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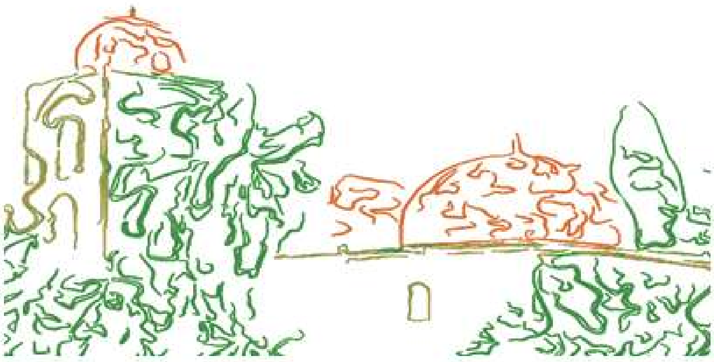

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A Bayesian study of temporal changes in seismicity

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Abstract. *The q-exponential distribution, solution of a maximum entropy problem in the frame of nonextensive statistical mechanics, is useful for describing complex, non-linear dynamic systems that emerge in many applications of environmental and social sciences, including seismology. In this study we analyze the seismic sequence of L'Aquila earthquake and investigate the ability of the q-exponential probability distribution to grasp the temporal variations of some seismic parameters, such as magnitude and spatial location of the epicentres. Bayesian inference is performed by processing data on sliding time windows, such that each window has a fixed number of events and shifts at each new event. Other distributions (tapered Pareto, generalized gamma) are also considered and the best fitting distribution in each time window is selected by comparing the evaluated values of the posterior marginal likelihood. We found that the best fitting distribution varies over time and can be a further indicator of the activation state of the systems.*

Keywords. *q-exponential distribution; Voronoi tessellations; Bayesian inference; Probabilistic forecasting; Statistical seismology*

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1 The q-exponential distribution

Recent studies have shown evidence that the Earth's crust behaves as a complex, non-linear dynamic system that is characterized by long-range correlations [1, 3]. As an expression of this complex system, seismic phenomena also show long-range correlations, critical instabilities, multifractal hierarchical structures, for which standard equilibrium Boltzmann-Gibbs statistical mechanics is not applicable [7]. In 1988 Tsallis introduced a nonextensive generalization of statistical mechanics based on the so-called Tsallis entropy [8]. This innovative formalism turned out to be surprisingly suitable for describing a large variety of complex systems, including seismicity. Denoting Boltzmann's constant by k_B, the Tsallis entropy is defined by:

$$S_q = k_B \frac{1 - \int f^q(x) dx}{q-1} , \tag{1}$$

under the assumption that $f(x)$ is a probability density function and the expectation with respect to the *escort* probability distribution $f_q(x) \propto f^q(x)$ is finite. According to the maximum entropy principle, the maximizing probability density function that satisfies the above conditions is the q-exponential distribution:

$$f(x) = \frac{1}{\beta} \left(1 - \frac{(1-q)}{(2-q)} \frac{x}{\beta} \right)^{1/(1-q)} \qquad \text{for} \quad x \geq 0 \quad \beta > 0 \quad \text{and} \quad 1 < q < 2 \tag{2}$$

where q is called the entropic index and β is the mean with respect to the *escort* probability distribution. The q-exponential density function (2) is a fat-tailed distribution, because it goes to zero for large x as the power function $x^{1/(q-1)}$. The q-exponential distribution is also heavy-tailed, being always bounded below by the exponential density function and having

$$\lim_{x\to+\infty} e^{tx}\bar{F}(x) = \lim_{x\to+\infty} e^{tx}\left(1+\frac{q-1}{2-q}\frac{x}{\beta}\right)^{-(2-q)/(q-1)} = +\infty \quad \forall t>0 \tag{3}$$

where $\bar{F}(x) = 1 - F(x)$. Noteworthy, the exponential distribution is recovered as q tends to 1, , i.e. the distribution that maximizes the Boltzmann-Gibbs entropy.
In the following, a sequence of earthquakes that occurred in central Italy is analysed by showing that the q-exponential distribution may indicate the state of criticality of the seismogenic system.

2 Data

The study area is centered on the epicentre of the L'Aquila earthquake occurred on 6 April 2009 (01:13:40 UTC, latitude 42.342°, longitude 13.380°) and moment magnitude M_w 6.1 [4]. It is a rectangular area of latitude $41.8° - 43.0°$ and longitude $12.8° - 13.8°$, and covers a temporal period from 7 April 2005 to the end of July 2009. Taking $m_0 = 2$ as the completeness magnitude, we obtain $N = 2725$ events drawn from the Italian Seismological Instrumental and Parametric Database (ISIDe Working Group 2007), of which 339 events occurred before L'Aquila earthquake (Fig.1, left panel) and 2386 after it (Fig. 1, right panel).

