Generating Discorrelated States for Quantum Information Protocols by Coherent Multimode Photon Addition

Nicola Biagi, Luca S. Costanzo, Marco Bellini, and Alessandro Zavatta*

It is demonstrated that the recently developed technique of delocalized single photon addition may generate discorrelation, a new joint statistical property of multimode quantum light states, whereby the number of photons in each mode can take any value individually, but two modes together never exhibit the same. By coherently adding a single photon to two identical coherent states of light in different temporal modes, the first experimental observation of discorrelation is provided. The capability of manipulating this statistical property has applications in scenarios involving the secure distribution of information among untrusted parties, like in the so-called "mental poker" games.

1. Introduction

When considering different modes of the electromagnetic field, the number of photons (NoP) populating each of them may exhibit a wide variety of distributions. Independently from the distribution in each single mode, correlations among the number of photons in the different modes are of particular interest and may come in different forms. Injecting a coherent state in one input port of a balanced beam-splitter, with vacuum in the other, the probability of observing a particular number of photons n in one output mode is independent (un-correlated) from that of measuring m in the other one (see **Figure 1**a). If a classical thermal state is split, correlations are observed, with a higher probability of measuring the same NoP in the two modes. These classical correlations have been already exploited in ghost imaging applications,[1] whereas non-classical correlations, such as those existing in twin beams generated by parametric down-conversion,[2,3] are known to bring additional and unique

Dr. N. Biagi, Dr. L. S. Costanzo, Dr. M. Bellini, Dr. A. Zavatta
Istituto Nazionale di Ottica (CNR-INO)
Largo Enrico Fermi 6, Florence 50125, Italy
E-mail: marco.bellini@ino.cnr.it; alessandro.zavatta@ino.cnr.it

Dr. N. Biagi, Dr. L. S. Costanzo, Dr. M. Bellini, Dr. A. Zavatta
LENS and Department of Physics & Astronomy
University of Firenze
50019 Sesto Fiorentino, Florence, Italy

advantages for quantum enhanced imaging and sensing.[4] In principle, an ideal two-mode squeezed vacuum exhibits perfect photon number correlations, meaning that two perfect detectors always measure the same number of photons in the two modes. Such perfect correlations are also important for quantum communication tasks, when common but random information has to be shared between two parties.[5,6] On the other hand, when a single photon is used at the input of the beam-splitter, perfect anti-correlations are observed between the outputs: if a single photon is measured at one output, then no photon is present in the other one, and viceversa. Equally anti-correlated outputs result from impinging two indistinguishable photons onto a balanced beam-splitter, in a so-called Hong-Ou-Mandel (HOM) scheme.[7]

Different from all the above, a new form of correlation in the NoP of different field modes has been recently discussed by Meyer-Scott et al.[8] It consists in the zeroing of particular outcome probabilities for the NoP in one mode for a given measurement result in the other mode. Appropriately named as discorrelation, it manifests itself in null elements of the joint photon number probability distribution P_{n_1,n_2}. In particular, one can have the case $P_{n,n} = 0$, where the probability of measuring the same number n of photons in both modes is always zero for all n, but the marginal distributions $P_n = \sum_{m=0}^{\infty} P_{n,m}$ are always nonzero (see Figure 1b).

Figure 1c presents an alternative way to visualize discorrelation. Here, the integrals along the diagonal direction of the joint photon number distributions show the probability $P(\Delta n)$ of a difference $\Delta n = n_1 - n_2$ in the photon counts between the two modes. Therefore, a separable state (Figure 1a) shows a maximum for $\Delta n = 0$, whereas the ideal discorrelated state (Figure 1b) has a zero there, meaning null probability of measuring the same number of photons in the two modes.

Opposite to quantum key distribution schemes, where common random keys need to be shared, "discorrelation" can be used to distribute unique randomness between parties. In particular, it could be useful in so-called "mental poker" problems, which are concerned with the fair dealing of cards between distant players without a trusted third party.

