F. Carrabs, R. Cerulli, C. D'Ambrosio, A. Raiconi, A Hybrid Exact Approach for Maximizing Lifetime in Sensor Networks with Complete and Partial Coverage Constraints. Journal of Network and Computer Applications, Volume 58, pp. 12-22, 2015.

https://doi.org/10.1016/j.jnca.2015.08.018

A Hybrid Exact Approach for Maximizing Lifetime in Sensor Networks with Complete and Partial Coverage Constraints

Francesco Carrabs[a,*], Raffaele Cerulli[a], Ciriaco D'Ambrosio[b], Andrea Raiconi[a]

[a] *Department of Mathematics, University of Salerno, Via Giovanni Paolo II 138, 84084, Fisciano, Italy.*
[b] *Department of Computer Science, University of Salerno, Via Giovanni Paolo II 138, 84084, Fisciano, Italy.*

Abstract

In this paper we face the problem of maximizing the amount of time over which a set of target points, located in a given geographic region, can be monitored by means of a wireless sensor network. The problem is well known in the literature as Maximum Network Lifetime Problem (MLP). In the last few years the problem and a number of variants have been tackled with success by means of different resolution approaches, including exact approaches based on column generation techniques. In this work we propose an exact approach which combines a column generation approach with a genetic algorithm aimed at solving efficiently its separation problem. The genetic algorithm is specifically aimed at the Maximum Network α-Lifetime Problem (α-MLP), a variant of MLP in which a given fraction of targets is allowed to be left uncovered at all times; however, since α-MLP is a generalization of MLP, it can be used to solve the classical problem as well. The computational results, obtained on the benchmark instances, show that our approach overcomes the algorithms, available in literature, to solve both MLP and α-MLP.

Keywords: Maximum Lifetime, Wireless Sensor Network, Column Generation, Genetic Algorithm.

1. Introduction

Wireless Sensor Networks (WSNs) are usually composed of a large amount of sensing devices (*sensors*) scattered over a region of interest. Each sensor is generally capable of monitoring a certain portion of the space around itself (usually called its *sensing area*, and defined by the *sensing range* of the sensor). While each individual device has obvious limits in terms of range extension and battery lifetime, a coordinated use of multiple sensors together allow them to perform complex monitoring activities in possibly large areas, in fields as diverse as environmental control, military and healthcare applications, among others (see, for example, [1], [14], [16]).

Given the limited power of the batteries that usually keep sensing devices operational, an issue which has generated intense research interest in the last years is related to the optimization of battery consumption. In particular, the problem of appropriately use sensors to monitor a set of specific points of interests, known as *targets*, for as long as possible has been widely studied;

*Corresponding author

Email addresses: `fcarrabs@unisa.it` (Francesco Carrabs), `raffaele@unisa.it` (Raffaele Cerulli), `cdambrosio@unisa.it` (Ciriaco D'Ambrosio), `araiconi@unisa.it` (Andrea Raiconi)

the problem is usually known as Maximum Network Lifetime Problem (MLP). It has been mainly approached with methods aimed at finding several, possibly overlapping sets of sensors (defined *covers*) which can individually provide coverage for all the targets, as well as an activation time for each of them, such that the sum of the activation times of the covers in which each sensor appears is not greater than its battery life. The idea is then to activate the covers one by one, where by activating a cover we intend to turn on all the sensors which belong to it, while keeping all other sensors turned off.

In [4] the authors showed that MLP can bring improvements with respect to previous approaches in which sensors were divided into disjoint sets (that is, each sensor could only belong to a single cover). They also proved that the problem is NP-Complete, and proposed an approximation algorithm to solve it.

A column generation approach aimed at solving the MLP was proposed in [12]. In this work the author proposes a hybrid approach where the separation problem of the column generation technique is either solved heuristically or optimally by means of an appropriate ILP formulation. More details about this technique are given in Section 3. For a survey on hybrid algorithms, including the embedding of heuristics and metaheuristics into column generation frameworks, the reader may refer to [3].

Several variants of MLP have been proposed as well, in order to adapt the problem to different contexts. Some of the proposed variants take into account cover connectivity ([2], [7], [8], [15], [20]), reliability issues ([10]), or consider sensors with adjustable sensing ranges ([5], [9], [17]). For many of these variants, efficient algorithms based on column generation have been proposed ([2], [6], [7], [8], [9], [10], [15], [17], [18]).

Another interesting variant of the problem is the Maximum Network α-Lifetime Problem (α-MLP), which was proposed and studied in [13]. In this variant, a predefined portion of the overall number of the targets can be neglected in each cover. As will be better investigated in Section 2, α-MLP generalizes MLP and therefore each method aimed at solving this problem can also be used to face the original one. In [13] the authors presented both a heuristic algorithm and an exact one, showing that large improvements in terms of the global network lifetime can generally be achieved by neglecting just a small percentage of targets in each cover. Furthermore, the authors also showed that such improvements can usually be mostly retained when some additional regularity conditions are taken into account, in order to guarantee a minimum global coverage level to each target.

