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ON THE IMPROVEMENT OF DIGITAL RADIO LINK PERFORMANCES USING SPREAD SPECTRUM TECHNIQUES

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1. Introduction

Well known are the performances of spread spectrum transmission techniques [1-4]; very interesting is, for example, their immunity with respect to interferring signals [5, 6].

This paper deals with a very common spread spectrum technique, that is with a binary direct sequence spread spectrum system (DS-SS) and for this case the improvement of the system performances in multi path communication channels are derived.

2. The assumed model for the transmission system

Let's refer to fig. 1, where the logic diagram of a binary DS-SS transmission system is shown.

The transmitted direct signal s(t) may be written as

$$s(t) = A\,d(t)\,PN(t)\cos\omega_0 t \tag{2-1}$$

where

A = Amplitude of the transmitted signal;

d(t) = random sequence of rectangular pulses, with + 1 or -1 amplitude and duration T (bit time);

PN(t) = pseudo-random sequence of rectangular pulses, with +1 or -1 amplitude and duration T_c (chip time)

d(t) and PN(t) may be written as

$$d(t) = \sum_{j=-\infty}^{+\infty} d_j\, P_T(t - jT) \tag{2-2}$$

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where $d_j \in (+1, -1)$, $P_T(t)=1$ for $0 \leq t \leq T$

$P_T(t) = 0$ elsewhere

and

$$PN(t) = \sum_{i=-\infty}^{+\infty} b_i P_{T_c}(t-iT_c) \qquad (2\text{-}3)$$

where $b_i \in (+1,-1)$, $P_{T_c}(t) = 1$ for $o \leq t \leq T_c$ and

$P_{T_c}(t) = 0$ elsewhere.

Let us also put

$N = T/T_c =$ Process gain

with N an integer number.

The whole received signal is

$r(t) = s(t) + s_M(t)+n(t)$

where

$s_M(t) = \alpha A d(t-\tau)\, PN(t-\tau) \cos(\omega_o t - \theta)$

with α, τ, and θ attenuation, time delay and phase delay respectively of the reflected signal.

$n(t)$ = channel noise process assumed to be a white Gaussian process with two-sided spectral density No/2.

In the case that the pseudo-randon sequence generated at the receiver is correctly synchronized with the sequence received through the direct path, the signal $g(T)$ at the output of the integrator can be expressed as ($\omega_o T \gg 1$)

$$g(T) = S(T) + S_M(T)+N(T) \qquad (2\text{-}4)$$

where

$$S(T) = \int_0^T 2Ad(t)\, PN^2(t) \cos^2 \omega_o t\, dt = ATd_o$$

$$S_M(T) = \int_0^T \alpha A \cos\theta\, d(t-\tau) PN(t) PN(t-\tau)\, dt \qquad (2\text{-}5)$$

$$N(T) = 2\int_0^T n(t)\, PN(t) \cos \omega_o t\, dt$$

It is to note that $S_M(T)$ depends on the reflected signal while $N(T)$ is a gaussian random variable with zero mean value and variance $\sigma^2 = N_o T$.

3. The error probability in the detected signal: an elementary approach

a) let us, firstly, assume that α, θ, τ are constant and τ such that

$$0 \leq \ell T_c < \tau < (\ell+1) T_c < T$$

and let us put

$$R_{PN}(\tau) = \int_0^{\tau} PN(t)\, PN(t-\tau)\, dt \tag{3-1}$$

$$\hat{R}_{PN}(\tau) = \int_{\tau}^{T} PN(t)\, PN(t-\tau)\, dt \tag{3-2}$$

Equation (2-4) may now be written as

$$g(T) = A d_0 T + \alpha A \cos\theta \left[d_{-1} R_{PN}(\tau) + d_0 \hat{R}_{PN}(\tau) \right] + N(T) \tag{3-3}$$

For the calculation we assume that d_{-1} takes the values of +1 and -1 with probability 1/2.

The error probability P_e in the detected signal is

$$P_e = P\left[g(T) > 0 \,\middle|_{d_0 = -1} \right] =$$

$$= \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_0}} \left(1 + \alpha \frac{R_T(\tau)}{T} \cos\theta \right) \right] + \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_0}} \left(1 + \alpha \frac{\hat{R}_T(\tau)}{T} \cos\theta \right) \right] \tag{3-4}$$

Where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

$E_b = A^2 T/2$ = Energy for each transmitted bit

$$R_T(\tau) = R_{PN}(\tau) + \hat{R}_{PN}(\tau)$$

$$\hat{R}_T(\tau) = \hat{R}_{PN}(\tau) - R_{PN}(\tau)$$

It may be shown that, without Spread Spectrum technique, P_e is expressed as

$$P_e = \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_o}}(1+\alpha\cos\theta)\right] + \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_o}}\left(1+\alpha\frac{T-2\tau}{T}\cos\theta\right)\right] \quad (3\text{-}5)$$

By comparing equations (3-4) and (3-5) the following considerations are allowed:

i) For $\alpha = o$, i.e. for a multipath free channel, equations (3-4) and (3-5) have a common value expressed as

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) \quad (3\text{-}6)$$

That is, in these conditions, no advantage is obtained through the use of a SS technique.

ii) For $\alpha \neq 0$, the attenuation is multiplied for $R_T(\tau)/T$ and for $\hat{R}_T(\tau)/T$, respectively, in the first and the second part of equation (3-4).