A possible (classical) solution to this class of problems, also related to blind signature for secure voting or electronic cash, was first provided by Rivest, Shamir, and Adleman (RSA) in 1981, in one of the first appearances of Alice and Bob as the two untrustworthy parties playing poker over the phone.[9] The proposed

**ADVANCED
SCIENCE NEWS**

www.advancedsciencenews.com

**ADVANCED
QUANTUM
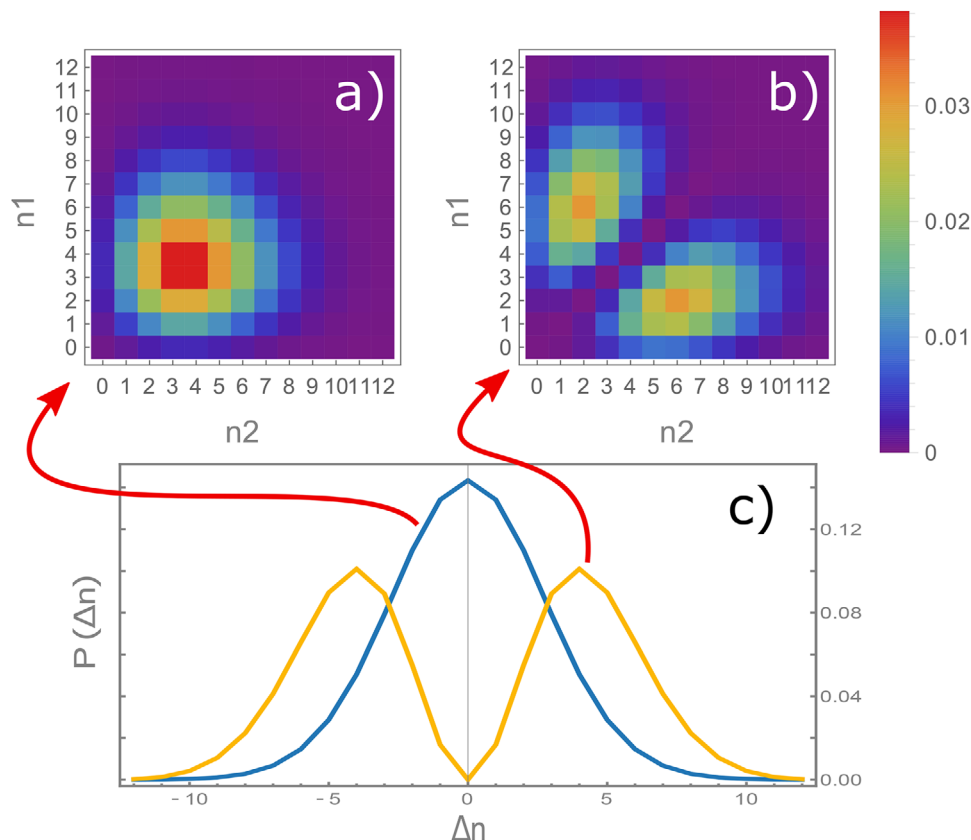TECHNOLOGIES**

www.advquantumtech.com

Figure 1. Joint photon number distributions P_{n_1,n_2} for: a) an ideal separable $|\alpha\rangle_1|\alpha\rangle_2$ state (with $\alpha=2$) as obtained, for example, by splitting a coherent state $|\sqrt{2}\alpha\rangle$ in a 50% beamsplitter. The numbers of photons measured in the two modes are not correlated in this case; b) the same state exhibiting discorrelation after delocalized single photon addition; c) integrals along the diagonal direction of the above joint photon number distributions (see the text for more details).

protocol relied on commutative encryption, that is the possibility to correctly retrieve a multiply encrypted message by performing decryption operations in a different order. However, just like for the RSA public key algorithm,[10] the security of the scheme was completely dependent on the difficulty of the mathematical problem of factorization. On the contrary, sharing a discorrelated multimode state would naturally guarantee the uniqueness of the distributed random numbers.

