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Behind the Scene of "The Holy Family with St. Anne and the Young St. John" by Bernardino Luini: A Computer-Assisted Method to Unveil the Underdrawings

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Abstract

Uncovering the underdrawings (UDs), the preliminary sketch made by the painter on the grounded preparatory support, is a keystone for understanding the painting's history including the original project of the artist, the *pentimenti* (an underlying image in a painting providing evidence of revision by the artist) or the possible presence of co-workers' contributions. The application of infrared reflectography (IRR) has made the dream of discovering the UDs come true: since its introduction, there has been a growing interest in the technology, which therefore has evolved leading to advanced instruments. Most of the literature either report on the technological advances in IRR devices or present case studies, but a straightforward method to improve the visibility of the UDs has not been presented yet. Most of the data handling methods are devoted to a specific painting or they are not user-friendly enough to be applied by non-specialized users, hampering, thus, their widespread application in areas other than the scientific one, e.g., in the art history field. We developed a computer-assisted method, based on principal component analysis (PCA) and image processing, to enhance the visibility of UDs and to support the art-historians and curators' work. Based on ImageJ/Fiji, one of the most widespread image analysis software, the algorithm is very easy to use and, in principle, can be applied to any multi- or hyper-spectral image data set. In the present paper, after describing the method, we accurately present the extraction of the UD for the panel "The Holy Family with St. Anne and the Young St. John" and for other four paintings by Luini and his workshop paying particular attention to the painting known as "The Child with the Lamb".

Keywords

Infrared reflectography, underdrawings, image processing, Bernardino Luini

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Introduction

The application of advanced techniques to study cultural heritage artifacts has been increasingly demonstrating to be fundamental to a deeper knowledge of artistic and historic objects, contributing thus to plan the conservation interventions. Infrared reflectography (IRR) is a well-established technique for the in situ investigation of paintings.[1] First employed by J.R.J. van Asperen de Boer in the 1960s,[2,3] today it is performed routinely in many museums worldwide. IRR is a key technique for both the comprehension of the artistic creative process and the examination of the painting conditions. On the one hand, the high penetration of the IR radiation through the matter allows the researcher to unveil features underneath the painted surface, the *pentimenti* (an underlying image providing evidence of revision by the artist) and the underdrawing (UD, a preliminary sketch made by the artist on a preparation ground, prior to painting) can be visualized.[4–6] On the other, retouches or re-paintings (generally speaking, non-original

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materials added at a later stage for modifying the design of artwork's figures) and the restoration interventions (the process of re-establishing the artwork readability through selective removal of patina, consolidation of the original materials and possible reconstruction of missing pieces) change the surface and, consequently, its response if exposed to IR radiation.[7,8] IRR relies on the fact that the paintings' surfaces partly reflect and partly transmit the infrared radiation through the paint's layers when illuminated by an IR source. The light propagating inside the paint's layers is scattered by the pigments' particles and can reach the preparation layer where it is either backscattered to the observer or adsorbed if the UD is present. The visibility of the drawing's traits depends on the paint layers' thickness and their chemical composition, as well as on the chemical composition of the materials present in both the UD and the preparatory layer. An UD drawn with a highly absorbent material in the IR spectral range (a carbon-based pigment for example) over a chalk- or gypsum-based ground (they both efficiently reflect the infrared radiation) would be visible. On the contrary, an UD sketched employing a low IR absorbent material such as iron-gallic inks or sanguine would be difficult to be discerned even if superimposed to a highly reflective ground. In other words, the detectability of the UD depends on the contrast between the radiation scattered by the preparatory layer and the one absorbed by the drawing itself. Sketches made by black carbon appear dark on a light ground and can be easily observed because of the high absorption from carbon-based pigments.[9] The transparency of paints depends on the size of the grains and the concentration of the pigment, the medium, the layer's thickness, and the reflectance, and generally it increases with the wavelength of the IR radiation, reaching its maximum for most pigments around $1.7\,\mu m$.[2] Therefore, near-infrared (NIR, $0.7–1.1\,\mu m$) and particularly short-wave infrared ($1.1–2.5\,\mu m$) radiation is well suited to shed light on UDs made with carbon-based pigments such as charcoal, graphite, soot ink, and black chalk,[4,5,7] which were typically used in 15th–16th century paintings.[9]

The pioneering use of IRR by van Asperen de Boer marked the involvement of applied sciences into the study of the implementation stages of the artworks.[2,3] By making available the UDs to a wider audience, IRR has quickly become a keystone for many research groups, either with historic or art-conservation expertise.[10] Recently, the introduction of multispectral IRR gave a new boost to the application of scientific methods to the analysis of the design of the works of art. Based on the acquisitions of the reflected IR radiation in several narrow spectral bands,[11] multispectral IRR has brought some advantages over the conventional broadband IRR modality (i.e., based on a unique large NIR band). Indeed, the reflectograms centered around the wavelengths in which the contrast between the pigments used for tracing the UDs and the preparatory layer is maximized help to visualize the features of the UD (i.e., the lines of the drawings are more visible in certain images). Portable scanning multiband IRR devices characterized by narrow acquisition bands[12,13] are particularly suitable for the study of the artists' materials, allowing the in situ acquisition of information about some pigments and their mapping across the paintings. Exploiting the collection of different reflectograms to read the features of the UD, multispectral IRR introduces the need to create tracings that collect the information from what seen in different images. This is usually done manually, is time consuming, requires strong knowledge in the field of cultural heritage, and is user dependent; therefore, the requirement for more automation is increasing and justifies the development of new computational methods. Since the first applications of IRR to works of art,[2,3] many papers reported on the study of UDs, but most of them focused on the experimental setup and its effectiveness[14] or found their rationale in the importance of the investigated painting.[15] Moreover, when the attention was centered on the image processing method, the adopted protocol was often too complex for users with little experience in computer vision or image analysis.[16–18] Over the past decades, a large range of sophisticated imaging techniques, each with its own sphere of application, strengths, and weaknesses, have been fine-tuned and used as complementary methods in the scientific characterization of cultural heritage materials.[19–21]

