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CHARACTERIZATION OF FRICTION ANGLES FOR STABILITY AND DEPOSITION OF GRANULAR MATERIAL

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ABSTRACT

The concept of friction angle as a measure of friction among bodies in static or dynamic conditions, is almost ubiquitous in Earth sciences. In spite of its importance, there is not a general agreement on its definition or standardization on the way to measure it.

This study goes back to the fundamentals of friction among granular particles, presenting results from laboratory tests performed in order to measure the friction angles of particles of different shape, density and material, getting indications on the role of interparticle friction on the stability of a mass of granular material and on its depositional features.

Several granular materials of different nature, natural and artificial, are studied in laboratory by means of a tilting flume. The aim of the performed tests is to measure the characteristic friction angles, both depositional (or repose) and stability limit (critical) taking into account the material characteristics: size, shape, density and roughness.

The granular materials are heaped inside the flume which is then tilted until destabilization of the mass and the gradient of the new deposition surface is then measured by means of a lab-size laser scanner and a digital still camera; video shots of the motion of granules while sliding have been taken as well.

The study shows that characteristic friction angles depend on size and shape of grains while when mixing granules of different size a sorting mechanism arises with less clear deposition angles. Although there are many other ways to measure friction angles (in particular by classical geotechnical apparatuses) the authors propose this way that is closer to the natural slope condition where granular mass flows originate.

KEY WORDS: granular material, friction angle, deposition process, yielding process

INTRODUCTION

The friction angle of a granular mass is of fundamental importance among the rheological parameters that rule the initiation, the resistance to motion and the deposition of a flowing mass composed by granules, with or without the presence of an interstitial fluid.

The term friction angle comes from the well established Coulomb equation (for dry non cohesive materials):

$$\tau = \sigma \tan \Phi \qquad (1)$$

where the term tan(Φ) is a parameter which, expressing the ratio of tangential and direct stresses, represents the friction among two bodies in contact. Initially referred to a solid body laying on a surface this empirical relation was extended to granular media (CASAGRANDE, 1936) both in static/quasi-static (e.g. in soil mechanics) and in dynamic conditions (e.g. HUNGR & MORGENSTERN, 1984) even if very different physical phenomena are involved in the two situations.

In spite of the simplicity of eq. (1), very often scholars and designers use the term "friction angle" with different adjectives, especially in earth sciences (e.g. repose, dynamic, static, critical, internal, natural, stopping, residual, neutral, bulk, etc.; (e.g. Fischer *et alii*, 2008; Pudasaini & Hutter, 2007; Calvetti *et alii*, 2000; Holz & Kovacs, 1981) with different meanings, according to different physical situations, following their field of interest. In other words, it is not possible to find in literature a univocal, unambiguous definition of friction angle for the different conditions. Moreover a generic "angle of internal friction" referred to bodies in static or quasi-static conditions can assume different values following the Coulomb equation as the shear and normal stresses can assume different values depending on the state of bodies in contact.

With reference to granular materials, where the influence of the packing state makes things more complicated than the simple case of a block slide, the terminology relative to friction among granules and to friction angle abounds with no standardization (e.g. Metcalf, 1965; Bagnold, 1966; Cundall & Strack, 1979; Metha & Barker, 1994). This aspect is relevant especially with regard to the special situations (or limit situations) in which a granular media can be found and in those that are of most interest in engineering design and natural hazard studies like the angle of deposition (or repose) and the angle of incipient movement for a mass of granular media.

Given the importance of friction parameters in the development of models for debris flow runout prediction (Hungr *et alii,* 2002; Rickenmann, 2005) many different ways to measure various types of friction angles for granular materials have been proposed by various scholars (e.g. Metcalf, 1965; Hutter & Koch, 1991; Pudasaini & Hutter, 2007). Thus the values of measured friction angles for a certain material generally differ from a study to another and are significant for the particular context in which those values were found.

The authors of the present work take into consideration the deposition (or repose) friction angle defined as the uniform angle at which a granular material arranges itself on a inclined slope coming at a stop after being mobilized and a critical (or yielding) friction angle defined as the gradient at which a granular mass becomes unstable and start to slide down. A measure of critical friction angles for movement initiation and for movement ceasing has already been proposed by Deganutti & Scotton (1997) by means of a cone and plate rheometer in controlled normal stress conditions; here we propose a methodology that reproduces in laboratory the natural conditions of loose granular rock materials (such as scree) on a slope, conditions potentially able to originate debris flows. In this sense authors' proposed methodology is strictly related to superficial instability, where there is no confinement pressure and the granules are free to move and to dilate (Reynolds, 1885); in order to test the effect of a vertical load on the tested material, authors have performed some shear box tests as well.