The aim of this work is to provide the first experimental evidence of such a valuable resource for quantum information protocols.

2. Discorrelation by Single-Photon Addition

The property of discorrelation had never been investigated and observed experimentally yet, therefore Meyer-Scott et al.[8] proposed some feasible schemes to generate discorrelated states in a laboratory. Their schemes were based either on the mixing of a coherent and single-photon state on a beam-splitter, or on conditional measurements after the mixing of an entangled HOM state with two independent multiphoton states. Here, we use a different approach to generate discorrelated states, based on the delocalized addition of a single photon.

Single-photon addition is the optical realization of the bosonic creation operator, and it operates by increasing an optical field mode by exactly one quantum of excitation. Experimentally, the photon addition operation is realized by taking advantage of stimulated emission in a two-mode degenerate parametric amplifier, provided that the parametric gain is low enough.[11] Owing to photon number correlations between signal and idler beams of the parametric emissions, the detection of an idler photon heralds the successful realization of the photon addition onto the light state injected along the input signal mode. Single-photon addition can generate nonclassical light states from classical ones and, once combined with photon subtraction, it is used for full light state engineering at the single-photon level. For instance, using sequences of photon addition and subtraction operations, it is possible to realize enabling tools for optical quantum technologies, such as noiseless linear amplification[12] and measurement-induced strong Kerr nonlinearity at the single-photon level.[13] Moreover, when the single-photon addition process is coherently applied to a multimode field, delocalized heralded addition can entangle two or more field modes.[14]

Here, we use this recently developed method of delocalized single photon addition to generate discorrelation between two modes of the electromagnetic field initially populated by uncorrelated coherent states. The effects of this optical manipulation are

**ADVANCED
SCIENCE NEWS**

www.advancedsciencenews.com

**ADVANCED
QUANTUM
TECHNOLOGIES**

www.advquantumtech.com

ideally depicted in Figure 1: the delocalized addition of a single photon digs a null diagonal into the joint photon number probability distribution of two coherent states of equal amplitude (initially exhibiting a maximum on the diagonal of P_{n_1,n_2}). After addition, although the NoP in each of the two modes can still take any value individually, the two modes together never exhibit the same, that is, $P_{n,n} = 0$. By experimentally reconstructing the output joint photon number probability distributions P_{n_1,n_2}^e of two identical coherent states that underwent the delocalized addition of a single photon, we provide the first evidence of discorrelation in a multiphoton two-mode state.

The coherent addition of a single photon to two distinct field modes, 1 and 2, by means of the balanced superposition $(\hat{a}_1^\dagger - \hat{a}_2^\dagger)/\sqrt{2}$ generates entanglement between the two modes independently of the kind of states originally populating them.[14] The simple coherent addition of a shared single photon to two vacuum states, produces a single-photon mode-entangled state[15] of the kind $(|1\rangle_1|0\rangle_2 - |0\rangle_1|1\rangle_2)/\sqrt{2}$, which clearly exhibits discorrelation, as discussed above. When delocalized single-photon addition acts on two input modes containing identical coherent states $|\alpha\rangle$, the final state is:

$$|\psi(\alpha)\rangle_{12} = (\hat{a}_1^\dagger|\alpha\rangle_1|\alpha\rangle_2 - |\alpha\rangle_1\hat{a}_2^\dagger|\alpha\rangle_2)/\sqrt{2}$$

$$= [\hat{D}_1(\alpha)\hat{D}_2(\alpha)(|1\rangle_1|0\rangle_2 - |0\rangle_1|1\rangle_2)]/\sqrt{2} \quad (1)$$

with the phase-space displacement operator defined as $\hat{D}(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$ and using the operator relation $\hat{a}^\dagger\hat{D}(\alpha) = \hat{D}(\alpha)\hat{a}^\dagger + \alpha^*\hat{D}(\alpha)$. Therefore, coherently adding a delocalized single photon to two identical coherent states is seen to coincide with the result of an equal phase-space displacement operation $\hat{D}(\alpha)$ on both modes of a single-photon mode-entangled state; the final state thus maintains constant maximum entanglement independently of the amplitude of the input coherent states. It is easy to see that this state can be equivalently obtained by mixing a single photon and a $|\sqrt{2}\alpha\rangle$ coherent state on a balanced beam-splitter, as proposed in ref. [8], therefore the two output modes should present perfect multiphoton discorrelation.