In this scenario, we propose a computer-assisted protocol, i.e., a protocol based on statistical and mathematical methods involving user feedback rather than over-complicated automated methods. In specific, our proposal exploits the capabilities of principal component analysis (PCA)[22–24] to rationally order the information contained in the raw data. Our method processes the principal components (PCs) relying on well-known image analysis procedures such as Otsu's method,[25] asks the user to check the obtained images and, finally, exploits the results for recovering the UDs. Note that the choice of PCA for investigating multi- or hyper-spectral data is not new.[6,8] The novelty relies in the role played by the PCs; here, they are not the result of the analysis process, but rather the starting point for the classification of the reflectograms' pixels into two distinct groups (those that belong to the UD and the others). In this paper, after illustrating the main idea behind the architecture of the protocol, we report the results obtained on five paintings by Bernardino Luini and his workshop, held in the Pinacoteca Ambrosiana in Milan (Italy). In particular, we describe in detail the data on the paintings, "The Holy Family with St. Anne and the Young St. John" (shortened to "The Holy Family" in the following) and "The Child with the Lamb" (shortened to "The Child" in the following), two oil on wood panels by Luini (Figs. 1a and 2a), and we compare the extracted UDs with the ones hand crafted by art experts (Figs. 1b and 2b).

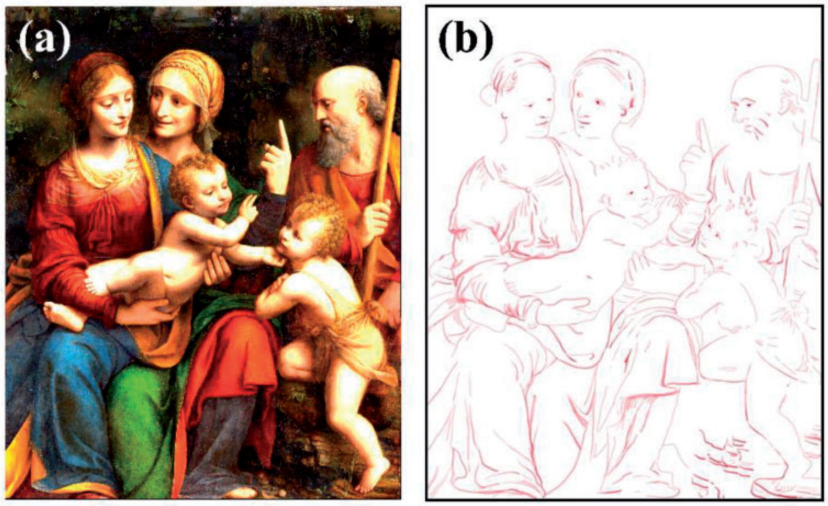

Figure 1. "The Holy Family with St. Anne and the Young St. John" by Bernardino Luini. (a) An RGB image of the painting, the contrast of the image has been deliberately enhanced for display purpose. (b) The sketch of the lines that have been identified as underdrawings by Valeria Villa on the base of careful observation of the IRR images, especially the reflectograms collected at 1230 and 1705 nm.

Figure 2. "The Child with the Lamb" by Bernardino Luini. (a) An RGB image of the painting, the contrast of the image has been deliberately enhanced for display purpose. (b) The sketch of the lines that have been identified as underdrawings by Valeria Villa on the base of careful observation of the IRR images, especially the reflectograms collected at 1230 and 1705 nm. (c) A false color image where the red component has been substituted by the reflectogram acquired at 1230 nm; the image shows the conditions of the painted surface.

In addition to the fact that "The Holy Family with St. Anne and the Young St. John" is the most important work by Luini held at the Pinacoteca Ambrosian, the choice of "The Holy Family" finds its rationale in the fact that it is an ideal candidate for testing the algorithm. The availability of a reliable handcrafted UD (Fig. 1b) together with the supposed presence of *pentimenti* and differences with respect to the final painting make the accuracy of the extracted UDs easy to be evaluated. Moreover, Luini worked in that circle of artists called *Leonardeschi* (i.e., those painters influenced by the painting style introduced by Leonardo da Vinci,[26] the *Leonardismo*) and, since "The Holy Family" is unequivocally inspired by the cartoon, "The Virgin and the Child with St. Anne and St. John the Baptist" (No. 6337) by Leonardo da Vinci[27] held at the National Gallery in London (U.K.) and also known as the Burlington House cartoon, the possibility to observe the UDs designed by Luini could shed light on the relationship between "The Holy Family" and Leonardo's cartoon. The interest around "The Child" is due to the fact that this small oil on wood work underwent at least one extended restoration event (Fig. 2c); even if this is not surprising for an ancient painting, it is a complex challenge for the capabilities of the algorithm especially if, as in this case, the retouching strongly modified the original layers of paint. The obtained results allowed for the underlining and discussion of the critical issues of the algorithm and the definition of its limits. The mentioned characteristics underpin the main interests behind the search for UDs, making these masterpieces by Luini the appropriate benchmark for our proposal. The application of the proposed protocol to the paintings, "The Holy Family with St. Anne and the Young St. John" and "The Child with the Lamb", is part of a larger project funded by Fondazione Cariplo, other national institutions, and Iperion CH.it (Molab access).

