision-making for multiple-robot systems can be expressed according to the multi-value logic underlying quantum computing. In our review, specific quantum circuits are sketched to give an idea of how these problems have been faced in computational terms. The article is enriched by references to sonification, a strategy adding one more sensory dimension to data representation, a human-friendly tool to navigate the complexity of swarm-robotic movements in a given arena.

This article is part of the theme issue 'The road forward with swarm systems'.

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1. Introduction

Taking a rest from purely scientific work, we might magine ourselves in a marine environment, sitting in front of the sea. Above our heads, a flock of seagulls, in front of our eyes, close to the surface of the water, we see a shoal of small fish. These are instances of natural swarms, that is, phenomena of spontaneous organization of different beings, where each component of the swarm is accomplishing a simple action, and an overall complex action is emerging from the coordination of small, simple individual actions. While a single small fish is vulnerable, a shoal of its peers can face a great predator, disorienting it and escaping its menace. For the sake of simplicity, we adopt the term 'swarm' to indicate a great number of small animals that are moving together, communicating and collaborating to let the correct behaviour towards a common objective emerge.

A fascination of natural swarms led us to develop mechanical and electronic imitations, through the development of robotic swarms. In particular, electronic imitations are helpful as benchmarks to test theories concerning the natural organization of swarms and their emerging properties. But they can also help achieve tasks that are complex or dangerous for humans, such as search-and-rescue missions during natural catastrophes.

Exploring further, considering the observation of atoms and their constituents, we deal with non-living beings that are subject to other rules. Limiting ourselves to the investigation of electrons and entrusting to them our messages, we have to face the rules of *quantum mechanics*. Our observations interfere with the states and, if we know a quantity with a certain degree of precision, for instance, momentum, we may not know another quantity with the same degree, for instance, position. We detect the energetic transitions between states as a jump from one quantum level to another.

Trying to catch the 'flying' photon emitted during one of these energy transitions, we might wonder how to adapt the physics learned in the quantum world to the objects that populate our world, including artificial swarms, for instance, the swarms of robots. We can borrow some of the mathematics required to formalize quantum physics and adapt it to our purposes. We focus on some properties, quantizing them, that is, choosing a 'level up' and a 'level down' as 0 and 1, respectively, and modelling the other configurations as intermediate states. By analogy with quantum physics, our observation is destructive, in the sense that it makes the wavefunction describing the quantum state collapse to a specific state. Information obtained by these means can be used for further computations.

This is the beginning of a branch of computer science derived from the laws of quantum mechanics: quantum computing. The starting point of quantum computing is the definition of the quantum bit, the *qubit*, the values of which are the real numbers ranging from 0 to 1, unlike the classical bit, which can assume only the values 0 and 1. If the states 0 and 1 correspond, for instance, to 'failure' and 'success', respectively, in a target search, then the superposition of 0 and 1 represents the intermediate states of success in reaching the target. Thus, the adoption of quantum logic is an instance of multi-value logic, as opposed to the dichotomic logic of true–false statements.

In this article, we present our research on the application of quantum computing to swarm robotics. A swarm of robots consists of numerous small, elementary units, each capable of performing simple tasks that collectively contribute to achieving the swarm's overall objective. The global behaviour and the swarm's ability to reach a common goal emerges from the interactions and communications between these robots, often described as 'emergent behaviour'. Given these characteristics, modelling and representing the global behaviour of a swarm cannot be confined to the framework of classical physics.

In addition to modelling the global behaviour of the swarm, which is crucial from an engineering perspective for the efficient design of the individual elements, many other motivations justify the application of quantum computing to swarm robotics, including the possibility of gaining another perspective on existing problems, and to improve computational time. It is also possible to 'translate' classic algorithms into quantum algorithms, defined in terms of quantum gates and circuits.

Here, we present a review of some recent examples of the application of quantum computing to swarm robotics. The majority of our examples concern an application of quantum computing to the definition of pairwise interactions between the robots of the swarm. The quantum circuit can be embedded inside a classic algorithm, and the global behaviour emerges from these local interactions. Such a mix of quantum–classical mechanisms can speed up the decision-making process of a simulated swarm in several applications, for instance in a search-and-rescue mission.

The article is organized as follows: in §2, we present how we applied quantum computing to robotic swarms, and in §3, we summarize the key ideas and discuss potential threads for further research developments.

2. Applications

The first insights concerning the idea of applying quantum computing to robotics were provided in a Physics Review article [1]. In this pioneering work, a theoretical model of robots, equipped with onboard quantum computers, perform computational and action-based tasks, interacting with their environment. Modelling took place in discrete space and time, to change the states or collect information.

Following this article, ideas sparked across the computer science and robotics communities, and other articles began to appear. According to the authors [2,3] one can envisage three core elements in quantum-based robots: multi-quantum computing units, quantum controllers/actuators and information acquisition units. In particular, it is proved that quantum search algorithms can lower search complexity from $O(N^2)$ for classical robots to $O(N\sqrt{N})$ for quantum robots. The topic of quantum advantage with respect to complexity is also faced in more recent works. In general, the application of quantum computing can also lead to improvements in machine learning, including efficiency and exponential improvements in performance over limited time periods [4]. Of course, such an advantage of quantum computing over classical computing can be inherited by quantum-based swarm-robotics developments.

Applications of quantum computing to swarm robotics concerns collaborative robotic tasks of autonomous robots (unmanned) as parts of a network whose agents are entangled [5]. The potential advantages of quantum-based cooperation of agents embodied in interacting robots deal with security against quantum attacks, thanks to entanglement, which constitutes a chapter of quantum mechanics in itself. It is proved that entanglement leads to the improvement of the collaborative behaviour of the robotic equivalent of ants [6]. Quantum computing is also helpful in the domain of optimization. A set of multiple, interacting robots can reach their target through swarm evolution mechanisms, and such a strategy is improved via a quantum-based optimization algorithm, jointly with a collision/obstacle avoidance scheme [7]. The quantum paradigm also helps improve learning approaches; quantum-enhanced clustering algorithms and deep self-learning approaches have been used to improve swarm intelligence algorithms, and for emergency vehicle dispatch management during the COVID-19 crisis [8].