3. Experimental Setup

We use a temporal-mode scheme (see **Figure 2**) for the experimental implementation of the delocalized single-photon addition. The coherent superposition of photon additions on two different traveling temporal modes, 1 and 2, is obtained by allowing the herald (idler) photon from a stimulated parametric down-conversion (PDC) process[16] to travel two indistinguishable paths of different length toward the herald detector.[15] The signal mode of the PDC is seeded with attenuated laser pulses coming from the main laser source (a mode-locked Ti:sapphire laser emitting 1.5-ps pulses at 786 nm and at a repetition rate of 81 MHz). The PDC crystal (a 3-mm long, bulk, type-I, BBO) is pumped by the frequency-doubled laser emission. The two paths for the idler photons are realized with an unbalanced, fiber-based, Mach–Zehnder interferometer placed after a set of spectral and spatial filters (F). A detection event by the single-photon detector (D) placed at one of the interferometer outputs heralds the successful implementation of a delocalized single-photon addition
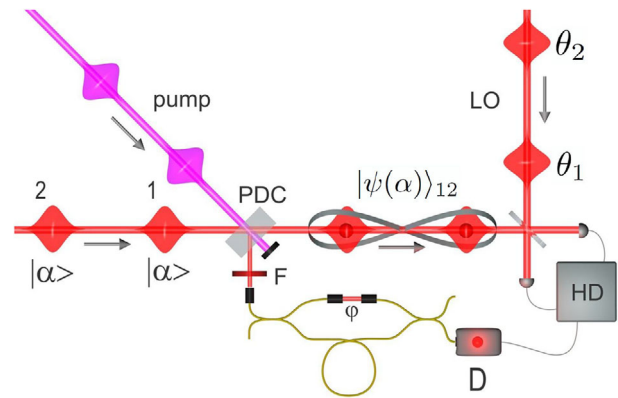

Figure 2. Schematic view of the experimental setup performing coherent single-photon addition on two different input temporal modes, both containing a coherent state $|\alpha\rangle$. A click in the single-photon detector D, placed after a Mach–Zehnder interferometer unbalanced by the time delay between the input temporal modes, heralds the delocalized addition of a single photon to the two output modes, which are then completely analyzed by means of time-domain homodyne detection (HD). See the text for further details.

between the two temporal modes. In principle, one can also freely adjust the phase between the two photon addition operations by varying the relative phase φ between the interferometer arms via the fine adjustment of an air-gap length. A feedback loop based on the interference of a counter-propagating pulse train injected in the unused interferometer output port provides phase stabilization. In practice, for this experiment investigating discorrelation properties, we are only interested in the odd superposition of Equation (1), but the experimental scheme is much more versatile than this and would easily allow one to explore the effects of different superposition phases and weights.

Ideally, in order to test the discorrelation properties of the final state, one should perform a joint photon number resolving measurement on the two consecutive wavepacket modes. In practice, due to the lack of proper photon number resolving detectors, we adopt a different strategy. The analysis of the output states is based on time-multiplexed homodyne detection, where pairs of local oscillator (LO) pulses with controlled phases $\theta_{1,2}$ in the two separate temporal modes are interfered with the signal modes on a 50% beam-splitter. Two proportional photodiodes detect the pulses at the outputs of the beam-splitter and the corresponding photocurrents are subtracted for each mode. Quadrature data points are thus obtained for each mode by integrating the corresponding difference photocurrent signal over a time window of about 10 ns.