Materials and Methods

Paintings

The following experimental data handling and analysis protocol was designed, checked, and applied to five paintings by Bernardino Luini and his workshop held at the Pinacoteca Ambrosiana in Milan. The results obtained on the paintings known as "The Holy Family with St. Anne and the Young St. John" (1520–1530, oil on wood panel, 118×92 cm, Fig. 1a) and "The Child with the Lamb" (1525, oil on wood panel, 28×25 cm, Fig. 2a) are discussed here. Those regarding "The Blessing Christ" (1520–1530, oil on wood panel, 43×37 cm, Figure S1a), "The Holy Mary that breastfeeds the Holy Baby" (1520, oil on wood panel, 51×41.4 cm, Figure S1c), and "The Noli Me Tangere" (1510–1520, oil on canvas, 95.5×91.5 cm, Figure S1e) have been summarized in the Supplemental Material. "The Holy Family" and "The Child" have been described in detail for different reasons: (i) they are respectively the main work by Luini held at the Pinacoteca Ambrosiana and a typical example where the presence of retouches modifies the original project of the author (Fig. 2c). (ii) For both the paintings it has been possible to compare the extracted UDs with one handcrafted by qualified art experts (Figs. 1b and 2b). (iii) The interest around the relationship between "The Holy Family" and the cartoon by Leonardo da Vinci which inspired Luini (the Burlington House Cartoon, 1499–1500, charcoal (and wash?) heightened with white chalk on paper, 141.5×104.6 cm).[27]

Data Acquisition–IR Reflectography (IRR)

The IRR data set has been collected by means of the multi-spectral visible–near-infrared (Vis–NIR) scanner recently developed at the National Institute of Optics (CNR-INO) in Florence[28] as an improvement of the experimental setup described in literature.[12] The device collects a set of 32 images in the range 395–2550 nm. The spectral resolution is 10–20 nm in the visible and 50–100 nm in the NIR spectral range, respectively. The maximum scanned area was 1 m^2 (carried out in 3 h) with 250 μm of spatial sampling; mosaicking was needed for bigger areas.[28] In the cases of "The Holy Family with St. Anne and the Young St. John" (0.92×1.16 m^2) and of "The Child with the Lamb" (0.28×0.25 m^2), a single set of images of 3690×4645 and 2074×2298 pixels, respectively, was acquired. The 16 images in the VIS range were combined (using the standard D65 illuminant and 1931 observer) in order to obtain the red-green-blue (RGB) images (Figs. 1a and 2a), whereas the 16 Vis images, in the range between 750 and 2550 nm, were used to feed the protocol presented herein. All 32 images were aberration-free and self-registered, allowing for a straightforward use without any post-processing.

Data Selection

After scrolling the 16 reflectograms acquired in the wavelength range 750–2550 nm, 10 bands centered at 1230, 1292, 1400, 1500, 1600, 1705, 1830, 1940, 2100, and 2200 nm (Fig. 3) have been selected for establishing the capabilities of our computer-assisted protocol. The range below 1100 nm has been left aside because most pigments are absorbent in this spectral range.[29] Moreover, images at 2345 nm and 2550 nm have been neglected due to high level of random noise and huge bandwidth, respectively. The 10 selected reflectograms correspond to a cube of 10 matrices that describe the light intensity reflected to the detectors at the wavelengths specified above; the cube constitutes the starting data set for extracting the UDs. The images have been processed by means of custom-made scripts developed in the macro language of ImageJ/Fiji, the freeware software for image analysis developed and supported by the National Institute of Health.[30]

Data Handling

The words data handling indicate the preliminary operations performed on the reflectograms before they were processed by the algorithm. All the 10 matrices constituting the data cube were first cropped and successively divided into sub-hyperspaces (SHSs, see the related sub-section). The crop operation has been performed to exclude those fragments of the frameworks that surrounding the painting have been accidentally imaged during the acquisition of the data sets and could influence the statistical features of the data. The consequence is that the width and the height of the analyzed matrices result slightly lower than those of the acquired reflectograms: 3636×4584 instead of 3690×4645 pixels for "The Holy Family" and 1884×2223 instead of 2074×2298 pixels for "The Child".

Principal Component Analysis

Principal component analysis is a well-known mathematical method employed to reduce the number of variables describing complex data sets. It is used in a wide range of applications[23] and it has been extensively reviewed elsewhere (see for example the paper by Wold et al.[22] or Bro and Smilde[24]). Briefly, PCA depicts the variables as vectors in a multidimensional space (the number of the dimensions equals that of the variables, i.e., the number of the reflectograms in the present case); it searches for the reference system which guarantees the largest variation for the vectors' components and orders the axes using the variance parameter as ranking criterion. The projection of

Figure 3. The 10 cropped reflectograms of "The Holy Family" used to search the UD; in the upper right corner of each IRR image has been reported the central wavelength of its acquisition band. The level of reflectance is expressed in arbitrary units as described by the gray color bar on the right.

the data along the axis characterized by the maximum variance constitutes the first principal component (PC1) and so on for the higher order axes and components. The result is a series of linear combinations of the original variables, which are independent and, since the first places of the ranking retain most of the information of the data set, the whole starting data set can be exhaustively represented by a relatively low number of PCs; the remaining PCs can be considered as redundant.[22,24]

Segmentation and Binarization: Otsu's Method

Segmentation is a process aimed at partitioning a digital image into sets of pixels that together cover the whole surface of the starting image. There exist a lot of segmentation techniques, which split the pixels on the base of certain features such as pixel intensity, color, texture, etc. All these techniques rely on a specific method (the review by Haralick and Shapiro[31] offers an exhaustive overview on the most applied techniques), but they can be classified into two basic types: global or local segmentation methods, depending on whether they concern the whole image (i.e., with a large number of pixels simultaneously) or with specific part(s) of it.[32] In this paper, the segmentation was performed applying standard image processing operators and an automatic thresholding criterion named "Otsu's Method" after its inventor[25] to the PCs. To reduce the noise in the flatten areas, the PCs have been first blurred by applying a spatial Gaussian operator[33] and then handled to emphasize the high gradient areas. This latter task was accomplished through the sequential application of a variance operator that stressed the edge of the objects in the image,[34] followed by the unsharp default ImageJ/Fiji operator that enhanced the contrast.[35] The method by Otsu assumes the histogram of the input image as a bi-modal distribution of gray tones and selects the best value for clustering the pixels into two groups by minimizing the intravariance of the resulting distributions, i.e., the method identifies the threshold that better divides the pixels into two groups. The identified threshold was used to binarize the PCs into foreground (FG) and background (BG). The binary images (also called masks) were refined by deleting those pixel's clusters that resulted in smaller than 250 pixels (they were considered noise), and those pixels' clusters that were characterized by an aspect ratio greater than 0.7 (the aspect ratio is defined as the ratio between the minor and the major axis of the cluster) because we supposed that clusters resembling a circle would hardly belong to the traits of the UDs. Except for the eyes of the characters, this hypothesis is reliable referring to the handcrafted UDs (Figs. 1b and 2b).