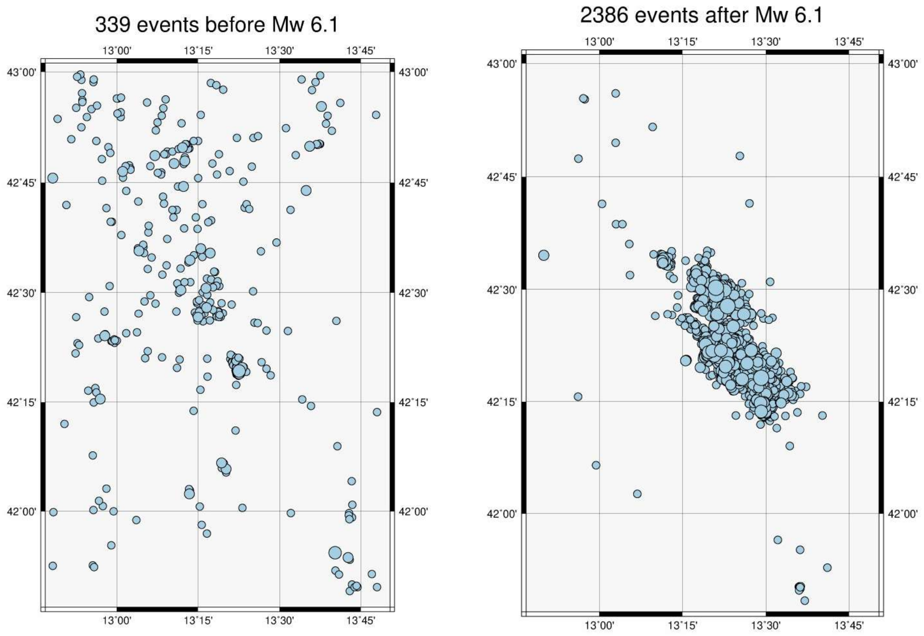
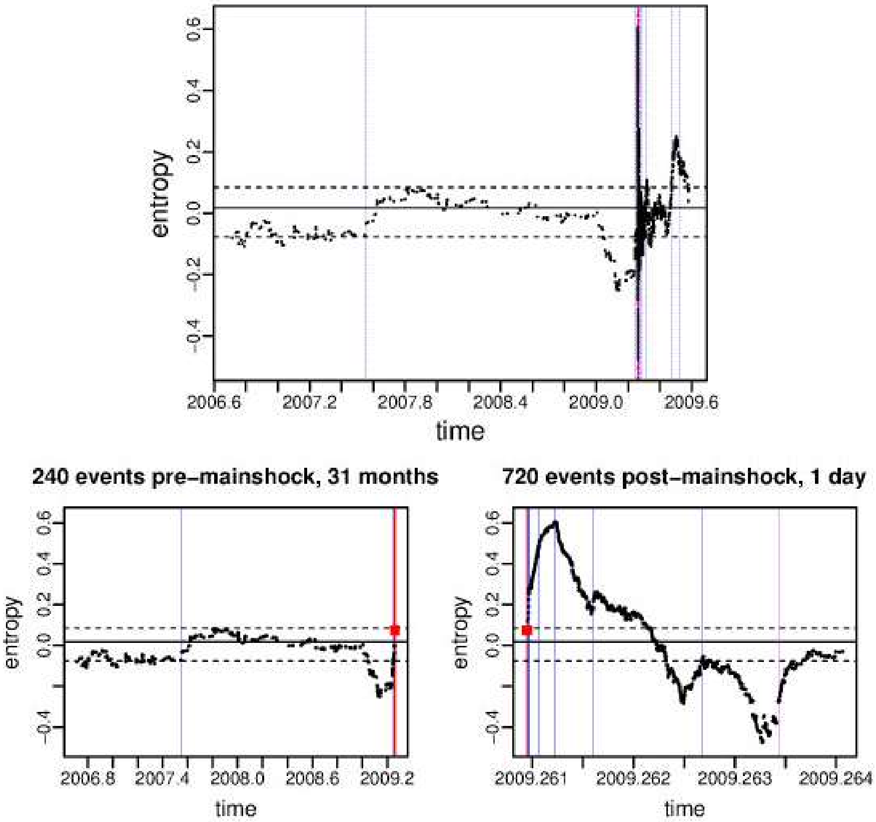


Figure 1: Earthquakes of $M_w \geq 2$ before (left) and after (right) the L'Aquila main shock on 6 April 2009, M_w 6.1. The circle size is proportional to the event magnitude.

3 Temporal variations of magnitude distribution

In this section the earthquake magnitude is assumed to follow a q-exponential distribution, which can be interpreted as a generalization of the Gutenberg-Richter law. Entropy and entropic q index are estimated by following the Bayesian paradigm. A Markov chain Monte Carlo method based on the Metropolis-Hastings algorithm is performed to approximate the posterior distribution of model parameters. To investigate the temporal evolution of the seismic phenomenon, we consider time windows of 100 events, a value obtained by varing the sample size in some pilot runs and chosen to balance reliability of the estimates and accuracy in examining the evolution of the physical process. The time windows are shifted at each new event through the seismic sequence, so as to have the best accuracy in revealing changes in entropy. The q-exponential model is fitted to data in each time window and the estimated entropy and entropic q index are associated, in the figures, with the time of the last event in that window.