4. Results and Discussion

Two-mode quadrature data are collected for different values of the two LO phases scanned in the $[0, \pi]$ interval by controlling their global phase via a piezo-mounted mirror, and their relative phase by means of an ultrafast electro-optic modulator. Applying an iterative maximum likelihood algorithm[17,18] to the collected data it is possible, at least in principle, to reconstruct the density matrix fully representing the optical two-mode state. This way, one can access not only the joint photon number distribution
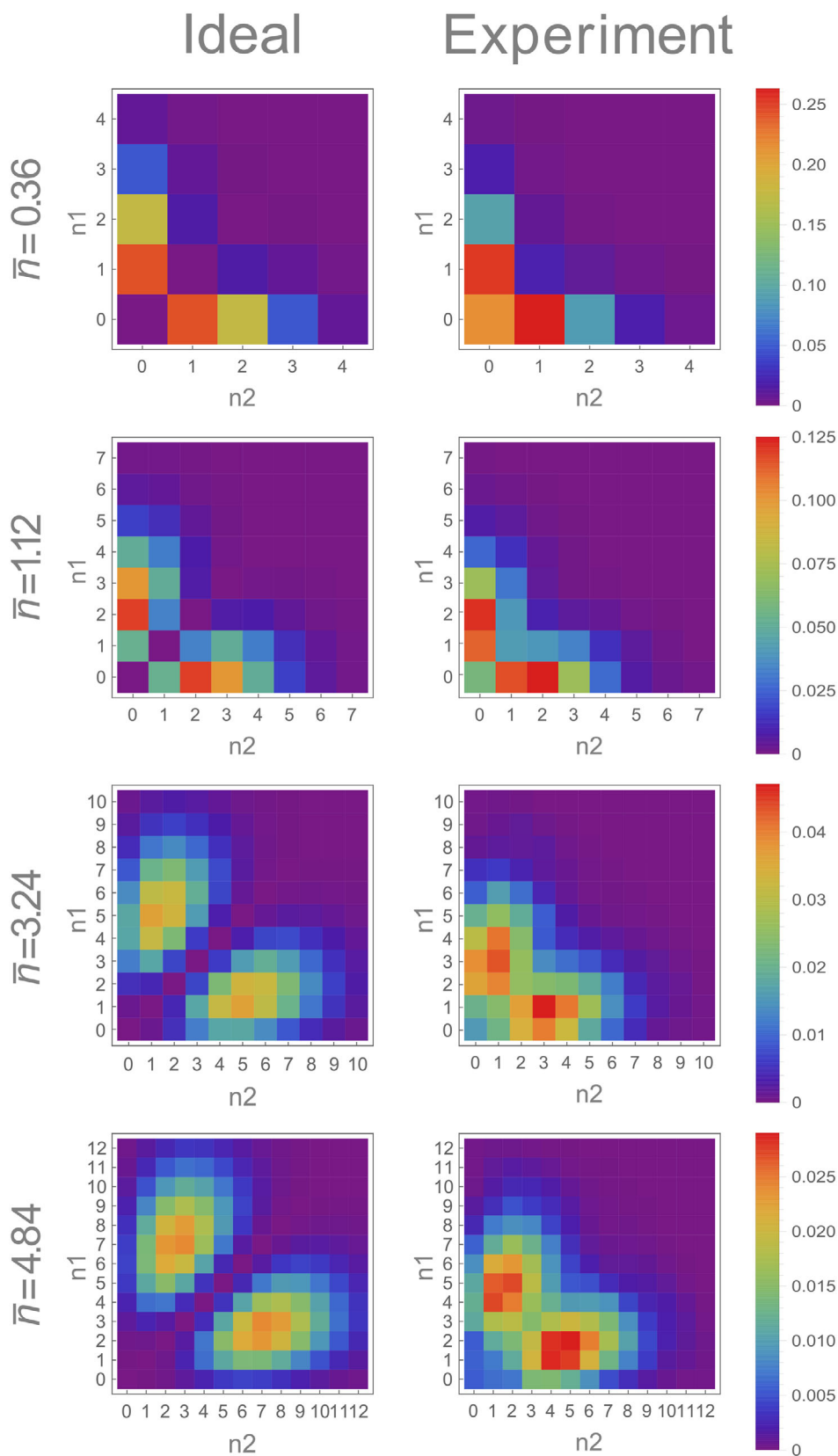
Figure 3. Ideal (left column) and experimental (right column) joint photon number distributions for different mean photon numbers $\bar{n} = |\alpha|^2$ in each of the input coherent states.