Some recent developments (Daer & Douady, 1999; GDRMiDi, 2004; Jop *et alii*, 2006) on dense granular flows are focused on some particular laboratory tests in which artificial granular material (generally glass beads) is poured on a tilted rough plane; in that particular experimental settings (in which the material is far from a natural one, given the ability to rotate and the very low particle-to-particle friction of glass spherical beads) those scholars found that there is a relation between the plane inclination and the thickness of granular material that remains stable on the plane. In that case the surface of the grain pile is parallel to the plane. While that finding is important for its relation to industrial applications in which dense granular flows are involved, present paper's approach is more focused on yielding and deposition of the superficial layer of a pile of granular material (that is not parallel to plane bottom) for its relation to natural conditions, e.g. of a mountain scree slope.

LABORATORY APPARATUS

The laboratory tests have been performed using an apparatus (Fig. 1), constituted by a 2 m long and 1.5 m wide tilting plane with an adjustable slope up to 38° from the horizontal. A 232 mm wide flume with one glass side wall was installed on the plane. The tilting plane was hinged to a fixed horizontal plane (1.5 m long and 1.5 m wide). To avoid slippage of material, coarse sand paper was glued on the flume bottom

The tilting movement of the plane was controlled by a synchronous electric motor which provides a constant (slow) rate of tilting (around 2.5×10^{-3} rad s^{-1}) through a robust rotating screwed bar; this system has proved to be very good for the smoothness of movement and for the absence of vibrations on the plane itself, both characteristics of great importance for the kind of test that have been performed.

The morphological measures on materials were performed by a laboratory class laser-scanner which was attached to a frame able to move along the flume and adjustable in height through a spherical joint; in this way the position of the measuring instrument was adjustable in every direction for the best accuracy of measurements. This instrument had just been added to the laboratory equipment and thanks to its ability to scan an object surface with millimetric precision proved to be very useful for the planned measurement where a mathematical definition of the granular surface easily yielded the average surface angle to the horizontal (with exclusion of the tests with fine material, see later in the text). The irregularities on the grain surfaces were generally within 2 particle diameters. In those experimental conditions the use of mechanical measuring devices would have been not easy and more time consuming.

TESTED MATERIALS

Seventeen different types of granular cohesionless material, both synthetic and natural, have been tested. The main characteristics of tested materials are presented in Table 1.

Materials m1 and m2 are PVC cylindrical granules whose density is respectively 1200 and 1500 kg m^{-3}.

Tested materials differ as regards the origin, the grain size distribution and the grain shape.

Materials from m3 to m8 and m15 are coarse, well rounded river gravel particles with equivalent sizes ranging on the whole from 31.5 to 4.0 mm and a mean density of 2700 kg m^{-3}.

Materials from m9 to m11 and m13 are coarse-grained sub-angular gravel particles, obtained by crushing limestone rocks, with equivalent sizes ranging on the whole from 16.0 to 4.0 mm and density between 2600 and 2700 kg m^{-3}.

Fig. 1 - Laboratory flume and laser-scanner apparatus testing low density PVC material

Materials m12, m14 and m16 are represented by graded sands with equivalent sizes ranging on the whole from 2.0 to 0.075 with a few fines (less than 5-6%); m14 is the finer fraction of m12.

Materials m8, m11 and m12 result by mixing different grain size classes: m8 by 20% of each a, b, c, d and e size class; m11 by 50% of c and d size class; m12 by 50% of g and h size class.

Material m17 is a coarse grained material obtained by mixing in equal proportions rounded and sub-angular gravels to have a grain size distribution from 4.0 to 2.0 mm (mean density 2650-2700 kgm^{-3}).

The roundness grade is based on the classification of Powers (1953).

METHODS

Some authors consider as "friction angle" of a granular material the maximum slope at which a heap of loose material will come to rest when dumped on a slope, but this way to measure the friction angle is subject to the way the granular material is dumped, and to the roughness of the surface on which the material is poured. Tests were then performed by accumulating a heap of dry material in the rising end of the flume, not considering the angle at which the material stopped. The plane was slowly and continuously tilted