In this work we propose a hybrid exact approach for the α-MLP problem, named GCG. While the overall structure of the algorithm is again based on the column generation technique, the main contribution of this work is the proposal of an appropriately designed genetic metaheuristic, which is used to solve its separation problem. As will be shown by discussing the results of our computational tests, the algorithm is proven to be highly efficient in terms of requested computational time, outperforming the algorithms presented in [12] for MLP and [13] for α-MLP.

The rest of the work is organized as follows. Section 2 formally introduces the problems and a mathematical formulation to describe them. Section 3 resumes the approaches presented in [12] and [13] to solve MLP and α-MLP. Section 4 describes our proposed genetic algorithm, while Section 5 presents the results of our computational experiments. Finally, Section 6 presents some final remarks.

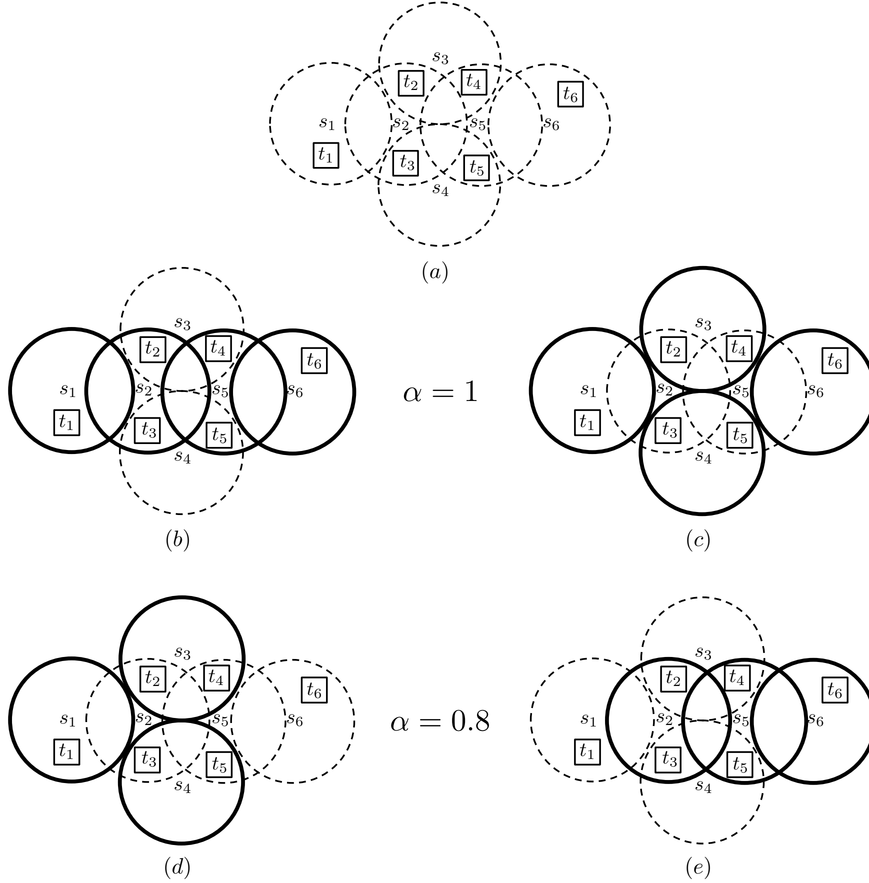


Figure 1: Example network

2. Problems Definition and Mathematical Formulation

Let $N = (T, S)$ be a wireless sensor network, with $T = \{t_1, \dots t_n\}$ being the set of the targets and $S = \{s_1, \dots, s_m\}$ being the set of the sensors. As previously introduced, each sensor is assumed to have a given sensing range, and to be powered by a battery that can keep it activated for a limited amount of time. In this work, we assume each sensor to be identical, therefore they all have sensing ranges of the same size and equal battery durations, normalized to 1 time unit. In Figure 1(a) a sensor network with sensors $s_1, \dots, s_6$, targets $t_1, \dots, t_6$ and sensing ranges represented by circles is shown.

For each target $t_k \in T$ and sensor $s_i \in S$, let δ_{ki} be a binary parameter equal to 1 if t_k is positioned within the sensing range of s_i (it is *covered* by the sensor), 0 otherwise. For a subset of sensors $S' \subseteq S$ and a target $t_k \in T$, let $\Delta_{kS'}$ be another binary parameter equal to 1 if $\delta_{ki} = 1$ for a given $s_i \in S'$, 0 otherwise.

Given a value $\alpha \in (0, 1]$, we define $C \subseteq S$ to be a *feasible cover* (or simply a cover) for the network if its sensors cover at least $T_\alpha = \alpha \times n$ targets, that is, $\sum_{t_k \in T} \Delta_{kC} \geq T_\alpha$. Furthermore, we define a cover to be *non-redundant* if it does not contain another cover as a proper subset.