Despite many potential advantages and first experimental confirmations, the application of quantum computing to swarm robotics is far from being fully explored. One of the open problems concerning a swarm of robots is *the modelling of local-to-global and global-to-local behaviour*. One possible strategy to face the issue is the definition of simple rules, from which global behaviour emerges. It is necessary, for instance, to define a self-modifying system for which the local rules are changing, to effect change in the global behaviour. In [9], an architecture is proposed where artificial swarms can change their behaviour through minimal intervention from humans, limited to some basic underlying principles. The authors, starting from principles of combinatorial approaches for multi-agent systems organization [10], focus on the particle swarm optimization algorithm, applied to search-and-rescue missions.

Proposed specifically for a search-and-rescue mission, the algorithm proposed in [11] lets global behaviour emerge from pairwise interactions, modelled through quantum gates. For a mission other than target searching, a different pairwise interaction has to be defined. A comprehensive mathematical description of a robotic swarm is attempted; for example, a nested matrix is defined in [11]. Its diagonal blocks contain the 'inner' information of single robots, and the off-diagonal blocks contain the terms of interaction between different robots. The information exchanged is that used for the previously discussed logic gates. However, such an approach is useful only for schematization purposes [11].

One of the key topics of quantum mechanics, also exploited for quantum-computing applications, is *entanglement*. It is used to model a robotic swarm as a theoretical simplification, to group and connect the behaviour of all robots of the swarm [12]. It is also used to model the communication strategy between two complex robots, not belonging to a swarm [13].

In particular, in [13], the emerging field of *quantum mechatronics*, with the combination of macroscopic mechatronic systems and microscopic quantum technologies, is used to investigate, for instance, the use of quantum effects in macroscopic actuators and for problem-solving applications.

The relevance of quantum computing in the framework of problem-solving optimization has been proved [14].

In this work, an integration of quantum computing with robotics is proposed to optimize order picking in warehouses. The developed algorithm minimizes travel distance and order batching. Among different quantum environments, the authors consider Qiskit, which we also adopted for some tests.

As mentioned in §1, one of the main objectives of our research is to model and represent the behaviour of a robot in general or a robot in a swarm to have means for going from conception to implementation. To achieve this, we study the possibility of exploiting the features of quantum computing. The main problem to be faced before designing an application of quantum computing to a field other than theoretical physics is: which elements do we *quantize*? For a robotic swarm, a possible choice is the quantization of the individual robot's position and success in target reaching, called 'reward' in some literature works [11]. In this article, in particular, the pairwise interactions between robots of the swarms, schematized as points in a rectangular arena, are modelled via a quantum circuit, embedded inside a classic algorithm. The result is a classic/quantum algorithm for a search-and-rescue mission, or, more precisely, for the search step. The classic step is a random reshuffling, and the quantum step is pairwise decision-making. The quantum circuit models an XNOR gate. If a robot explores a portion of a segment (for which the extremes are 1 and 0) and finds the target, its reward is 1 (success) and this information is sent to the other robot. If it does not find the target its reward is zero (fail), the other robot searches in the next portion of space. Degrees of space position and success in target search are expressed as combinations of 0 and 1. On the *xy*-plane, if a robot is unsuccessful, the other robot can search in three other quadrants. The truth table for two robots moving along a line is listed in table 1; the truth table for two robots moving in the *xy*-plane is listed in table 2; the quantum circuit implementing the second case is proposed in figure 1.

Let us imagine a one-dimensional scenario, as a linear path followed by two robotic ants. The robots move along the path and exchange information concerning their position and their perceived proximity to the target. On the left-hand side of table 1, two qubits are used as input, q_0 and q_1, and two qubits as output, q_0 (repeated) and q_2. The qubit q_0 denotes the one-dimensional position of robot R_1, and q_1 denotes its proximity to the target, called here *reward*. Qubit q_2 denotes the one-dimensional position of the second robot, R_2. This means that the inputs of the circuit are the position and degree of success of the first robot, which are considered by the second robot to modify (or not) its position q_2. On the right-hand side of table 1, the situation is switched: robot R_2 reaches the new position q_2 and communicates this value and degree of target proximity q_3 to R_1, which meanwhile had been waiting. Then, according to the exploration outcome of R_2, R_1 decides how to modify its position q_0, and the cycle starts

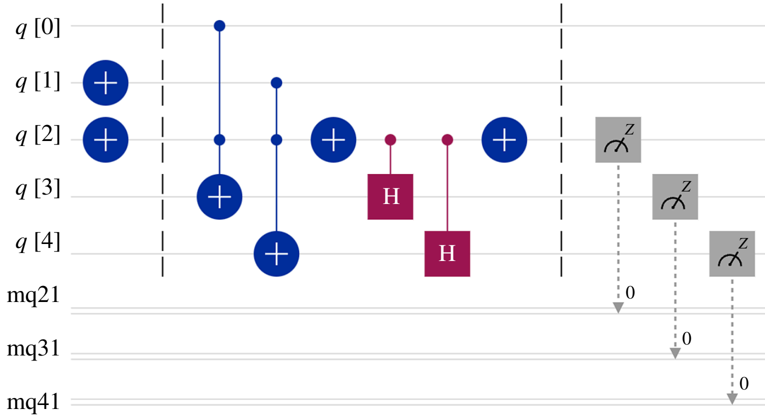

Figure 1. Quantum circuit that implements the truth table of table 2 from [11], with NOT, Toffoli and Hadamard gates. As an example, the initial configuration with $|q_0\rangle = 0$, $|q_1\rangle = 1$, $|q_2\rangle = 1$ is shown. The "plus" in the blue circle indicates the NOT gate; the same symbol connected with two smaller blue circles indicates the Toffoli gate. The symbol containing the red square with the H indicates the Hadamard gate. The boxes with the Z letter characterize the measurement operation. Each line indicate a qubit (q[0], ..., q[4]), and a classical bit (mq21, mq31, mq41), where the results of the measurements are stored.