For the L'Aquila sequence, the obtained time-entropy values are represented in the top panel of Fig. 2



240 events pre–mainshock, 31 months

720 events post–mainshock, 1 day

Figure 2: Entropy of the q-exponential distribution evaluated for each shifting window (top). Bottom panel: magnifications of the entropy values around the main shock, where the red square indicates the value of the entropy associated with the window ending on the main shock.

and their magnification around the main shock is shown in the two panels below: on the bottom left the values from the 7 April 2005 up to 6 April 2009, and on the bottom right the values associated with the 720 events that followed the main shock and covered just over one day. The vertical lines in Fig. 2 correspond to the occurrence times of the stronger events of $4 \leq M_w < 5$ (blue), $5 \leq Mw < 6$ (magenta) and $Mw \geq 6$ (red), and the horizontal lines are the mean (solid line) and the first and third quartiles (dashed lines) of the entropy values.

Entropy decreases slowly for a relatively long time before the main shock (starting from September 2008) and drops significantly below the first quartile (21 February 2009); this behaviour can be associated with a preparatory phase of strong earthquakes. Then entropy starts to increase quite rapidly and it reaches the maximum 2.5 hr after the main shock when all of the events of the window belong to the aftershock sequence; this sudden increase after the main shock corresponds to an increase in the diffusion of the energy that is also observed in correspondence to the subsequent strong shocks, and in particular to the pair of earthquakes of M_w4.4 (22 June 2009) and M_w4.2 (12 July 2009).

Similar trends are also observed for the estimated entropic q indices.

4 Temporal variations of the spatial distribution of epicentres

The sliding window method, that we applied in previous section, is now used to analyze the temporal variations in the spatial distribution of earthquakes. In each time window, we consider the cell areas of the Voronoi tessellation generated by the epicentral coordinates of the earthquakes that occurred in that time window. Then the q-exponential model is fitted to each set of cell areas and the estimated values of the posterior marginal log-likelihood are associated with the time of the last event in the corresponding window. In the literature other probability models have been suggested to describe the distribution of Voronoi cell area for earthquakes, such as the generalized Gamma distribution, the tapered Pareto distribution, and the exponential distribution [6]. Similarly to the q-exponential model, these distributions are fitted to data and estimated values of the posterior marginal log-likelihood are obtained for each time window. The best model in each time window is then selected by comparing the estimated posterior marginal likelihoods according to the Jeffreys scale [2].

Fig. 3 shows the value of the posterior marginal log-likelihood of the best model for each time win-

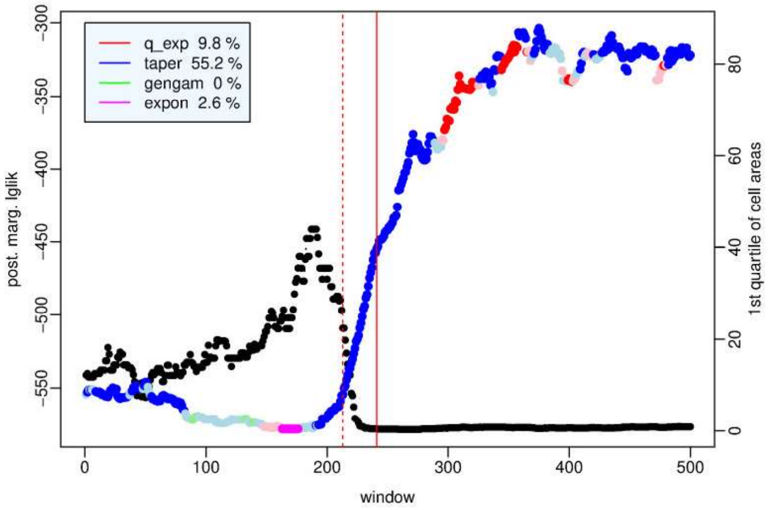


Figure 3: Posterior marginal log-likelihood of the probability distribution of Voronoi cell area that fits better than the other distributions to the dataset of each time window in the period from April 2005 to 6 April 2009 (hr 8). The red vertical lines indicate the mainshock (solid line) and the M_w 4 30 March earthquake (dashed line) respectively. The black dots indicate the first quartile of the set of cell areas in each time window.

dow: red dots correspond to the q-exponential probability distribution, blue dots to the tapered Pareto distribution, and green dots to the generalized gamma distribution. Dots with bold color indicate that the specific best model outperforms the others with strong evidence, while dots with less intense color denote less strong evidence. These results allow us to discriminate different regimes of seismicity: - the exponential distribution exceeds weakly the other probability distributions in a preparatory phase, which is characterized by a spatially diffuse seismicity in the region; - the tapered Pareto distribution becomes the best model when the seismic activity tends to concentrate around the mainshock area; - the outperformance of q-exponential distribution characterizes the period of maximum concentration of the seismicity, identified as clustering phase. Finally, in Fig. 3 the black dots represent the first quartile of the set of cell areas at each window. Noteworthy, this quartile reaches the maximum in mid-February 2009 and then it begins to decrease in correspondence to an increase of seismic activity before main shock with augmenting number of small cells due to the concentration of epicentres around the L'Aquila epicentre.

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