The Maximum Network α-Lifetime Problem (α-MLP) consists then in finding a collection of pairs (C_j, w_j) where each $C_j \subseteq S$ is a feasible cover and each $w_j \geq 0$ is an activation time, such

that the sum of the activation times is maximized and each sensor is used for an amount of time that does not exceed its normalized battery duration. It is easy to understand that an optimal solution can always be found by only considering non-redundant covers.

Assuming to be able to compute the whole set of feasible covers $C_1, \dots, C_\ell$ in advance, α-MLP could then be represented using the following Linear Programming formulation, where the binary parameter $a_{ij} = 1$ if $s_i \in C_j$, 0 otherwise:

$$[\mathbf{P}] \max \sum_{j=1}^{\ell} w_j \tag{1}$$

s.t.

$$\sum_{j=1}^{\ell} a_{ij} w_j \leq 1 \qquad \forall i = 1, ..., m \tag{2}$$

$$w_j \geq 0 \qquad \forall j = 1, ..., \ell \tag{3}$$

Objective function (1) maximizes the sum of the activation times, while constraints (2) enforce the respect of the lifetime constraints for each sensor.

In the classical Maximum Network Lifetime Problem (MLP), each cover has to provide information on the whole set of targets in order to be feasible; therefore, MLP corresponds to the α-MLP with $\alpha = 1$ (and hence $T_\alpha = n$). Under these assumptions, the problem definition and the **[P]** formulation presented above represent the classical problem as well.

It is interesting to observe that, on the same wireless sensor network, the maximum lifetime for α-MLP is often greater than the maximum lifetime for MLP. For instance, let us consider again the network in Figure 1(a). It is easy to see that the only two non-redundant feasible covers for MLP would be $\{s_1, s_2, s_5, s_6\}$ (Figure 1(b)) and $\{s_1, s_3, s_4, s_6\}$ (Figure 1(c)). In this case, it is possible to obtain a network lifetime equal to 1 time unit by activating them for any couple of activation times $w_1, w_2 \geq 0$ such that $w_1 + w_2 = 1$. However, after this operation, the batteries of sensors s_1 and s_6 are exhausted, and no more feasible covers can be obtained by using the remaining sensors, therefore the final solution is equal to 1 as well. Let us consider now on the same network an α-MLP problem with $\alpha = 0.8$, that is, 1 out of 6 targets can be neglected. In this case, four non-redundant covers exist, namely $\{s_1, s_3, s_4\}$ (Figure 1(d)), $\{s_2, s_5, s_6\}$ (Figure 1(e)), $\{s_1, s_2, s_5\}$ and $\{s_3, s_4, s_6\}$. We can easily obtain a lifetime equal to 2 time units by activating in sequence the covers $\{s_1, s_3, s_4\}$ and $\{s_2, s_5, s_6\}$, for 1 time unit each.

On instances of real-world size, formulation **[P]** can not be expected to be directly applicable due to the high (potentially exponential) number of covers. This can be especially true for lower values of α; indeed, it is straightforward to observe that given $(\alpha_1, \alpha_2) \in (0, 1]^2$ with $\alpha_2 < \alpha_1$, each cover for α_1-MLP is also feasible for α_2-MLP. For this reason, it is necessary to apply different approaches, such as column generation which was proposed by [12] for MLP and by [13] for α-MLP. We use the same type of approach in this work; however, we focus our attention on solving efficiently the separation problem, since it is a key component to obtain an effective algorithm. To this end, we design a fast genetic metaheuristic whose main characteristic is the ability to return several good covers at once. As will be shown in Section 5, this feature is able to bring significant improvements, in terms of computational time, with respect to the previous algorithms.

3. Column Generation Approaches for α-MLP and MLP

Given a Linear Programming formulation with a large number of variables (in our case, formulation **[P]**), the Column Generation (CG) technique starts by considering a version of the formulation which only uses a subset of these variables (in our case, a subset of feasible covers) in the so-called *master problem*, and by solving it to optimality. The optimal solution of the master problem is clearly a feasible solution for [**P**]. The CG then considers a specific optimization problem (defined *separation problem* or *subproblem*) which either produces an attractive cover to be added for a new iteration of the master problem, or certifies that the last solution found by it (that we denote as *incumbent solution* from now on) is indeed optimal for [**P**]. The procedure iterates until the above described optimality condition is met. In this way, the CG approach implicitly discards most of the variables that will be nonbasic in the optimal solution.

An *attractive cover* is a feasible cover corresponding to a nonbasic variable with a negative reduced cost, which could therefore improve the incumbent solution if introduced in the master problem. Conversely, such a solution can not be improved if the reduced costs associated with the nonbasic variables are all non negative. More in detail, given the dual prices π_i associated to each constraint of the master problem, that is, to each sensor, the incumbent solution is optimal if $\sum_{i:s_i \in C_j} \pi_i - c_j \geq 0$ for each nonbasic cover C_j, which can be rewritten as $\sum_{i:s_i \in C_j} \pi_i \geq 1$ since the coefficients in the objective function (1) of the original LP formulation are all equal to 1.