Table 1. Truth table (XNOR) of the interaction between robots R_1 and R_2, from [11]. The qubits are position q_0, reward q_1 of R_1, and position q_2 and reward q_3 of reward. The first qubit is copied in the output information to have the same number of inputs and outputs, which is one of the conditions required for reversibility.

q_0	q_1	q_0	q_2
0	0	0	1
0	1	0	0
1	0	1	0
1	1	1	1
q_2	q_3	q_2	q_0
0	0	0	1
0	1	0	0
1	0	1	0
1	1	1	1

again. In fact, in the first application, table 1, it is considered a waiting time after each step of exploration: a robot explores, sends the information to the other and then stops. The other robot receives the information, decides where to go according to its position and the information from the other robot, and then moves. At this point, it in turn sends the information on position and degree of success to the other robot, and the cycle starts again. In a more recent application, with more than two robots in motion along a plane, there are no waiting times. The robots start from a 'nest', a small region of space, by analogy with an ants nests. Then they are randomly scattered in the arena, classically, not in a quantum way. They send each other information on position and proximity to the target. Each robot decides where to go next according to the result of the quantum circuit called for each pair of robots. And each robot follows the more successful one. Thus, if one of the robots is already close to the target, the other robots also reach it. If not, there is another round of 'classic' scattering, and the cycle starts again. All robots exchange information and move only if their reward is lower than the reward of the

Table 2. Extension of table 1 and two-dimensional space from [11]. Reversibility is lost given the indeterminacy of one of the three remaining quadrants of the plane to be explored in response to a low degree of success of the robot whose information is used as input to the circuit. When more than two robots are participating and exchanging information, then such an indeterminacy is experimentally solved, as some of the robots may be more successful than others, and the situation with a low-success degree can be avoided or postponed.

q_0	q_1	q_2	q_4	q_3	q_2
***x*-pos**	***y*-pos**	**reward**	***y*-pos**	***x*-pos**	**reward**
R_i	R_i	R_i	R_j	R_j	R_i
0	0	0	0/1	0/1	0
0	0	1	0	0	1
0	1	0	0/1	0/1	0
0	1	1	1	0	1
1	1	1	1	1	1
1	0	0	0/1	0/1	0
1	1	0	0/1	0/1	0
1	0	1	0	1	1

more successful robot. As an extension of this code, the quantum-pairwise algorithm can be recursively called inside a Grover-based path-planning strategy [15], which will be described later.

In an ideal set-up, each robot may contain an integrated quantum computer that processes the information from robotic peers and its own position and target proximity. However, this might be impossible to implement, owing to the size limitations of the small robots constituting a swarm, their simplified architecture and the constraints of real quantum computers (size, temperature, isolation, to name but a few). Thus, we can imagine that our robots send and receive information from an external quantum computer. In addition, to make the decision system more efficient, it is not the overall swarm information that enters the quantum circuit exploited as a decision system, but only a part of it. More precisely, each robot can send its own position and reward information to an external computing centre. The first step of classical computation would be the comparison between reward values. The information of position corresponding to the robot with the highest reward can thus enter the quantum circuit. The output is an indication of possible regions to explore. For instance, the swarm robots can simply reach the most successful peer, if the degree of success is high, or they can be scattered throughout the arena, exploring other places of the two-dimensional scenario, if the reward of the most successful robot is below a certain threshold. This idea can be modelled without a quantum circuit. However, after a step of preliminary random scattering out of the nest, the use of a quantum circuit to undertake decisions allows the swarm to reach the target faster. This can be proved by comparing the number of quantum-algorithm cycles against those of a completely classical algorithm and the travelled distance in each step [11]. In fact, in the application of [11], the quantum circuit is not called for each pair of robots, but only when one of the robots in its preliminary, random exploration of the arena, shows a higher level of reward. The information of the more successful robot enters the quantum circuit, and its output provides an indication of the approximate region of the arena for the further step of exploration. Then, the robots reach (approximately) the new region, with a randomization degree that avoids the superposition of all robots. The comparison between levels of success in target finding is again compared (through an ideal message exchange that is implemented for real in further simulations [16,17]), and the position information of the more successful robot enters again the quantum circuit. The procedure goes on until the swarm reaches the target.

Another simplification is strategically proposed in [11]: the robots do not detect their proximity to the target through a simulated sensor system nor there is any actual message exchange between the robots. The platform is an omniscient system where the reward is the distance between the target and the position of the robot. In [18], a more realistic system is implemented with message exchange between robots and computation of the reward according to robotic observations. The circle-like robots in Jupyter/Qiskit collect information from robots modelled in Webots, with a sensory system able to detect the approximate distance from the target and its 'visibility' in terms of angle of sight. Also, the robots send each other messages with the required information; also implemented is a connection between the quantum simulation and the robotic swarm simulation. In this study, the input of the quantum circuit is provided by the parameters of a swarm simulated using the software Webots, through an automatic system of file writing and reading. The important parameters are robot position and measured target proximity (perceived through their sensors). During the run of the circuit, the robots of the simulation are stopped and then wait for new information. After the run of the circuit (on Qiskit-Jupyter), the output of the measurement (as the most likely states to be obtained) is written into a text file and sent to Webots. There, the average of the most likely results of the quantum measurement are mapped into positions to be reached by the robots on the simulation platform. Once the robots start their movement again and reach the designated positions, perceiving their proximity to the target via their sensors, they update their values of both position and reward. This information is again coded into a text file, sent to Qiskit-Jupyter and the cycle starts again until the target is reached by the swarm. Through this file-exchange system, we verify the advantage of the quantum-pairwise interaction against the completely classic approach. However, the time interval needed for file exchanges between Webots and Jupyter suggests that a more efficient strategy, or a different robotic platform, should be considered.

Modelling in Webots, or other robotic platforms, requires the setting of physical constraints. In addition, a more precise modelling of robotic message exchange requires the definition of telecommunication protocols. A protocol for communication for a swarm of robots inside an aquatic scenario is proposed in [16], with an application of graph theory to the swarm. In such a scenario, the quantum-pairwise interaction strategy is also instantiated [17]. Other instances of quantum algorithms thought for aquatic scenarios include a study on robotic fish [19].