Sub-Hyperspaces

The dimension (and shape) of the SHSs could be, in principle, arbitrarily settled by the user, but practically it must be tailored considering that both the protocols (PCA and Otsu's method) and the operators of the image processing depend on the statistical properties of the matrices under investigation and, therefore, on the size (and shape) of the SHSs. Take the contrast enhancing operator (i.e., the ImageJ/Fiji default unsharp operator) for instance: The bigger the area it processes, the more important the influence due to local peculiar groups of pixels (such those related to artifacts or to irregularities in the shape of the painted surface) on the whole matrix could be. In shed of these considerations, it is straightforward that changing the size (and shape) of the SHSs corresponds to influence the algorithm's final output (i.e., the extracted UD). Since works of art as paintings are unique pieces, it is not possible to determine a general rule to adjust the dimension (and shape) of the SHSs; however, it is useful to establish some general guidelines:

(i) Limit the involvement asked to the user, the number of SHSs must be kept as small as possible. In fact, the lower the number of SHSs, the lower will be the number of PCs to be checked.
(ii) In cases in which there exists a reliable handcrafted UD (at least a partial one), it is appropriate to compare the output of the algorithm with the work done by art experts.
(iii) The presence of large restoration areas and of hidden works (i.e., traces of projects sketched and then rejected by the artist but that remain underneath the painting surface) must be considered and, if their presence hardly affects the final result, their exclusion from the analysis must be considered.
(iv) Further information as false color images or the reflectograms themselves can support the evaluation of the extracted UD and must be considered.
(v) The number of compromised PCs must be kept as low as possible.

Practically speaking, these guidelines give rise to an iterative approach. During the first attempt to recover the UD, the reflectograms are processed as a single SHS and the output is checked (when possible against a reliable handcrafted UD); then, if there is no evidence of areas of restoration or of hidden works under the surface, the data set can start to be divided into SHSs. The result is then again checked and the data set is eventually divided into smaller SHSs and so on. If large restorations or hidden works affect some parts of the painting, before going forward with each division it is possible to re-apply the algorithm excluding the affected portions of the surface from the analysis. In the case the handcrafted UD is not available, the critical observation of the IRR data set and of further images such as the false color ones becomes the leading references for the design of the SHSs. This is because the reflectograms, if correctly understood, allow identification of the traits of the UD while false color images unveil the main retouches and artifacts. Finally, if there is no possibility of correctly understanding the IRR data set and the other available images, the minimization of the compromised PCs becomes the main criterion to design the size (and shape) of the SHSs. This is justified by the fact that the lower the number of discarded components is, the lower the losses of information will be.

Following the described approach, the number of SHSs was settled to nine SHSs of the dimensions of $1230 \times 1548 \times 10$ pixels for "The Holy Family", four SHSs of $942 \times 1111 \times 10$ pixels for "The Child with the Lamb", four SHSs of $721 \times 814 \times 10$ pixels for "The Blessing Christ", four SHSs of $810 \times 1002 \times 10$ pixels for "The Holy Mary that Breastfeeds the Holy Baby", and four SHSs of $1750 \times 1824 \times 10$ pixels for "The Noli Me Tangere". The SHSs cover the whole surface of all the paintings with a regular grid.

Manual Rendering System for Tracing the Underdrawing

The manual rendering of the UDs was executed using Adobe Photoshop CS5.1 on a Wacom graphic tablet equipped with the Intuos3 Grip pen (Intuos3), a solution commonly used for image editing or for reproducing pencil or brush strokes in a realistic way.[36] Put together, the tablet and the pen constitute a graphic system that returns the position of the tip on the tablet, the pressure at the tip, and the tilt angles (azimuth and altitude) of the pen with respect to the surface of the tablet. The pressure at the tip is linearly adjusted into a range decided by the user; low- and high-pressure corresponds respectively to thin and wide ink lines and therefore changing the pressure while tracing creates a trait with variable width. The tilt angles render the behavior of a brush; high tilts correspond to a large area covered by the stroke; flat brushes, for example, draw thin lines when the stroke is perpendicular to the azimuth of the pen; a change of the tilt angles while tracing corresponds to a twist of a real brush during the stroke.[36] The possibility of adjusting these parameters by managing the grip pen's position, orientation, and pressure with respect to the tablet's surface allows one to draw lines as faithfully as possible to the features of the traits recognized looking at the images of interest. Moreover, the use of the Wacom together with Photoshop allows one to superimpose different transparency layers, simultaneously or one a time, to the selected image, which is a useful tool when the traits are characterized by unique features. The tracing has been executed switching the monitor display zoom between the 100%, to allow maximum precision in drawing lines and lower percentage values to check the overall view against the whole reflectograms.

Results and Discussion

Handcrafted Underdrawings

The handcrafted UDs of "The Holy Family with St. Anne and the Young St. John" (Fig. 1b) and of "The Child with the Lamb" (Fig. 2b) have been obtained by a work in tracing overlay, executed exploiting the manual rendering system on the IRR data sets. By observing the reflectograms at high magnification, the ones collected at 1230 nm and 1705 nm were selected as the principal references for tracing the handcrafted UD because the art experts recognized these two images as the most suitable to recover the peculiar traits of the UD. Both the UDs are characterized by two main groups of traits. The elements of the first group seem to have been executed with a dry medium, and they are all remarkably similar to the traces of a pencil even if they show differences for what concern the hardness of the lines. The elements of the second group probably have been drawn by a brush with a fluid medium, they are similar to strokes, and present a complex shape; sometimes they are transparent at the center and dark at the edges, sometimes they are homogeneously dark or transparent. In shed of these considerations, two different transparency levels have been overlapped to the IR images, one to show the dry medium's drawn lines, the other for those drawn by the brush.