Material	type	Size (mm)	Size class
m1	Cylindrical black PVC grains	2r = 3.1, h = 3	
m2	Cylindrical white PVC grains	same size as m1	
m3	Well rounded gravel	22.4 < d < 31.5	a
m4	Well rounded gravel	16.0 < d < 22.4	b
m5	Well rounded gravel	11.2 < d < 16.0	c
m6	Well rounded gravel	8.0 < d < 11.2	d
m7	Well rounded gravel	4.0 < d < 8.0	e
m8	Well rounded gravel	4.0 < d < 31.5	a+b+c+d+e
m9	Subangular gravel	11.2 < d < 16.0	c
m10	Subangular gravel	8.0 < d < 11.2	d
m11	Subangular gravel	8.0 < d < 16.0	c + d
m12	Sand	d < 2.0	g+h
m13	Subangular gravel	4.0 < d < 8.0	e
m14	Sand	d < 0.5	h
m15	Well rounded gravel	4.0 < d < 8.0	e
m16	coarse sand	0.5 < d < 2.0	g
m17	mixed shape gravel	2.0 < d < 4.0	f

Table 1 - Tested Material - 2r, h: cylinder diameter and height; d: particle equivalen

until the material slid; at this moment the plane raising motor was stopped and the slope of the flume was measured.

Since the material was simply poured into the flume and the heap resulted in a loose and chaotic state, the first sliding was not considered as "natural".

The plane was further tilted up until material slid again and the tilting increase necessary to cause the instability of the material was recorded, its value added to the deposition angle gives the critical angle for stability of the tested material; depending on material characteristics, up to seven successive slides were obtained.

After each sliding stopped forming a uniform slope, gradients to the horizontal (deposition or repose angle) were measured by least squares interpolation of the mathematical surface described by the laser scanner (Fig. 2), along the middle longitudinal section, in order to minimize possible disturbances due to "side-effects" of flume walls.

The recording accuracy of the laser-scanner was 0.38 mm with a measuring range from 38 to 45 cm. After every scan a digital photo of the material was taken normal to the glass wall of the flume to check angle measurement and the photos were digitally processed to get the material slope.

RESULTS

A total of 143 tests have been done, some in the same experimental conditions in order to check their repeatability.

In Table 2 synthetic resulting data are reported: for every tested material the average values of the critical and repose angles and their difference, a delta value which is a measure of the yield strength of the granular media, are given.

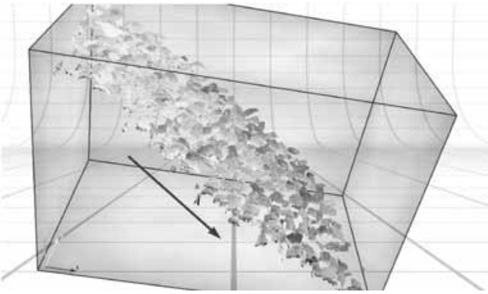

Fig. 2 - 3-D Laser Scanner survey restitution of the surface of material m3 after a slide in the flume

The standard deviation for the values of angles is less than one degree for around 60% of tests performed for every material. Repeated tests in the same experimental conditions gave differences generally smaller than 2 degrees, a value which is about our apparatus and measurement system accuracy range.

For comparison, a series of direct shear tests were conducted using direct shear box tests to determine the consolidated-drained shear strength of some of the materials tested on the flume (Standard Reference: ASTM D 3080 - Standard Test Method for Direct Shear Test of Soils Under Consolidated Drained Conditions). PVC materials (m1 and m2), coarse natural sands (m16) and mixed rounded and sub-angular gravels (m17) were tested.

Mohr's failure envelopes have been obtained using the peak stresses recorded by shear box tests. PVC (m1 and m2) materials gave quasi-static friction angles ranging as a mean from 30.6° to 33.9°, differences being due mainly to difficulties in obtaining the same initial density and to the distribution of particles axis with respect to the shear plane. High values of the quasi-static friction angles were obtained for m16 (35.0°) and m17 (46.8°) materials. In these cases, however, the materials showed a contractive behavior during shear that increased the initial bulk density to quite high values and this occurred in spite of the relatively high value of the normal stress (50

		size class	mean deposition angle	mean critical angle	Angle Delta Crit-Dep
mat	mat. type		(°)	(°)	(°)
m1	PVC LD		31.0	37.2	6.2
m2	PVC HD		28.6	35.2	6.6
m3	rounded river gravel	a	35.0	40.8	5.8
m4		b	35.1	39.9	4.8
m5		c	33.7	38.4	4.7
m6		d	33.2	37.5	4.3
m7		e	31.5	36.2	4.7
m15		e	30.9	34.0	3.1
m8		a+b+c+d+e	34.0	36.8	2.8
m9	angular gravel	c	35.3	42.2	6.9
m10		d	35.6	39.4	3.8
m13		e	35.2	39.1	3.8
m11		c+d	36.0	40.3	4.3
m17	mixed shape	f	30.8	34.3	3.5
m12	sand	g+h	irregular behaviour		
m14		h	irregular behaviour		
m16		g	32.1	34.4	2.3

Table 2 - Average values of deposition and critical angles.

kPa). The contractive behavior is the consequence of an initial void ratio higher than the corresponding one for that normal stress. Besides, it should be noted that a contractive behavior has already been found for field conditions in Acquabona site (Genevois *et alii*, 2000).