Natural inspiration can help solve problems in different fields, such as the generation and distribution of electric energy. In an improved version of the quantum-fish algorithm, called the Improved Quantum Artificial Fish Swarm Algorithm [19], the goal is to solve distributed network programming problems, including distributed generation, through a combination of quantum computing with an artificial-fish swarm-optimization algorithm. The results of that study showed an advantage in terms of convergence speed and accuracy. The passages of the algorithm involve random movement, preying, swarming and following. The exploitation of quantum computing offers advantages in computational parallelism and increased storage capacity. In particular, in [20], the position of each 'quantum fish' is encoded in a matrix of phases, which can be decomposed according to the space coordinates. The position of each fish is periodically updated, at each step of the search. This is not directly related to our work but constitutes a further example of quantum-swarm algorithm development.

The model of the aquatic robot proposed in [16,17] is shown in figure 2, simulated inside the software Webots. The robot, called *RoboWood*, comprises a tablet of wood, with four propellers visible by clicking on them, four GPS position sensors on each side (above the level of water), distance sensors on each side and a camera on the bottom face to collect images of the seabed, for a subsequent step of analysis. The physics of the aquatic system, where the robot acts is also simulated. The Webots simulation allows the operator to perform estimations of speed, battery requirement and to project a communication protocol for a swarm of RoboWoods [16].

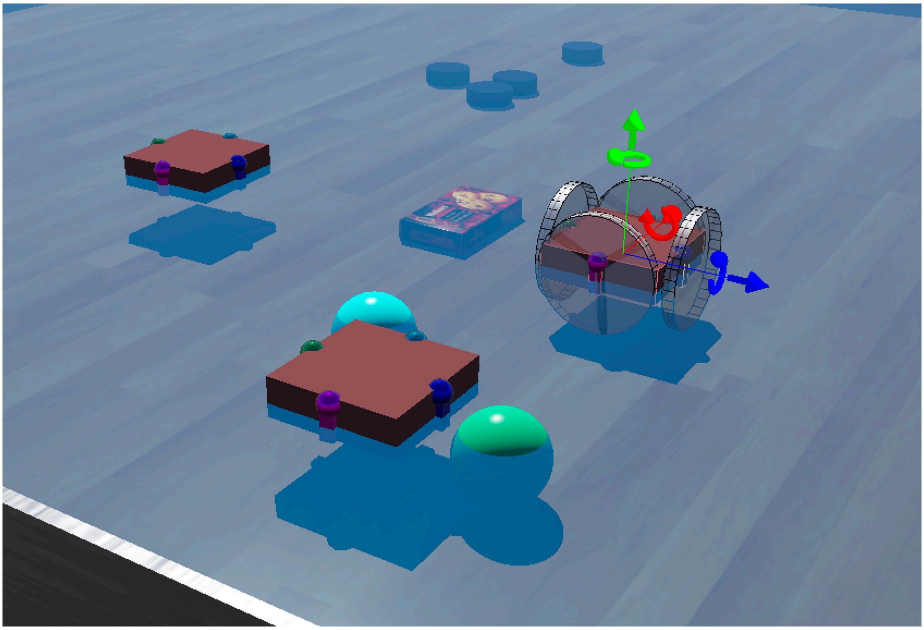

Figure 2. The robot *RoboWood*, proposed in [16].

As mentioned previously, one of the applications of quantum computing is the improvement of strategies for path planning. As an example, we describe here the core idea for Grover-based path planning. Figure 3 shows the circuit for a single Grover iteration, as described in [22].

First, the oracle is applied to mark the solutions to the problem. This is achieved by inverting only the phase of a solution state once one gets detected, leaving the rest of the superposition intact. Then, the diffuser completes the amplitude amplification procedure by having the marked superposition interfere with the starting state, constructively enhancing the amplitude of solution states and consequently dampening non-solution ones. This operation is performed through a conditional phase shift targeted at the states that are orthogonal to the starting state, or, equivalently, orthogonal to the $|0\rangle$ state in the Hadamard-transformed space. Finally, the oracle workspace register contains additional qubits that may be needed to perform the computation in the search space. The implementation of the quantum circuit for the Grover procedure applied to the path-planning problem in the 2×2 environment is shown in figure 4.

At the beginning, the search space is encoded as a uniform superposition of the inputs through Hadamard gates. Then, a V-shaped, problem-specific oracle is applied on to the uniform state to mark solutions through the oracle qubit, displayed at the bottom wire. Note that since this problem instance requires combinatorial logic to produce searchable outputs out of the input states, additional oracle workspace qubits are employed in the circuit. Finally, the upper right-hand side purple block is the diffusion operator. Its representation is synthesized as a single black box as its structure is fixed, depending just on the number of qubits it is applied to. In this case, only one iteration of the algorithm is required to complete the search, and therefore just a single oracle–diffuser pair is appended to the encoded input. The circuit of figure 4 is applied to a robotic swarm, called iteratively at each step of the swarm exploration and after each step of message exchange; see [15] for its union with the quantum-based pairwise interaction proposed in [11].

Another application of quantum computing to robots is proposed in [23], where the starting point is the concept of an *agent*. This work focuses on multi-agent systems; a swarm of robots can be considered one of their instances. Each robot has a mechanism of sensory-based control and reaction to the environment perception. Briefly, the robot perceives the environment through sensors and performs actions such as identifying a ball, carrying a ball, detecting

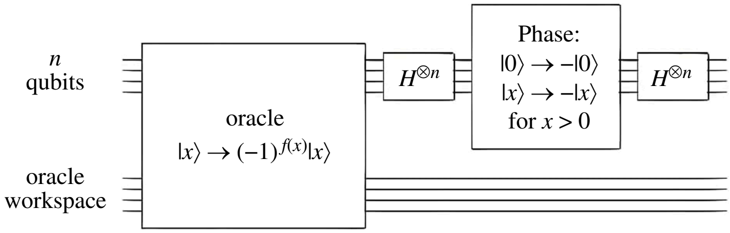

Figure 3. Quantum circuit for a single Grover iteration, from [21], adapted from [22].