We can therefore define as subproblem the following formulation **[SP]**, where objective function (4) minimizes the sum of the dual prices in the sensors chosen to be part of the newly produced cover, while constraints (5)-(8) define a feasible cover:

$$\textbf{[SP]} \qquad \min \sum_{i=1}^{m} \pi_i x_i \tag{4}$$

s.t.

$$\sum_{i=1}^{m} \delta_{ki} x_i \geq y_k \qquad \forall k = 1, ..., n \tag{5}$$

$$\sum_{k=1}^{n} y_k \geq T_\alpha \tag{6}$$

$$x_i \in \{0,1\} \qquad \forall i = 1, ..., m \tag{7}$$

$$y_k \in \{0,1\} \qquad \forall k = 1, ..., n \tag{8}$$

For each sensor s_i, the binary variable x_i represents the choice on including it in the new cover, while, for each target t_k, the variable y_k represents whether the target is monitored in the cover. Constraints (5) make sure that each y_k can have value 1 only if at least one of the sensors that cover the target has been added, while constraints (6) impose that at least T_α targets are covered. The incumbent solution is then optimal if the value of objective function (4) is greater or equal than 1, otherwise the new attractive cover is added to the master problem.

When $\alpha = 1$, that is we are considering the MLP problem, constraints (5) reduce to $\sum_{i=1}^{m} \delta_{ki} x_i \geq 1 \ \forall k = 1, ..., n$, and constraints (6) as well as variables y_k are not necessary.

3.1. Heuristics to enhance CG

The main disadvantage of the above presented CG approach is that [**SP**] is strongly NP-hard, being a specialization of the Set Covering problem. For this reason, it is advisable to limit as much as possible the number of times in which it is required to be solved. In [12] the author faces the problem by introducing a constructive heuristic to quickly solve the subproblem. This heuristic iteratively builds a cover by first choosing, in a random way, an uncovered target and then by selecting the sensor that can cover it, with the minimal dual price value. This process is repeated until a complete coverage has been obtained. The author introduces three column generation-based approaches named Exact, Heur and Mixed, respectively. The first algorithm solves the subproblem in an exact way, while the second one solves the subproblem by invoking the above described constructive heuristic. When the heuristic does not find attractive covers, Heur stops without certifying the optimality of the incumbent solution. For this reason, the solutions provided by Heur can be suboptimal. Finally, in the Mixed algorithm the attractive covers are provided by the constructive heuristic and, when it fails, by solving the exact subproblem, which is also used to prove the optimality of the solution in the last iteration of the algorithm.

In [13], the authors propose instead a heuristic meant to independently produce a complete solution for α-MLP (that is, a collection of covers and activation times). Each cover in this approach is again built iteratively, adopting some heuristic criteria to favor the coverage of sensors which have been covered for fewer amounts of time so far in the partial solution. Each newly produced cover is assigned a predefined amount of time, and the algorithm ends when the residual energy in the sensors do not allow to produce a new feasible one. Finally, the set of produced covers is used as initial restricted set for the master problem.

In this work, we attempt to heuristically solve [**SP**] at each iteration, by using a genetic metaheuristic instead of a simple constructive heuristic as the one proposed in [12]. As in the Mixed algorithm, the exact subproblem formulation is used when the genetic algorithm fails in order to guarantee that an exact solution is always found. We define this hybrid exact approach GCG. As is shown in the next sections, our approach is able to significantly outperform the previous algorithms for MLP and α-MLP proposed in the literature.

4. A Genetic Algorithm to address the [SP] Subproblem

A genetic algorithm is a naturally randomized technique that emulates the typical steps of the biological evolution. It is based on the concepts of *natural selection*, *crossover* and *mutation*. Each problem solution is expressed by an element, named *chromosome*, that represents the structure of an individual. Given a starting population P of chromosomes, the genetic algorithm iteratively produces new chromosomes by means of the crossover operator which combines, in a probabilistic manner, the genetic information of typically two or more randomly selected elements of the population. On each newly generated chromosome a mutation operator is applied, in order to provide a perturbation of the solution and add diversity. The natural selection process and the *fitness* function, which is used to rank each solution, aim at introducing in the population new chromosomes which are better adapted to the environment. Genetic algorithms take typically into account stop conditions, which could be for instance a maximum number of iterations, a time limit, a lack of improvements in the fitness function of the best individual for a given number of iterations, or a combination of some of the above. For a complete and detailed description of genetic algorithms and their characteristics, the reader can refer to [11].