The Algorithm

The protocol extracting the UD consists of two main steps: the first splits the cropped matrices of the 10 selected reflectograms into SHSs and performs the PCA exploiting the ImageJ plug-in by Cutrona and co-workers:[37] The second step processes the PCs, asks the user to check the processed images, and finally recovers the extracted UD for each SHS. The whole UD is obtained by mosaicking the results of the single SHSs.

The Role of Principal Component Analysis

The multidimensional space defining the input variables for the PCA is constituted time by time by the matrices of each SHS (Fig. 4a-l shows the matrices of the SHS number 2 of "The Holy Family"). Even if IR reflectography can emphasize the traces of the UDs,[25] it cannot exclude the effect of other characteristics of the painting. If present, features such as restorations or the *craquelure* can be spread on the whole panel and can affect all matrices. In this scenario, PCA appears to be an ideal tool for concentrating the information into the lowest number of images without losing details.[38] However, PCA cannot be asked for distinguishing different features into separate variables[39] or for isolating the useful PCs because PCA is not a classifier and there is no general rule to decide how many and which components contain the traits of the UD. These drawbacks seem to advise against the use of PCA to extract the UDs; however, the fact that PCA organizes the information following a well-defined criterion, the variance, offers a rationale base that results useful for searching the traits of the UDs. Since all the PCs (Fig. 4al' shows the PCs for the SHS number 2 of "The Holy Family") are linear combinations of the starting matrices, even those unnecessary for the data representation are not deprived of information, they rather contain information which can be defined as redundant. Thanks to this redundancy, the components can be seen as a multiple representation of the main characteristics of the data. The consequence is that those components dominated by obvious artifacts (and therefore useless for any kind of analysis) can be discarded without significant losses of information. The selection of the components is usually accomplished basing on statistical constraints[22] or following a direct qualitative or quantitative examination of the PCs.[38] Dealing with paintings, the heterogeneity of the materials, the low number of starting variables and, mostly, the unpredictability and non-uniform distribution of the peculiar details limit the possibility of distinguishing between the components applying statistical or mathematical constraints. Therefore,

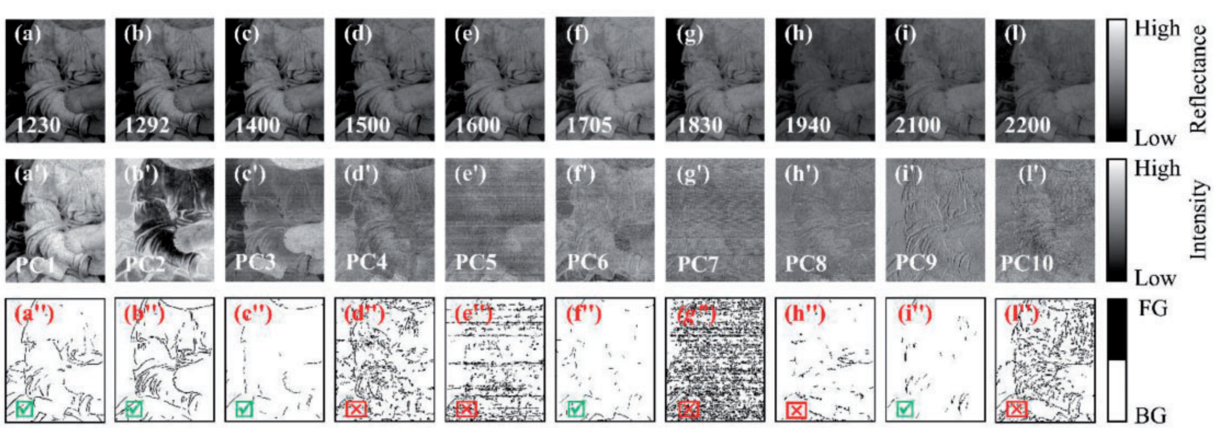

Figure 4. The application of the algorithm to one of the sub-hyperspaces of "The Holy Family". (al) The reflectograms of the considered sub-hyperspace as detected by the instrument. (al') The PCs. (a'l") The binary masks, i.e., the images a'l' after that they have been processed to highlight the traits of the underdrawing. The gray color bars refer to al and al', they describe respectively the reflectance of the portions of the reflectograms included in the selected sub-hyperspace (al) and the gray levels' distribution of the PCs (al'). The black and white bars refer to panels a'l" and distinguish the FG from the BG within the binary masks. The red x-marks highlight the discarded components, while the green checkmark the ones used to recover the UD.

the number of the PCs is kept equal to that of the input variables and the user is asked for manually checking the components of each SHS. The check is devoted to prevent the extracted UDs to be affected by the presence of obviuos artefatcs and not to the identification of the traits of the UDs. Therefore, the user's involvement is postponed after the PCs have been processed because image processing could emphasize their criticality in terms of artefacts.

Besides the motivations strictly related to the study of paintings and the fact that PCs are not necessarily relevant for clustering[39] (i.e., a single component cannot fulfill the extraction of the UD), one more issue enhances the complexity of pointing out the significant components: the artifacts generate high variation making one or more components useless.[40] Small defects either due to the detection, such as shot noise, or intrinsic, such as small paint detachments, can be so dramatically amplified by PCA that they can compromise the PCs. Even if a visual inspection of the reflectograms of Fig. 3 reveals no remarkable defects, some PCs are affected by artifacts. For example, the fifth component of SHS number 3 (Fig. 5b) and the third component of SHS number 5 (Fig. 5c) show stripes patterns that could most likely be ascribed to the data collection's process rather than to real details of the painting realization.