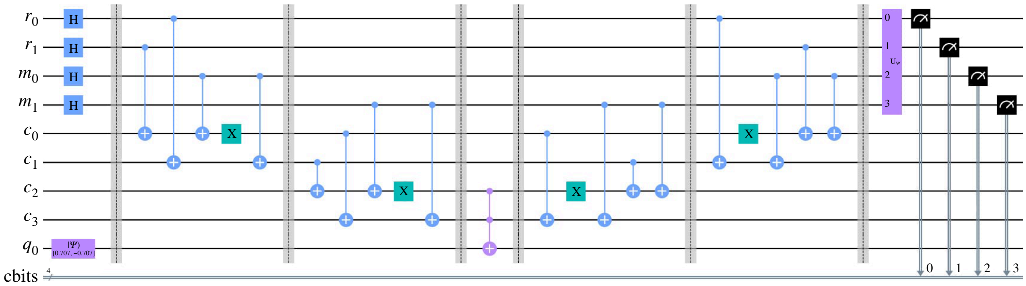

Figure 4. Quantum circuit for path planning, from [21], exploited for a robotic swarm in [15] with similar notation and symbols to figure 1.

an obstacle and reaching a deposit area. The choice of action by the robot depends upon the verification of conditions, e.g. the detection of an obstacle, or the deposit being reached. The robot will move randomly if none of the conditions is verified. Quantum modelling enters the definition of the reactive agent's state, defined as a superposition of tensor products of perception and action vectors. Different qubits are considered for perception, and other qubits for actions. When an action is not performed, the associated probability amplitude is zero. The robotic decision-making is accomplished through a quantum circuit, having as inputs the perception qubits, and as output the action qubits and their measurements. Figure 5 shows one of the steps for the procedure described in [23]. The gates CNOT and CCNOT are used for the perception; CNOT gates are used for an XOR logic operation, they activate actions requiring only one perception. The CCNOT gate (Toffoli gate) activates actions requiring two valid perceptions. Qubits $|x_i\rangle$ are ancillary qubits.

One of the open problems concerning swarm robotics is the control *before* the robotic movement and *during* the movement itself. The first point can be addressed by manipulating the simple rules of motion to shape the emerging behaviour, as discussed in [9]. The second point requires real-time data collection from the robots, possibly including human-friendly information; data that can quickly be processed by humans include visualization and *sonification*. A recent advance in the field of human–robot interaction involves the mapping of robotic parameters to sound. The human ability to process complex auditory data, retrieve patterns, localize sources and identify problems is very useful when dealing with quantum-driven robots. In [24], a strategy is proposed where the positions of the robots, in terms of slices of two- or three-dimensional space, are mapped to the notes of the Western chromatic scale (figure 6). The trajectory of each robot thus becomes a melody, and polyphonic music emerges from the simultaneous motion of a swarm of robots that explore an arena and converge on a target. In figure 7, an example of the sonification output is shown. The two-dimensional arena is divided into slices, and each slice is associated with a pitch of the chromatic scale. Through this association, the positions of the robots are mapped to sound. In the figure, we can see the representation of pitches in the chromatic circle and the corresponding steps of a robotic simulation in the Webots software.

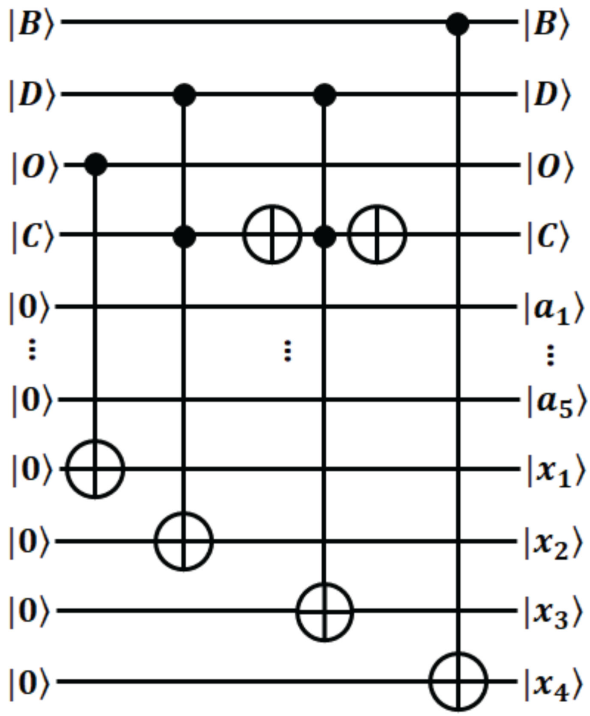

Figure 5. Quantum circuit developed for the activation of actions of an agent in response to external stimuli (from [23]).

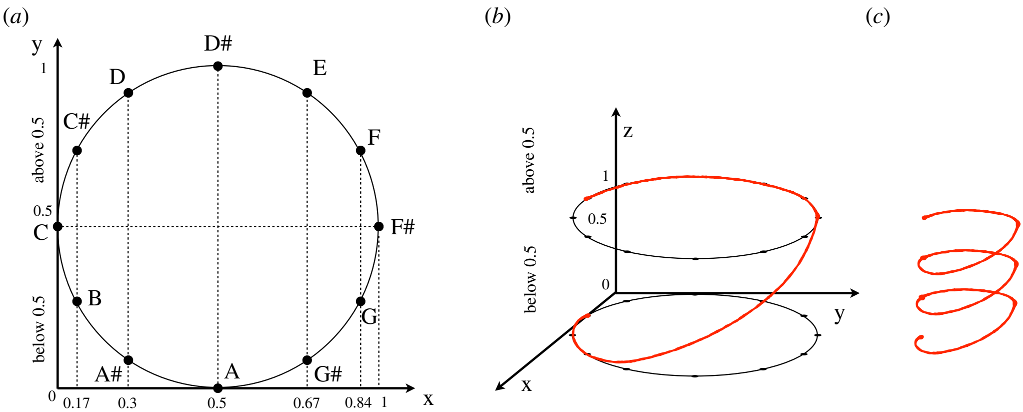

Figure 6. Pitch-choice structure, from [24].