**ADVANCED
SCIENCE NEWS**
www.advancedsciencenews.com

**ADVANCED
QUANTUM
TECHNOLOGIES**
www.advquantumtech.com

but, for example, it is possible to measure the entanglement of the state by means of the negativity of the partial transpose, as done in ref. [14]. However, it is worth noting that, increasing the mean photon number of the state, the number of density matrix elements to reconstruct rapidly increases too. This quickly makes such an approach unfeasible (e.g., a two-mode separable coherent state, with $|\alpha|^2 = 1$ in each mode, may already require the reconstruction of up to 2400 density matrix elements for a faithful representation). In order to overcome this issue, we performed a tomographic reconstruction by averaging the LO global phase θ and acquiring about 50 000 quadrature values for nine equidistant relative phase values in the interval $[0, \pi]$. The phase-averaged density matrix elements in the Fock basis are given by

$$\frac{1}{2\pi} \int_0^{2\pi} d\theta \langle n_1 | \langle n_2 | \hat{U}_1^\dagger(\theta) \hat{U}_2^\dagger(\theta) \hat{\rho} \hat{U}_1(\theta) \hat{U}_2(\theta) | m_1 \rangle | m_2 \rangle \qquad (2)$$

where $\hat{U}(\theta) = e^{-i\theta\hat{n}}$ describes a phase shift of θ, and $\hat{\rho}$ is a generic density matrix describing the state before the phase averaging. Using $\hat{U}(\theta)|n\rangle = e^{-i\theta n}|n\rangle$, it is apparent that the uniform averaging of the LO global phase θ, over the interval $[0, 2\pi]$, has the effect of nullifying all the elements such that $(n_1 - m_1 + n_2 - m_2) \neq 0$. This reduces the number of matrix elements needed to be reconstructed by the maximum likelihood algorithm because the only non-zero ones are those diagonal with respect to the LO global phase, which include the ones needed to calculate $P^e_{n_1,n_2}$ (those for which $n_1 = m_1$ and $n_2 = m_2$); this procedure has also been validated by numerical simulations.

The joint photon number distributions $P^e_{n_1,n_2}$ obtained from experimentally reconstructed density matrices are shown in the right column of **Figure 3** for different amplitudes of the input coherent states. Note that no correction for detection efficiency was included in the reconstruction procedure. Although the experimental distributions do not exhibit the perfect discorrelation of the ideal states (also shown as a reference in the left column), due to experimental imperfections (limited detection efficiency and imperfect delocalized photon addition operation), they nevertheless present an evident decrease in the magnitude of the diagonal elements, a clear signature of discorrelation.

This is better illustrated in **Figure 4**, where the integrals of the experimental joint photon number distributions of Figure 3 along the diagonal direction are presented. The experimental marginal distributions $P(\Delta n)$ show pronounced local minima for $\Delta n = 0$, clearly witnessing the discorrelation of the states. It is also evident that there is a very good agreement between the experimental and the corresponding theoretical marginals for the states described by Equation (1), calculated while keeping into account several imperfections according to the following model:

$$\hat{\rho}_{exp} = \text{Tr}_r \left\{ \hat{B}_1^\dagger(\eta) \hat{B}_2^\dagger(\eta) \left[\eta_p \hat{\rho}_{id} + (1 - \eta_p) \hat{\rho}(\alpha) \right] \hat{\rho}_{vac} \hat{B}_1(\eta) \hat{B}_2(\eta) \right\} \qquad (3)$$