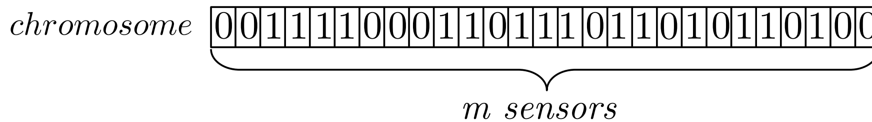


Figure 2: Chromosome representation

In order to overcome the hardness of the [**SP**] problem, we decided to solve the subproblem heuristically through the design of a specific genetic algorithm, defined GA from now on.

The aim of GA is to quickly find attractive covers, and return them to the master problem. An interesting feature of our approach is its ability to potentially produce several attractive covers at once, reducing dramatically the number of required iterations. If GA fails in finding any attractive cover, GCG solves the [**SP**] formulation instead, in order to either find a new attractive cover or prove the optimality of the current solution. It follows that the greater is the effectiveness of GA, the better are the performances of the whole GCG framework. As will be shown in Section 5, GA appears to be very effective, since on the considered set of benchmark instances it often fails only once, i.e. when the optimal solution is found.

Sections 4.1-4.5 describe in detail the different GA components, while Section 4.6 presents a general overview of the procedure.

4.1. Chromosome Representation and Fitness Function

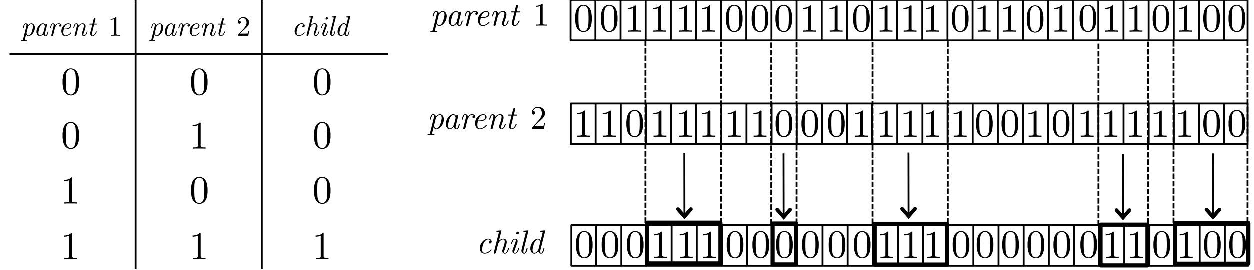
In GA, the binary vector representation shown in Figure 2 is used for the chromosomes. Each chromosome contains $m = |S|$ elements (defined *genes*), which are associated to the sensors of the network. A chromosome represents a feasible cover, meaning that each gene i ($i = 1, ..., m$) is equal to 1 if the related sensor s_i belongs to the cover (in which case the sensor is said to be *active*), and 0 otherwise. It can be observed that the value of gene i corresponds in GA to the value assigned to binary variable x_i in the [**SP**] formulation. Analogously to covers, a chromosome is defined to be redundant if it is possible to switch off at least one of its active sensors without losing feasibility. Since, as already mentioned, an optimal solution can always be found by only considering non-redundant feasible covers, during the GA execution we only allow non-redundant chromosomes to be part of the population.

The fitness function value for a given chromosome is equal to the dot product of the binary chromosome vector and the dual prices vector deriving from the last iteration of the master problem (and therefore it corresponds to objective function (4) for [**SP**]). At the end of the GA procedure, each chromosome with a fitness value that is lower than 1 is included in the master problem as a new column.

4.2. Crossover

One of the main aspects that influence the effectiveness of a genetic algorithm is the crossover operator. This operator allows the creation of new chromosomes starting from previous individuals. In particular, the crossover usually selects two chromosomes of the population (defined *parents*), and generates a new one starting from them (the *child*), which hopefully inherits good features. During the evolutionary process of a genetic algorithm, special care should be taken in order to avoid the case in which several identical chromosomes exist in the population; indeed, in this case the crossover operator has failed to create offspring that differs from the parents. This situation penalizes the effectiveness of the algorithm and therefore the quality of the final solutions.

crossover operator $child_i = parent1_i \wedge parents2_i \; \forall i \in \{1, ..., m\}$



parent 1	*parent* 2	*child*
0	0	0
0	1	0
1	0	0
1	1	1

Figure 3: Crossover operator

In our crossover, the selection of the parents is carried out through a typical binary tournament. To this end, two chromosomes are randomly selected, and the one with the best fitness value among them is chosen as first parent. Then, the second parent is chosen in the same way, avoiding the first parent to be chosen among the participants of the second tournament.

Our crossover operator works exactly like the bitwise AND logical operator. Figure 3 shows, on the left, the AND truth table and, on the right, two sample parent chromosomes *parent* 1 and *parent* 2, as well as the building process of the *child* chromosome starting from them. The operator is meant to bring to the new child the genetic heritage which is common to the two parents. The crossover operator runs in $O(m)$.

4.3. Mutation

The mutation operator alters one or more genes in a chromosome in order to introduce some perturbation, and thus provides diversification in the new generated chromosomes.