The motion through a three-dimensional arena can be approximately sonified considering changes of octave. A simplification of this approach, shown in the middle of figure 6, shows a jump of octave when the robotic motion involves the upper side of the three-dimensional space. An example of the musical output is presented in figure 8. With each robot are associated two musical instruments, of similar timbre to allow auditory recognizability, but playing in different ranges, according to the portion of three-dimensional space through which the robot is moving.

Sonification can also involve the other parameters for each robot, such as motor activation and sensory activation. As formalized in [24], the movement of the robots and their inner parameters (such as motor activation and sensory-distance activation) can be expressed as the tensor product of the parameter spaces seen as Hilbert spaces. Their sonification is obtained via a suitable mapping (based on perceptive similarities, gestural similarity and crossmodal correspondences) to the tensor product of motion through musical spaces, obtaining a more

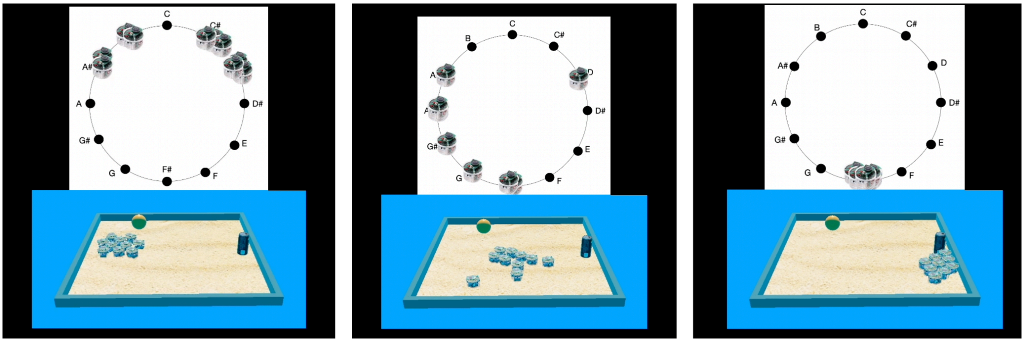

Figure 7. An example of swarm-robotic motion sonification, from [24]. Top: circle with the chromatic scale; bottom: screenshots of a robotic simulation in Webots, with initial (*a*), middle (*b*) and final (*c*) steps. The video can be accessed here.

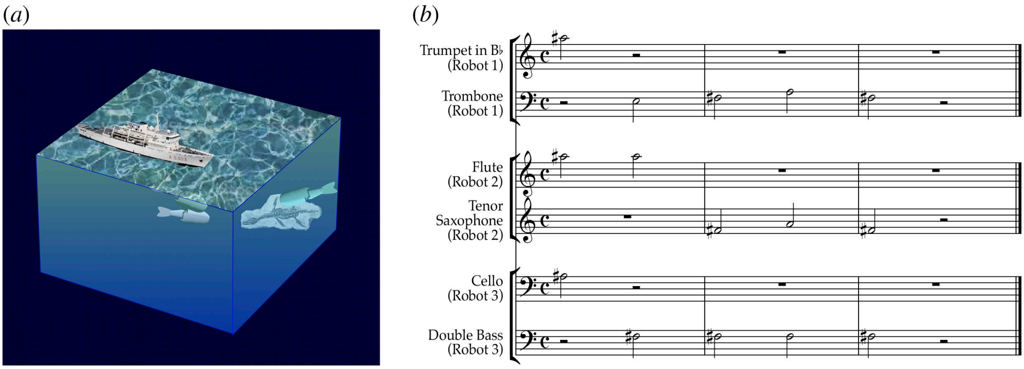

Figure 8. An example of three-dimensional robotic motion (*a*) and the corresponding sonification output (*b*), from [24]. The complete video can be accessed here.

comprehensive sonification strategy. Such an approach is formally justified by the use of groupoids on the structure of the tensor product of Hilbert spaces [25]. Also exploiting hints from mathematical music theory [26], the generalized musical space is defined in [24] as

$$\bigotimes_{i=1}^{4} \mathcal{H}_i^M(t) = \mathcal{H}_P^M \otimes \mathcal{H}_L^M \otimes \mathcal{H}_T^M \otimes \mathcal{H}_A^M, \tag{2.1}$$

where

— $\mathcal{H}_P^M$ is the pitch space;
— $\mathcal{H}_L^M$ is the loudness space;
— $\mathcal{H}_T^M$ is the timbre space;
— $\mathcal{H}_A^M$ is the articulation space.

Similarly, in [24], the swarm-robotic space is defined as

$$\bigotimes_{j=1}^{4} \mathcal{H}_j^R(t) = \mathcal{H}_C^R \otimes \mathcal{H}_M^R \otimes \mathcal{H}_i^R \otimes \mathcal{H}_S^R, \tag{2.2}$$

where

— $\mathcal{H}_C^R$ is the space of positions through coordinates;
— $\mathcal{H}_M^R$ is motor options space;
— $\mathcal{H}_i^R$ is the robot identities space;

— $\mathcal{H}_S^R$ is the sensors space.

With a more compact notation:

— $\bigotimes_{i=1}^{4}\mathcal{H}_i^M(t)$ is the generalized musical space;
— $\bigotimes_{j=1}^{4}\mathcal{H}_j^R(t)$ is the swarm-robotic parameter space.

The sonification is thus an arrow mapping swarm movements in the generalized robotic space (for the considered parameters) to the generalized musical space:

$$S\colon \bigotimes_{i=j}^{4}\mathcal{H}_j^R(t) \rightarrow \bigotimes_{i=1}^{4}\mathcal{H}_i^M(t)\,. \tag{2.3}$$

Conversely, sound-based control of robotic movements is expressed as

$$C\colon \bigotimes_{i=1}^{4}\mathcal{H}_i^M(t) \rightarrow \bigotimes_{i=j}^{4}\mathcal{H}_j^R(t). \tag{2.4}$$

The use of sonification opens new scenarios not only for the technical investigation of swarm parameters but also for their aesthetic applications. This consideration leads us to our conclusions.

3. Discussion and conclusions

We began our journey with a double inspiration from nature: swarm behaviour emerging from simple rules and the laws regulating the nature of quantum mechanics, and the branch of computer science derived from it, quantum computing.