For this respect it is to note that in this case the error probability P_e is such that

$$P_e \leq \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_o}}\left(1-\alpha\frac{|R_T(\tau)|}{T}\right)\right] + \frac{1}{2} Q\left[\sqrt{\frac{2E_b}{N_o}}\left(1-\alpha\frac{|\hat{R}(\tau)|}{T}\right)\right] \quad (3\text{-}7)$$

Expression (3-7) depends on the autocorrelation properties of the used sequence; particularly, with reference to m-sequences [7, 8, 9] and for the non limitative assumption that

$$T_c \leq \ell T_c \leq \tau \leq (\ell+1)T_c \leq T - T_c \quad (3\text{-}8)$$

we have

$$\frac{|R_T(\tau)|}{T} = \frac{1}{N} \quad (3\text{-}9)$$

and

$$\frac{|\hat{R}_T(\tau)|}{T} = \frac{1}{N}\left|\left[2c(\ell)+1\right]+2\left[c(\ell+1)-c(\ell)\right]\right|\frac{\tau-\ell T_c}{T} \quad (3\text{-}10)$$

where

$$c(\ell) = \begin{cases} \sum_{j=0}^{N-\ell-1} b_j \cdot b_{j+\ell} & 0 \leq \ell \leq N-1 \\ \sum_{j=0}^{N-1+\ell} b_j \cdot b_{j-\ell} & 1-N \leq \ell < 0 \\ 0 & |\ell| \geq N \end{cases}$$

In the further assumption that the used sequence is an auto-optimal one (AO/LSE)[10, 11], for equation (3-10) one obtains

$$\frac{|\hat{R}_T(\tau)|}{T} \leq \frac{\hat{M}}{N} \tag{3-11}$$

Where $\hat{M}/N$ value depends on N as shown in table I [11]

$\hat{M}/N$	0.22	0.17	0.11	0.098
N	31	63	127	255

Eq. (3-9) and (3-11) clearly show that for sufficiently high values for N the error probability expressed by (3-4) can be made to approach the value expressed by (3-6), i.e., by using suitable S.S. techniques, the effects of multipaths on the error probability can be greatly reduced.

b) Let us now assume that α and τ are constant while θ is a random variable with uniform distribution in the range $(0,2\pi)$ and independent from d(t) and n(t).

The error probability now becomes

$$P_{\theta e} = \frac{1}{2\pi} \int_0^{2\pi} P_e \, d\theta \tag{3-12}$$

Where P_e is again expressed by equations (3-4) or (3-5).

In this case also, the reduction of the effective attenuation factor α due to the use of suitable S.S. techniques, as shown in the above point a), causes $P_{\theta e}$ to approach the expression (3-6), relative to a multipath free communication channel.

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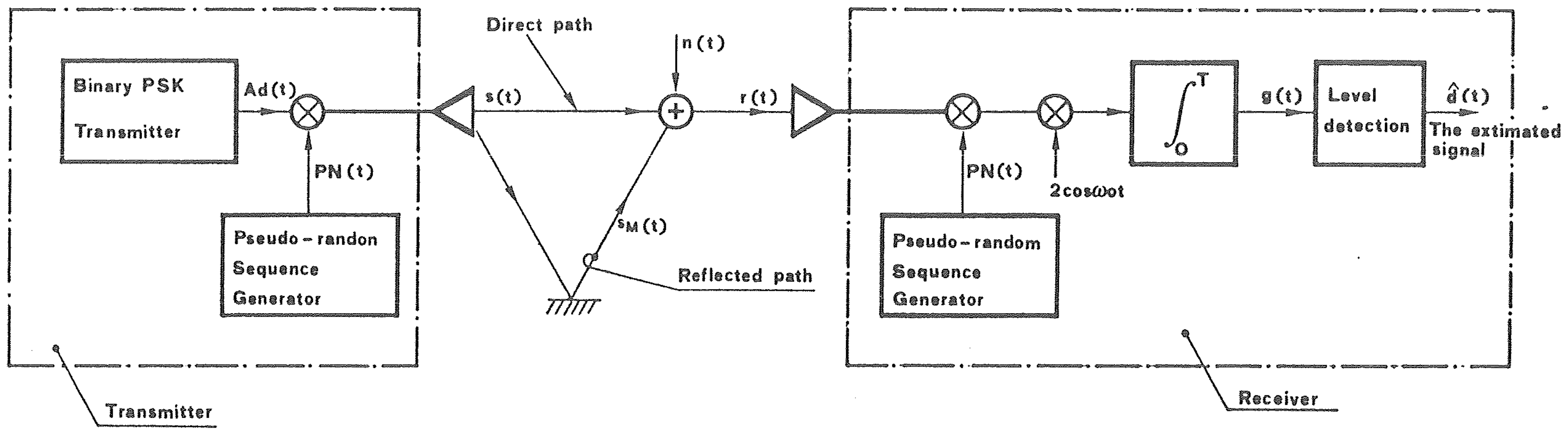

Fig. 1 - Logic diagram of the assumed model for the transmission system.