As previously introduced, in our genetic algorithm no duplicated chromosomes are allowed in the population. This means that building a duplicate chromosome would be a waste of computational time, since it would be rejected. While building each new chromosome we do not take into account all the previous ones in the population, therefore we try to differentiate it from at least both its parents. If two parent chromosomes have mostly identical genes, which could be a common case especially towards the end of the procedure, the child results to be very similar to them as well, and therefore it is not uncommon that the final operations carried out on it in order to guarantee feasibility and remove redundancy (see Section 4.4) could make it exactly identical to one of its parents. In order to face this problem, we use mutation to change the value of a random single gene in the child whose value is identical into its parents (if it exists). This allows to differentiate the child from both parents. This gene will be switched back only if strictly needed by the feasibility operator or by the redundancy operator, which are described in the next section. A list of the genes with a common value in the two parents is computed during the m AND operations performed by the crossover. Therefore, the mutation takes constant time to select a random element of the list and switch its value.

4.4. Feasibility and Redundancy Operators

It is easy to see that the chromosome produced by the crossover and mutation operators could be unfeasible, since it is not guaranteed that the T_α coverage requirement is satisfied. For this

reason, it is necessary to apply another operator, that we call *feasibility*. The operator selects a random gene whose value is equal to zero in the child and whose related sensor could cover some new targets, and switches its value to one (thus activating the sensor). This process is repeated until the T_α threshold is satisfied. Algorithm 1 shows the pseudocode of this operator. The while loop of lines 2-7 is repeated until the threshold is reached. The procedure individuates the set of uncovered targets $\hat{T}$ (line 3), and randomly selects one of them, t (line 4). Then it randomly selects and activates a sensor s which can cover t (line 5). Finally, in the last two lines inside the loop the operator updates the child chromosome and the related set of covered targets.

The operator has a complexity equal to $O(mn + n^2)$ in the worst case. For each target, we consider a coverage counter that reports the amount of sensors that cover it. During the initialization phase (line 1), iteratively for each active sensor, the counters of all its covered targets are updated. The initialization of all the n counters takes at most $O(mn)$ operations. The while loop is repeated at most $O(n)$ times, in the case of $\alpha = 1$ and all n targets initially uncovered. The $\hat{T}$ set is built in $O(n)$ steps, by checking the coverage counter for each target. The operations on lines 4-5 have constant time complexity. The last two operations of the loop (lines 6-7) have complexity $O(1)$ to change the value of the selected gene and at most $O(n)$ to update all the counters of its covered targets.

Algorithm 1: Feasibility operator

Input: unfeasible *Child* chromosome;
Output: feasible *Child* chromosome;

```
1 T_covered ← coveredTargets(Child);
2 while |T_covered| < T_α do
3     T̂ ← T \ T_covered;
4     t ← randomSelect(T̂);
5     s ← randomSelect(S, t);
6     Child ← switch(s);
7     T_covered ← update(s);
8 return Child;
```

The application of the feasibility operator can produce a redundant chromosome. Therefore we apply a final operator called *redundancy*, in order to switch off eventually redundant sensors. Algorithm 2 shows the pseudocode of the procedure. In the first step (line 1) the operator builds a set $\hat{S}$ containing the sensors that could be switched off in the chromosome without compromising feasibility. The while loop (lines 2-5) is repeated until the are no redundant sensors in the chromosome. The operator randomly selects a sensor from $\hat{S}$ (line 3) and switches it off (line 4). Note that when a sensor $s \in \hat{S}$ is switched off, some elements of $\hat{S} \setminus \{s\}$ may become needed to retain feasibility. For this reason, the operator re-evaluates $\hat{S}$ (line 5).

The redundancy operator has a time complexity equal to $O(m^2 n)$ in the worst case. More in detail, $\hat{S}$ is built by checking for each active sensor if the counter of all its covered targets is strictly greater than 1; in this case, the sensor is added to $\hat{S}$. The $\hat{S}$ set is built in at most $O(mn)$ operations. The loop is executed at most $O(m)$ times. Line 3 has a constant time complexity for the selection of a random sensor. The operation referring to line 4 has $O(n)$ complexity since the procedure has to update all the counters related to its covered targets. Finally, re-evaluating the

$\hat{S}$ set by looking for new possible non-redundant sensors has again $O(mn)$ complexity in the worst case.

Algorithm 2: Redundancy operator

Input: feasible *Child* chromosome;
Output: feasible non-redundant *Child* chromosome;

```
1 Ŝ ← redudantSensorsList(Child);
2 while isEmpty(Ŝ) = false do
3     s ← randomSelect(Ŝ);
4     Child ← switch(s);
5     Ŝ ← redudantSensorsList(Child);
6 return Child;
```

4.5. Building the Initial Population

Each individual belonging to the initial population P is randomly built by applying in sequence the feasibility and redundancy operators, starting from a chromosome whose genes are all set to zero.

As soon as a feasible chromosome is obtained, it is added to the population if it is not already contained in it and is rejected otherwise, until a fixed desired number $Size_P$ of different chromosomes is obtained. Finally, the population is sorted, in ascending order, according to the fitness values of the chromosomes.