In this review article, we have summarized open problems concerning robotic swarm organization, control and optimization. Some of the new solutions that can be hypothesized include the use of quantum computing mainly to enhance computational efficiency, to exploit the power of quantum computing concepts, such as superposition and entanglement for modelling, representing and simulating the behaviour of a swarm of robots.

The main scope of this article is to provide an overview of recent approaches. The key advantage of quantum computing is the reduction of computational time, as proved in specific references, such as [15,21]. The details of these tests are, however, not provided here, where we concentrate on the flow of ideas and methods. The disadvantage of quantum computing is the requirement of real quantum computers which, even though accessible remotely, have limitations in terms of costs (except for a limited number of qubits) and portability. It is not yet possible to equip a small robot in a swarm of an autonomous quantum-based computing unit. Perhaps this could be the vision of the future: how to miniaturize quantum computers, maintaining their temperatures and allowing portability.

While we discussed applications of swarm robotics and quantum computing mostly in the framework of search-and-rescue missions, that is, for *utility* scopes, we consider that the new technologies can also be adopted in the domain of the *arts*. In fact, in addition to task-oriented purposes of quantum computing applied to robotics, creative and aesthetic applications are also worthy of interest and constitute a promising field of future research. New hints for swarm robotics may come from the research fields of the Internet of Things and the related Internet of Sounds. The decision-making approach of individual, distant performers interacting between these fields can be approximated with a quantum logic gate [27]. The same approach can be exported to artificial musicians, also in the frameworks of the Internet of Things and the Internet of Sounds. And it could be instantiated as robotic performers or tiny musical agents in a swarm of robots. The union of quantum technology for swarm robotics leads to futuristic applications also in the domain of the Internet of Things [28], with potential social and economic effects.

Quantum computing is more and more applied to model the thinking mechanism of the human mind, including artistic creation. We mention a classic quantum dialogue to model robotic gesture production (as a choreography) in response to an external multi-sensory stimulus [29], inspired by the quantum approach to cognitive *qualia* by Faggin [30]. The use of qualia is an addition to the world of quantum computing, as a further step towards the modelling of complex 'thoughts' as a superposition of simple units and of thinking flow as a succession of steps of superposition and measurements. With reference to qualia and their use in formalization, one paves the way towards an enriched computational modelling of some human-thinking features. This thread of research may help model, and thus understand, our way of creating arts in dialogue between our inner world and the world outside of us.

From our minds to our physiological reality, we also mention here the promising field of microrobotics and nanorobots, the use of which may lead to a revolution in medicine [31]. Substantial novelties in this domain may be fostered through the application of quantum computing.

We started our exploration of the flourishing field of quantum computing as applied to robotics with a citation from Physics [1], and we want to end it with another reference to Physics; the physics of complex networks. In [17], a system of quantum-driven robotic swarms and their computing centrals is formalized as a multilayer network. In recent research, a network has been proposed in which the nodes are quantum circuits [32]. Thus, the role of quantum circuits may not be limited to the 'rules of action' of some objects but can constitute the 'object' themselves. In particular, in [32], an optimization approach is proposed in the framework of quantum machine learning. The authors, in particular, propose a quantum-architecture search algorithm, based on a Monte Carlo graph search for importance-measure sampling. The method can be applied to the discovery of new quantum circuits. This in turn can be related to the choice of the best circuits for robotic interaction. Can the robots find their local and pairwise rules, according to the general need of the swarm? Animals can do this, but they are *alive*. Can the robots of the swarm become as autonomous?

Perhaps the time has come to dedicate ourselves to the contemplation of nature.

Data accessibility. The links to data and video are indicated in the paper.

Declaration of AI use. We have not used AI-assisted technologies in creating this article.

Authors' contributions. M.M.: conceptualization, data curation, formal analysis, investigation, methodology, software, validation, visualization, writing—original draft, writing—review and editing; V.S.: conceptualization, investigation, methodology, project administration, supervision, visualization, writing—review and editing; A.C.: conceptualization, funding acquisition, investigation, project administration, supervision, writing—review and editing.

All authors gave final approval for publication and agreed to be held accountable for the work performed therein.

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References

1. Benioff P. 1998 Quantum robots and environments. *Phys. Rev. A* **58**, 893–904. (doi:10.1103/PhysRevA.58.893)
2. Dong D, Chen C, Zhang C, Chen Z. 2006 Quantum robot: structure, algorithms and applications. *Robotica* **24**, 513–521. (doi:10.1017/S0263574705002596)
3. Dong D, Chen C, Li H, Tarn TJ. 2008 Quantum Reinforcement Learning. *IEEE Trans. Syst. Man. Cybern. B* **38**, 1207–1220. (doi:10.1109/TSMCB.2008.925743)