where $\hat{\rho}_{id}$ represents the ideal pure state of Equation (1). Due to unwanted single-photon detector clicks, the heralded state is described by a mixture of the ideal state with the input uncorrelated two-mode coherent state $\hat{\rho}(\alpha)$ with amplitude α. The mixture weight is the preparation efficiency η_p that depends on: 1) the spatial-temporal widths of filters (F in Figure 2) along the heralding arm of the experiment; 2) the dark counts of the single-photon
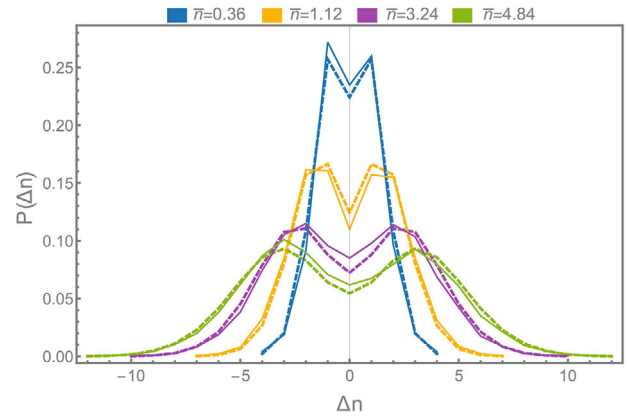

Figure 4. Experimental (solid curves) and theoretical (dashed curves) probability distributions $P(\Delta n)$ for a difference $\Delta n = n_1 - n_2$ in the photon counts between the two modes resulting from delocalized photon addition onto two identical coherent states of mean photon number $\bar{n}$.

detector D; and 3) imperfect matching of the addition mode to the detection one.[19] According to the standard quantum model of losses,[20] each mode of the generated mixed state is attenuated by a beam splitter with transmission η and described by the unitary transformation $\hat{B}(\eta)$. The two-mode vacuum term $\hat{\rho}_{vac}$ accounts for the extra quantum noise introduced by the loss process, and the unused reflected modes of each beam splitter are traced over by the partial trace Tr_r. In particular, the theoretical curves of Figure 4 consider an independently determined detection efficiency $\eta = 0.70$ (due to optical losses, quantum efficiency of the photodiodes and electronic noise in the homodyne detector) and a preparation efficiency of $\eta_p = 0.92$.[19]

Note that, for the extreme case of delocalized photon addition to two vacuum states (i.e., for $|\alpha|^2 = \bar{n} = 0$), resulting in the production of the single-photon mode-entangled state of [15], the ideal $P(\Delta n)$ curve would just consists of two equal peaks at $\Delta n = \pm 1$. If the effect of a finite global efficiency is taken into account, the central dip would only be visible for $\eta_{tot} \gtrsim 0.67$, whereas the curve would flatten and the dip turns into a peak for lower efficiencies. This is the reason why the curves of Figure 4 seem to show shallower dips for decreasing $\bar{n}$. With the present experimental inefficiencies, the central dip would disappear for input coherent states with $\bar{n} \lesssim 0.2$.

5. Conclusion

In conclusion, we have shown that our recently developed method based on the delocalized addition of a single photon, besides allowing one to entangle arbitrary states of arbitrarily large size,[14] can generate discorrelated states. Heralded single-photon addition on two different traveling temporal modes containing identical, uncorrelated, coherent states leads to a discorrelation in their joint photon number distribution, that is to the impossibility of measuring the same NoP in both modes. We experimentally verified the generation of discorrelated states by performing a reduced two-mode tomographic reconstruction that allowed us to manage the increasing computational effort in the reconstruction of states with larger mean photon numbers. Our experimental results clearly show the signature of discorrelation as a dip at

2000141 (5 of 6)

the center of the probability distributions $P(\Delta n)$ as a function of the difference in the photon counts between the two modes, even in the presence of limited detection efficiency, imperfect state preparation, and statistical noise.