In order to avoid the procedure to iterate indefinitely, a *maxinitDB* threshold is taken into account. If the number of rejected chromosomes reaches the threshold, the procedure stops and $Size_P$ is updated to be equal to the current value of $|P|$.

4.6. GA Structure and GCG initialization

This section describes the overall structure of GA. The pseudocode is listed in Algorithm 3. The input consists of a wireless sensor network (S, T), where S is the set of sensors and T is the set of targets, as well as a vector of dual prices *DP* coming from the last iteration of the current restricted master problem. The GA first generates a starting population P of feasible solutions and identifies the initial best chromosome through the evaluation of the best initial fitness value, named $BestFit$. During the evolutionary process, $BestFit$ stores the value of the *incumbent* solution and is used as a comparison parameter throughout the procedure. The population has a fixed size, named $Size_P$, throughout the algorithm execution, and it is initialized as described in Section 4.5. The genetic algorithm builds iteratively new chromosomes one by one, by executing the steps reported in Sections 4.2-4.4. The new child produced at each iteration is inserted in current population P only if it does not already belong to it, and in this case it replaces an individual which is chosen randomly among the $|P/2|$ individuals with the worst fitness values.

The procedure iterates until one of two stopping criteria is reached. The first criterion is based on a *MaxIT* parameter, representing the maximum number of iterations without improvements with respect to the $BestFit$ value, and the second one is the maximum number of consecutive duplicate chromosomes, named *MaxDB*.

Algorithm 3: Genetic Algorithm

Input: $(S, T), DP$;

Output: a subset of chromosomes(i.e. columns) for the MasterProblem;

```
1  P ← InitP();
2  BestFit ← bestFitness(P, DP);
3  criteria ← setCriterion(MaxIT, MaxDB);
4  while check(criteria) do
5      (p1, p2) ← tournament(P);
6      C ← Crossover(p1, p2);
7      C ← Mutation(C);
8      C ← feasibilityOperator(C);
9      C ← redundancyOperator(C);
10     if C ∉ Pop then
11         Insert(C, P);
12         if fitness(C) ≥ BestFit then
13             update(criteria);
14         else
15             BestFit ← fitness(C);
16     else
17         update(criteria);
18 Chromos ← chromosomes with fitness ≤ 1;
19 return Chromos;
```

The chromosomes in the final population P whose fitness value is less than 1 are then introduced in the master problem as new columns.

The GA algorithm was also used in our tests to provide the initial set of columns which is required by the first step of the master problem. In this case, however, the vector of dual prices which is used to evaluate the chromosomes is not available. For this reason, in this first iteration a random positive value is used as dual price for each sensor. The whole set of $Size_P$ individuals is returned to the master problem in this case.

Given the complexity of the single operators, it follows that the generation of each chromosome has a complexity equal to at most $O(m^2 n + n^2)$. However, given our stop condition based on the number of iterations without improvements with respect to $BestFit$, as well as the exponential size of the search space, it is not possible to determine a polynomial bound on the maximum number of iterations in the worst case. As well known, this is a common issue for metaheuristics, including simple local search schemes [19], and could be overcome by considering an additional stopping criterion related to a maximum global number of iterations. However, as will be shown in the next section, both GA and the GCG algorithm, in which it is embedded, have very good performances in practice, converging in very reasonable amounts of time.

Name	Role	Value(s)
	GA parameters	
$Size_P$	Population size	50
$maxinitDB$	Maximum number of rejected chromosomes during initialization	100
$MaxDB$	Maximum number of consecutive duplicate chromosomes	100
$MaxIT$	Maximum number of iterations without improvements for the incumbent optimum	2000
	Parameters of the [12] instances (MLP)	
m	Number of sensors	50, 100, 150, 200
n	Number of targets	30, 60, 90, 120
	Parameters of the [13] instances, Group 1 (α-MLP)	
m	Number of sensors	25, 50, 100, 150
n	Number of targets	15
T_α	Number of targets to be covered	8, 11, 13, 15
	Parameters of the [13] instances, Group 2 (α-MLP)	
Inst. Subgroup	Sensors placement type	Design, Scattering
n	Number of targets	100
T_α	Number of targets to be covered	100, 99, 97, 95, 93, 85, 75, 50