4. Dunjko V, Taylor JM, Briegel HJ. 2016 Quantum-Enhanced Machine Learning. *Phys. Rev. Lett.* **117**, 130501. (doi:10.1103/physrevlett.117.130501)
5. Khoshnoud F, Esat II, de Silva CW, Quadrelli MB. 2019 Quantum Network of Cooperative Unmanned Autonomous Systems. *Unmanned Syst.* **07**, 137–145. (doi:10.1142/S2301385019500055)
6. Iimura I, Takeda K, Nakajama S. 2014 An Experimental Study on Quantum-Entangled Cooperative Behavior in Swarm Intelligence. *Int. J. Emerging Technol. Adv. Eng.* **4**, 580–586.
7. Li RG, Wu HN. 2021 Multi-Robot Plume Source Localization Based on Swarm Evolution Mechanisms. In *Proceedings of the 2021 1st International Conference on Control and Intelligent Robotics*, p. 692. New York, NY, USA. https://dl.acm.org/doi/proceedings/10.1145/3473714.
8. Drias H, Drias Y, Houacine NA, Bendimerad LS, Zouache D, Khennak I. 2022 Quantum OPTICS and deep self-learning on swarm intelligence algorithms for Covid-19 emergency transportation. *Soft comput.* **27**, 13181–13200. (doi:10.1007/s00500-022-06946-8)
9. Rodriguez S, Hilaire V. 2023 A methodological approach for the analysis and design of Human–Swarm interactions based upon feedback loops. *Expert Syst. Appl.* **217**, 119482. (doi:10.1016/j.eswa.2022.119482)
10. Mella P. 2008 Observing Collectivities: The Combinatory Systems Approach in Social Sciences. *Int. J. Interdiscip. Soc. Sci.* **3**, 213–224. (doi:10.18848/1833-1882/cgp/v03i01/52499)
11. Mannone M, Seidita V, Chella A. 2023 Modeling and designing a robotic swarm: A quantum computing approach. *Swarm Evol. Comput.* **79**, 101297. (doi:10.1016/j.swevo.2023.101297)
12. Ivancevic V. 2016 Entangled swarm intelligence: Quantum computation for swarm robotics. *Math. Eng. Sci. Aerosp.* **7**, 441–451.
13. Lamata L, Quadrelli MB, de Silva CW, Kumar P, Kanter GS, Ghazinejad M, Khoshnoud F. 2021 Quantum Mechatronics. *Electronics* **10**, 2483. (doi:10.3390/electronics10202483)
14. Atchade-Adelomou P, Alonso-Linaje G, Albo-Canals J, Casado-Fauli D. 2021 qRobot: A Quantum Computing Approach in Mobile Robot Order Picking and Batching Problem Solver Optimization. *Algorithms* **14**, 194. (doi:10.3390/a14070194)
15. Chella A, Gaglio S, Mannone M, Pilato G, Seidita V, Vella F, Zammuto S. 2023 Quantum planning for swarm robotics. *Rob. Auton. Syst.* **161**, 104362. (doi:10.1016/j.robot.2023.104362)
16. Mannone M, Seidita V, Chella A, Giacometti A, Fazio P. 2023 Energy and SNR-Aware Robotic Swarm Coordination for Aquatic Cleaning Operations. In *IEEE 97th Vehicular Technology Conference (VTC2023-Spring)*, Florence, Italy. vol. **See**. (doi:10.1109/VTC2023-Spring57618.2023.10200327)
17. Mannone M, Marwan N, Seidita V, Chella A, Giacometti A, Fazio P. 2024 Entangled Gondolas. Design of Multi-layer Networks of Quantum-Driven Robotic Swarms. In *XVII international workshop on artificial life and evolutionary name: computation (WIVACE 2023)* (eds M Villani, S Cagnoni, R Serra), pp. 177–189. Cham: Springer. (doi:10.1007/978-3-031-57430-6_14)
18. Thomas A, Mannone M, Seidita V, Chella A. 2023 Swarm Robotics and Quantum Computing in Simulated Environments. In *10th Italian workshop on artificial intelligence and robotics (AIRO)*. (doi:10.3390/math10030372)
19. Zhu K, Jiang M. 2010 Quantum artificial fish swarm algorithm. In *Proceedings of the 8th World Congress on Intelligent Control and Automation*. (doi:10.1109/WCICA.2010.5553761)
20. Du T, Hu Y, Ke X. 2015 Improved Quantum Artificial Fish Algorithm Application to Distributed Network Considering Distributed Generation. *Comput. Intell. Neurosci.* **2015**, 851863. (doi:10.1155/2015/851863)
21. Chella A, Gaglio S, Pilato G, Vella F, Zammuto S. 2022 A Quantum Planner for Robot Motion. *Mathematics* **10**, 2475. (doi:10.3390/math10142475)
22. Nielsen MA, Chuang I. 2000 *Quantum information and quantum computation*. Cambridge, UK: Cambridge University Press.
23. Koukam A, Abbas-Turki A, Hilaire V, Ruichek Y. 2021 Towards a Quantum Modeling Approach to Reactive Agents. In *2021 IEEE International Conference on Quantum Computing and Engineering (QCE)*. (doi:10.1109/QCE52317.2021.00029)
24. Mannone M, Seidita V, Chella A. 2023 The Sound of Swarm. Auditory Description of Swarm Robotic Movements. *ACM Trans. Hum. Robot Interact.* **12**, 1–27. (doi:10.1145/3596203)

25. Baez J, Hoffnung E, Walker C. 2008 Groupoidification Made Easy. *arXiv* https://arxiv.org/pdf/0812.4864.pdf
26. Hughes JR. 2022 Generalizing the Orbifold Model for Voice Leading. *Mathematics* **10**, 1–16. (doi:10.3390/math10060939)
27. Mannone M, Turchet L. 2023 Theoretical Quantum Modeling of Improvisation in Networked Music Performances to Regulate the Behaviour of Artificial Musicians. In *4th International Symposium on the Internet of Sounds, IS2*, pp. 1–7. (doi:10.1109/IEEECONF59510.2023.10335489). https://doi.org/10.1109/IEEECONF59510.2023.10335489.
28. Kumar A, Augusto de Jesus Pacheco D, Kaushik K, Rodrigues JJPC. 2022 Futuristic view of the Internet of Quantum Drones: Review, challenges and research agenda. *Veh. Commun.* **36**, 100487. (doi:10.1016/j.vehcom.2022.100487)
29. Mannone M, Chella A, Pilato G, Seidita V, Vella F, Gaglio S. 2024 Modeling Robotic Thinking and Creativity: A Classic–Quantum Dialogue. *Mathematics* **12**, 642. (doi:10.3390/math12050642)
30. D'Ariano GM, Faggin F. 2022 Hard Problem and Free Will: An Information-Theoretical Approach. In *Artificial intelligence versus natural intelligence*, pp. 145–192. Cham: Springer. (doi:10.1007/978-3-030-85480-5_5)
31. Bijli MK, Verma P, Singh AP. 2024 A systematic review on the potency of swarm intelligent nanorobots in the medical field. *Swarm Evol. Comput.* **86**, 101524. (doi:10.1016/j.swevo.2024.101524)
32. Rosenhahn B, Osborne TJ. 2023 Monte Carlo graph search for quantum circuit optimization. *Phys. Rev. A* **108**, 062615. (doi:10.1103/PhysRevA.108.062615)