COMMENTS

1 - Apart from the mixed sands the tested materials showed a regular behaviour characterized by constant (in cited limits) deposition and critical angles, with the deposited granular mass forming a regular and uniform plane surface (Figg. 1 and 2).

2 - The lowest deposition angle was showed by high density PVC (28.6°) due to the smoothness of grain surface and to the cylindrical shape, giving to grains low resistance to sliding and some rotation ability.

A bit surprisingly, the lowest yielding angle is given by the size class *e* (4.0 < d < 8.0) rounded gravel with 34.0°, against 35.2° of high density PVC. This fact shows that the difference between the two characteristic angles (angle "delta") is not constant for different grain size.

This can be considered another proof that the yielding and deposition processes are physically different and that it is generally incorrect to consider a single generic "friction angle" when applying the Coulomb equation to the different conditions in which a granular mass can be.

3 - The highest value for critical angle was showed by size *c* (11.2<d<16.0) angular gravel showing a high resistance to yielding (due to its angular shape) while its deposition angle (35.3°) is a bit higher than that of rounded gravel of the same size (33.7°) showing that the grain shape is relatively less important in the deposition process.

4 - The size of grains of similar shape has the clear effect of giving a higher stability to bigger grains: values of critical angle for rounded gravel go progressively from 34.0° to 40.8° with size going from 4 to 31.5 mm. The same effect is shown by the same gravel for the deposition angle even if with slighter variations. As a consequence, the "Delta" between yielding and deposition angles decreases with size.

The angular gravel has a similar behaviour with regard to yielding angles, while its deposition angles seem to be unaffected by size.

5 - Tests with a mix of different grain size gave results for characteristic angles which are in the range of the values shown by the single size classes, but in different order (the deposition angle was close to the low ones and the yielding angle was close to the high ones) with the result that the angles "Delta" of m8, resulting from a mixture of 20% in weight from a, b, c, d, e class sizes, is the lowest for rounded river gravel.

Again, mixed angular gravel (m11) showed behaviour similar to river gravel, with smaller variations.

Apart from the mixed sands (m12 and m14) no sorting mechanism or "Brazilian nut effect" (Herrmann & Luding, 1998) was noted for different size grains, given the quasi-static experimental conditions.

6 - Materials m12 and m14, formed by a mixture of sand and finer particles, showed an irregular behaviour with a size mechanism of separation: when the bigger particles slid, the finer (silt) remained in their previous position with a slope higher than the deposited sand grains and forming a surface of irregular shape. This behaviour is probably due to an electrostatic interaction (Hungr & Morgenstern, 1984) among the fine particles giving them a cohesion effect.

7 - The density of material does not have an evident role in the yielding and deposition of material with the low density PVC having higher angles than the high density one, both for critical and deposition conditions; this can be explained by the higher friction showed by the "rubbery" surface of the low density PVC granules.

8 - The amount of mobilized particles during a slide is the minimum necessary for material to achieve the new repose profile, an example of "natural economy".

9 - From video shots of the granular slope failures it was possible to recognize two failure mechanisms: a general yielding of the slope starting from the top and, with materials m12 and m14 (mixture of sand and silt) the failure started from the toe climbing back the slope in a negative wave fashion.

CONCLUSIONS

The authors presented a series of laboratory tests in order to characterize two values of friction angles showed by different materials in particular limit conditions: a stability critical one and the deposition angle.

Critical angle measurements with an adapted tilting plane gave repeatable results within 1-2 degrees and were not affected by the volume of employed material.

The use of natural cohesionless granular material is of particular interest for its close relation with the stability of scree slopes which are prone to debris-flow triggering, especially in the Dolomites area.

These angles, here defined and measured, are of paramount importance for the rheology of granular materials and in particular for debris flow studies, where the consideration of a generic friction angles (e.g. in debris flow numerical models) as characteristic for yielding and deposition of a flowing mass, could lead to results far from reality and in particular to an underestimation of the maximum runout of debris flows and other granular flows.

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