Underdrawing Extraction

The segmentation is performed by applying the Otsu's method to each PC generated by each SHS. This choice finds its rationale in the fact that the observation of the reflectograms of the paintings and of the two handcrafted UDs suggests that the artist's UDs should be constituted by well-defined traits on a flat preparation layer. Since the image processing operators should highlight such hypothetical UDs, the method should identify the threshold that better divides the pixels into two groups (ideally, the pixel which constitute the traits of the UDs and the others) and that, therefore, should be used for generating the stack of binary masks for each SHS. The putative traits of the UD are expected to appear as black FG on a white BG (Fig. 1al", Figure S2). Even if the processing of the PCs and the method by Otsu have been selected for emphasizing the pixels expected to belong to the UD,[18] it is evident that spurious strokes heavily affect some masks, avoiding their engagement for the extraction of the UD. The user is required to select the clearly compromised masks for being removed from the respective stack (red x-marks in Fig. 4a1", Figure S2). Figures 4d", 4e", 4g", 4h", and 4l" are examples of useless components due to artifacts (Figs. 4e" and 4g") or to the unsolvable mix between the traits of the dress of the Holy Mary and probably the *craquelure* (Figs. 4d", 4h", and 4l"). The remaining components (green checkmarks in Fig. 4al" and in Figure S2) do not show obvious artifacts and, therefore, can be used for recovering the UD. Note that together with the compromised masks, the eighth masks of the SHSs number 8 and number 9 have been discarded because they do not contain information at all (masks identified by * in Figure S2). After that the masks have been checked, the putative traits of the UD should be confined in at least one of the surviving slices (green checkmark in Fig. 4al" and in Figure S2) and, therefore, the whole UDs are supposed to emerge mosaicking the sums of these slices into a single image that constitutes the extracted UD (Figs, 6a and 7a; Figures S1b, S1d, and S1f). The pixels of the UDs assume a value, n, in the range [0, N], where n depends on the times they have been identified as parts of the UDs and N equals the number of the starting variables (10 in these case studies). Since the FG of the masks is obtained exploiting the information redundancy of the PCs, the projection of the remaining

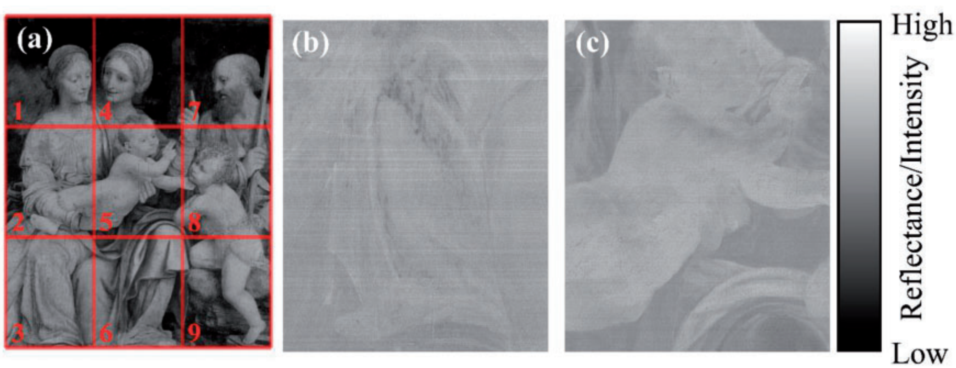

Figure 5. The effect of the defects on the input images on the PCs. (a) The organization of the sub-hyperspaces used to analyze the data cube of "The Holy Family". (b–c) The PC5 and the PC3 for the sub-hyperspaces numbered as (3) and (5) in (a); the horizontal stripes along the images demonstrate how the presence of artifacts, even if invisible at the naked eye, can affect the PCs. All the panels refer to an arbitrary gray level look up table; for (a) the gray color bar represents the level of reflectance of the reflectogram at 1705 nm, while for (b) and (c) it describes the gray levels' distribution of the PCs.

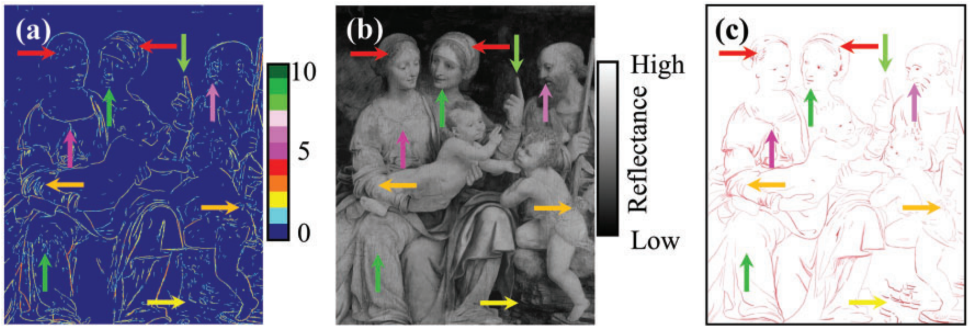

Figure 6. The extracted underdrawings of "The Holy Family with St. Anne and the Young St. John". (a) The extracted UD as it results from the algorithm; the multicolor bar indicates how many times a pixel has been detected as belonging to the traits of the UD. (b) The reflectogram collected at 1705 nm; the gray color bar indicates the reflectance distribution within the reflectogram. (c) The handcrafted UD. The colored arrows point out the main details of the UD discussed in the text.

binarized components could be confused with a sort of probability distribution for the pixels: i.e., the more are the components in which a pixel belongs to the FG, the more is the probability that it belongs to the UD. However, this speculation does not find clear confirmation. Some details given as parts of the UD by art experts have been detected as FG a number of times comparable with that of some of the pixels in the over-segmented areas, e.g., compare the double profile of the nose of St. Joseph and the region around the knee of the Virgin Mary for example (Fig. 6). Therefore, the times a pixel has been identified as a part of the FG cannot be adopted as an effective "validation tool" for the algorithm.