The peculiar statistical properties of discorrelated quantum states, experimentally verified here for the first time, might make these states useful for new specific applications. For example, the possibility of distributing random numbers among two parties while being certain of their uniqueness, that is the impossibility for the two parties to receive the same number, could be used in distributed voting schemes[21] or for fair card dealing in "mental poker" games.[9,22]

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

[1] A. Gatti, E. Brambilla, M. Bache, L. A. Lugiato, *Phys. Rev. Lett.* **2004**, *93*, 093602.
[2] A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, G. Camy, *Phys. Rev. Lett.* **1987**, *59*, 2555.
[3] D. T. Smithey, M. Beck, M. Belsley, M. G. Raymer, *Phys. Rev. Lett.* **1992**, *69*, 2650.
[4] A. Meda, E. Losero, N. Samantaray, F. Scafirimuto, S. Pradyumna, A. Avella, I. Ruo-Berchera, M. Genovese, *J. Opt.* **2017**, *19*, 094002.
[5] A. K. Ekert, *Phys. Rev. Lett.* **1991**, *67*, 661.
[6] C. H. Bennett, G. Brassard, N. D. Mermin, *Phys. Rev. Lett.* **1992**, *68*, 557.
[7] C. K. Hong, Z. Y. Ou, L. Mandel, *Phys. Rev. Lett.* **1987**, *59*, 2044.
[8] E. Meyer-Scott, J. Tiedau, G. Harder, L. K. Shalm, T. J. Bartley, *Sci. Rep.* **2017**, *7*, 41622.
[9] A. Shamir, R. L. Rivest, L. M. Adleman, in *The Mathematical Gardner* (Ed: D. A. Klarner), Springer, Boston **1981**, pp. 37–43.
[10] R. L. Rivest, A. Shamir, L. Adleman, *Commun. ACM* **1978**, *21*, 120.
[11] A. Zavatta, S. Viciani, M. Bellini, *Science* **2004**, *306*, 660.
[12] A. Zavatta, J. Fiurasek, M. Bellini, *Nat. Photon.* **2011**, *5*, 52.
[13] L. S. Costanzo, A. S. Coelho, N. Biagi, J. Fiurášek, M. Bellini, A. Zavatta, *Phys. Rev. Lett.* **2017**, *119*, 013601.
[14] N. Biagi, L. S. Costanzo, M. Bellini, A. Zavatta, *Phys. Rev. Lett.* **2020**, *124*, 033604.
[15] A. Zavatta, M. D'Angelo, V. Parigi, M. Bellini, *Phys. Rev. Lett.* **2006**, *96*, 020502.
[16] M. Bellini, A. Zavatta, *Prog. Opt.* **2010**, *55*, 41.
[17] Z. Hradil, J. Řeháček, J. Fiurášek, M. Ježek, in *Quantum State Estimation* (Eds: M. Paris, J. Řeháček), Lecture Notes in Physics, Vol. *649*, Springer, Berlin **2004**.
[18] A. I. Lvovsky, *J. Opt. B: Quantum Semiclass. Opt.* **2004**, *6*, S556.
[19] A. Zavatta, S. Viciani, M. Bellini, *Phys. Rev. A* **2004**, *70*, 053821.
[20] U. Leonhardt, *Measuring the Quantum State of Light*, Cambridge University Press, Cambridge, UK **1997**.
[21] J.-M. Bohli, J. Müller-Quade, S. Röhrich, in *E-Voting and Identity* (Eds: A. Alkassar, M. Volkamer), Springer, Berlin, Heidelberg **2007**, pp. 111–124.
[22] P. Golle, in *Proceedings of the International Conference on Information Technology: Coding and Computing*, Vol. *1*, IEEE, Piscataway **2005**, pp. 506–511.