"The Holy Family" Extracted Underdrawings

Figure 6 shows the UD extracted by the algorithm (Fig. 6a) and the handcrafted UD (Fig. 6c) of "The Holy Family" besides the reflectogram collected at 1705 nm (Fig. 6b) in order to compare the traits obtained by our method with those expected by art experts and to collocate the discussed features in the starting data set. Both the UDs point out the presence of an extensive UD and unveil some differences between the designed and the realized painting. The extracted UD confirms the art experts' expectation for a detailed plot of the hair of the Virgin Mary and of the head of St. Anne; portions of the braid of the Holy Virgin and of the shape of St. Anne's veil are evident (red arrows in Fig. 6) in both the UDs. However, the UD extracted by the method is more fragmented and shows some traits that do not appear in the handcrafted one (in particular on the forehead of St. Anne); we can speculate that these differences are probably due to the fact that the method can neither complete the uncomplete traits nor exclude the influence of the painted layers' thickness and of the surface's defects (i.e., the *craquelure*), while art experts can (see the further consideration on the algorithm sub-section for a detailed discussion). The traces ascribable to the clothing of the characters resemble the folds and the wrinkles the dress would assume in a live scene; the region near the right elbow of the Virgin Mary and, with lower evidence for what concern the extracted UD, the knot of the tunic of the Young St. John (orange arrows in Fig. 6) are examples of the Master's attempt to create a realistic portrayal of the reality. Moreover, the extracted UD confirms the deduction of the art experts that the UDs in the space surrounding the characters appears to have been just sketched by Luini. The algorithm, as well as the handcrafted UD, identifies the features of the setting with short disjointed traces as for the profiles of the stones in the bottom-right corner of the painting (yellow arrow in Fig. 6, under the knee of the Young St. John). Both the handmade and the extracted UD also highlight some discrepancies between the painted and the formerly drawn scene; the main detected differences concern the veil of the Holy Mary and the face of St. Joseph (dark and light pink arrows in Fig. 6, respectively). The veil appears much less complex with respect to the shape it assumes in the painting, whereas St. Joseph shows a double profile (in correspondence with the nose) suggesting a *pentimento* of the author. Even if also both these features found confirmation in the UD traced by art experts, the agreement between the extracted and the handcrafted UD is not complete. On the one hand, the algorithm over-segments some regions of the painting such as the areas around the right knee of the Virgin Mary or the mouth of St. Anne (dark green arrows in Fig. 6). On the other, the method fails to detect some details identified by art experts; the most important in the case of "The Holy Family" is the double outline of the St. Anne's index finger which is only partly recovered in the extracted UD (light green arrow in Fig. 6).

"The Child with the Lamb" Extracted Underdrawings

"The Child with the Lamb" belongs to those cases stigmatized by the guidelines enumerated in the sub-section dedicated to the design of the SHSs. The painting underwent at least one significant restoration event, and more than half of

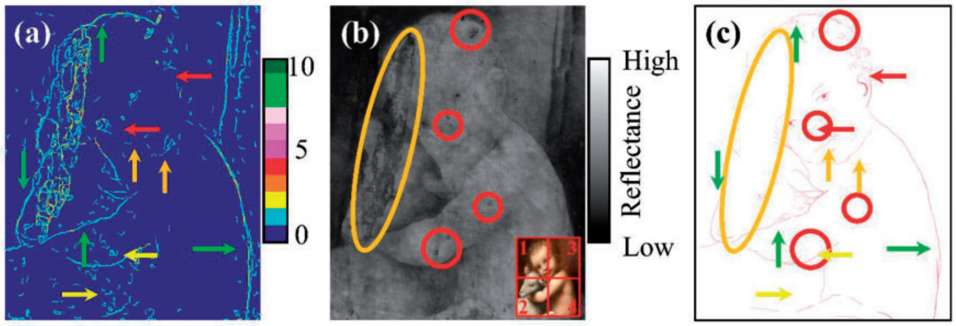

Figure 7. The extracted underdrawings of "The Child with the Lamb". (a) The extracted UD as it results from the algorithm; the multicolor bar indicates how many times a pixel has been detected as belonging to the traits of the UD. (b) The reflectogram collected at 1705 nm; the gray color bar indicates the reflectance distribution within the reflectogram; the inset illustrates the organization of the sub-hyperspaces used for the analysis of this painting. (c) The handcrafted UD. The red circles and the orange ellipse in (b) and (c) indicate, respectively, the presence of deep detachments on the painted surface and the heavily restored area on the left side of the painting and together with the colored arrows in (a) and (b) they point out the main details of the UD discussed in the text.

the painted surface has been hardly retouched as highlighted by the false color image (Fig. 2c). The left side of the panel is so compromised that the art experts did not risk the reconstruction of the UD (orange ellipse in Fig. 7b). Moreover, also the remaining part of the painted surface displays critical issues spread over the whole person of The Child. The area of the nose, the hair, and even the left arm and the back are affected by deep detachments (red circles in Fig. 7b). The situation is so "critical" that even the attempt of designing the SHSs to limit the influence of these artifacts does not produce significant improvements. However, the availability of a handcrafted reference provides the opportunity for evaluating the algorithm's performances in a hard context. Aside from the UDs within the heavily restored area on the left, the comparison between the extracted UD (Fig. 7a) and the hand-crafted one (Fig. 7c) is interesting. On the one hand, the general profiles of the characters have been correctly recovered by the algorithm: the details of the face of the Lamb, the back of both the figures and the upper profile of the Child's head substantially match the one drawn by art experts (green arrows in Figs. 7a and 7c). On the other, the method misses some details and over-segments some regions with respect to the handcrafted UD. The Child's lineaments in the extracted UD are almost undefined; the ear and the nose are nearly lost (red arrows in Fig. 7a), while the mouth and the chin of the Child appear as few confused traits (orange arrows in Fig. 7a); an over-segmented area can be observed around the elbow of the Child (yellow arrows in Fig. 7a). It could be argued that the artifacts laying over the flat regions weaken the effectiveness of the protocol, this is probably due to the gradient enhancements performed during the second step of the algorithm. This speculation finds confirmation as well in the other case studies considered in this work (refer to the Supplemental Material online) and points out that the presence of defects that systematically affects the starting data set can compromise, at best partially, the effectiveness of the algorithm.

Further Observations About the Algorithm

User involvement in the checking of the binary masks is a consequence of the design of the algorithm. Because the selection of the FG is performed exploiting the threshold identified by the Otsu's method, all the pixels with similar values are combined into FG or BG independently from what originates that particular level of reflectance.[25] This means that the tracings cannot be completely isolated as done by art experts (Figs. 1b and 2b). As discussed above, some details of the paintings (light green arrow in Fig. 6; orange arrows in Fig. 7a) get partially lost, while other regions appear over-segmented (for instance see the area indicated by the dark green arrows in Fig. 6 or by the yellow arrows in Fig. 7a). The introduction of machine learning to classify the traces could improve the performance of the algorithm and even avoid the engagement of the user,[41] but at the expense of a lack of simplicity. Indeed, a basic knowledge of machine learning mechanisms would be required and portions of the few IRR images should be used to train the machine. Therefore, we prefer asking the user for a contribution, rather than overload the algorithm to pursuit an accuracy that this discussion demonstrates as not so essential. Moreover, we point out that the algorithm neither accounts for the effects of the layers of paints nor distinguishes within traits drawn by the artist with different media. Ideally, most of the painting materials should be transparent to the radiation in the range above 1200 nm. Practically speaking the thickness of the layers and the pigment inclusions inside the layers act as scattering centers (depending on their size) and affect both the incoming and the reflected radiation. These effects cannot be quantified or isolated by the method (PCA is not a classifier).

Therefore, it is not possible to exclude the physical properties of the painting layers (e.g., the presence of dark areas and lines) from partially affecting the reflectograms, and consequently the extracted UDs). Art experts found that Luini used different media to design the UDs and they pointed out this finding using different transparency levels for the traits of the handcrafted UDs (Figs. 1b and 2b). The algorithm cannot perform a similar distinction; it would require at least one measurable feature (the thickness, the shape, the times the pixels have been detected as part of the extracted UDs, or whatever) able to split the traits of the extracted UDs into different groups (ideally one group for each identified drawing medium). Unfortunately, such a feature does not exist, which means the presence of different drawing media cannot be addressed by the algorithm. Finally, it is important to notice the relevance of the availability of a handcrafted UD to evaluate the performances of the algorithm. The accuracy of the extracted UDs can be effectively evaluated only in presence of a handcrafted counterpart; in the absence of a reference manually sketched by art experts, the extracted UD has to be seen as a proposal of how the UD could be and considered as the starting point to plan further analysis rather than the conclusive result (this is the case of the paintings reported in the Supplemental Material).

Conclusion

The increase of available experimental data justifies the request for analytical tools as possibly fast, user friendly, and reliable. The present work contributes an answer to this need in the field for multispectral IRR as applied to the surface of paintings with the purpose of recovering the UDs. The described protocol is a semi-automatic, quantitative method, and therefore it entails user engagement to best exploit the application of automated mathematical and statistical methods. Briefly, PCA rationalizes and concentrates the information of the data set, the image processing provides the putative traits of the UD, and finally, the UD is recovered once the user has checked the processed images. Even if the user is asked to supervise the partial output of the protocol, this does not require a high level of technical competencies (i.e., mathematical knowledge, computer skills, or expertise in fine arts) since the selection is aimed only to discard those images that are clearly useless (red x-marks in Fig. 4aI" and in Figure S2) and not to validate the ones used for recovering the UD which are constituted by the remaining components (green checkmarks in Fig. 4aI" and in Figure S2). Besides missing some details and partially over-segmenting flat areas, the method proved to be effective on "The Holy Family with St. Anne and the Young St. John". The extracted UD is comparable with the handcrafted one based on the visual analysis of the IRR data, i.e., probably the most widespread technique used for extracting the UDs. Moreover, the protocol directs the attention to the main debated features of the painting such as the *pentimenti* or the differences between the designed and the realized painting. Finally, the computational complexity has been kept at the minimum. The case of "The Child with the Lamb" is more complex with respect to that of "The Holy Family". Indeed the painted surface shows a large restoration area on the left side and small detachments spread all over the panel (Fig. 7c). In the presence of these artifacts, the enhancement of the edges of the PCs' features penalizes the low-defined traits of the UD with the consequence that only the marked parts of the UD can be effectively recovered (Fig. 7a). The extracted UD shows mismatch areas with respect to the handcrafted one and even if some of its main characteristics are still highlighted, the results indicate that the presence of systematic artifacts affecting the reflectograms suggests the revision, at least partial, of the algorithm's design.

In summary, we demonstrated the effectiveness of a semi-automatic protocol for recovering the UD of different paintings by Luini. At the expense of a little involvement of the user, the method guarantees many benefits with respect to the standard hand-driven procedure. The extracted UDs are obtained by quantitative methods; if the starting data set does not present systematic artifacts, the results are in good agreement with the expectation of the art experts (i.e., the handcrafted UD), and, if a handmade counterpart is not available, the ectracted UDs even provide a starting point for discussing the design of the paintings. Moreover, the whole analysis process requires so little knowledge of art works or computer science that it can be fully managed by users with little experience in these two research fields. Finally, the fact that the algorithm has been kept as simple as possible ascribes two more attributes to the method. On the one hand, it is easily adjustable to comply with the features of new paintings; on the other, the traits identified as the UDs can be employed as references for the development of new analysis strategies aimed to deepen the understanding of the technique used to realize the paintings under investigation.

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Supplemental material

The supplemental material mentioned in the text, consisting of Figures S1 and S2, is available in the online version of the journal.

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