Table 1: Values of the parameters in the computational tests

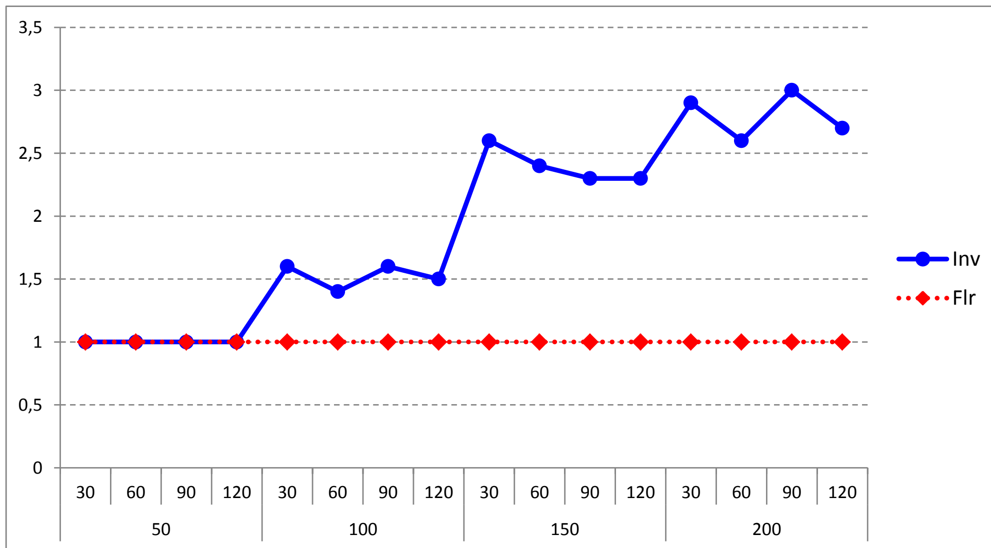


Figure 4: Number of invocations and failures of GA within the GCG algorithm on the on the benchmark instances proposed in [12]

5. Computational Results

The purpose of the computational experience presented in this section is to study the performance of our algorithm GCG with respect to column generation approaches proposed in the literature by [12] for the case $\alpha = 1$ and by [13] for the general case. The algorithm proposed in [13] will be called GR from now on. The computational tests are carried out on the same set of instances used in these two papers. Our algorithm was coded in C++ on a (SUSE) Linux platform running on a Intel Core2 Duo 2.4GHz processor with 4GB RAM (single thread mode). Mathematical formulations within the GCG framework were solved using the Concert library of IBM ILOG CPLEX 12.5.

We ran a preliminary tuning test phase to determine the values used in the tests for the GA parameters. Their chosen values can be found in Table 1.

Let us start our comparison from the benchmark instances proposed in [12]. In Table 2 the results of GCG are reported. Each line in the table represents a *scenario* composed of 10 instances with the same characteristics but different topologies. Therefore, the results reported in each line

m	n	Lifetime	Time	Inv	Col	Flr
50	30	3.80	0.21	1.0	0.0	1.0
	60	3.00	0.31	1.0	0.0	1.0
	90	2.80	0.40	1.0	0.0	1.0
	120	2.70	0.51	1.0	0.0	1.0
100	30	8.70	0.44	1.6	10.1	1.0
	60	7.20	0.65	1.4	6.1	1.0
	90	6.90	1.11	1.6	8.5	1.0
	120	6.70	1.57	1.5	7.4	1.0
150	30	14.70	0.80	2.6	20.4	1.0
	60	12.30	1.41	2.4	18.8	1.0
	90	11.80	2.40	2.3	19.6	1.0
	120	11.30	3.38	2.3	19.9	1.0
200	30	19.60	1.24	2.9	24.4	1.0
	60	17.30	2.39	2.6	23.2	1.0
	90	16.60	4.10	3.0	24.5	1.0
	120	15.50	5.14	2.7	24.4	1.0
Avg				**1.93**	**12.96**	**1.0**

Table 2: Results obtained by the GCG algorithm on the benchmark instances proposed in [12]. Lifetime values expressed in time units, time values expressed in seconds.

are the average values on these 10 instances. Each scenario corresponds to a different combination of values for the number of sensors and targets, chosen among the possible values reported in Table 1; for a detailed description of these scenarios, see [12]. The first two columns report the number of sensors and targets in the scenarios. The columns $Lifetime$ and $Time$ report the solution values measured in time units and the CPU times in seconds, respectively. The last three columns Inv, Col and Flr report how many times the genetic algorithm is invoked by the restricted master problem after the initialization phase, the average number of columns (i.e. attractive covers) returned by the genetic algorithm at each invocation (again excluding the starting one), and how many times the genetic algorithm returns zero columns (i.e. the number of failures), respectively. Finally, the last line of the table reports the average values of the last three columns. The values of the Time column show that GCG is very fast on the considered instances, with a running time that is always lower than 6 seconds on all the scenarios. A more accurate analysis of the GCG performance will be carried on while discussing the results reported in the Table 3. However, let us first analyze the impact of the genetic algorithm within the column generation approach. To this end, we focus on the values reported in the last three columns of Table 2. To better show the trend of the number of GA invocations and failures, these values are also plotted in Figure 4. The x-axis reports the

instance characteristics, while the Inv and Flr values are reported on the y-axis. The invocations trend and the failures trend are represented by the continuous and the dotted line, respectively. The values of Inv show that the genetic algorithm is invoked very few times, with an overall average equal to 1.93. In particular, on the scenarios with 50 sensors, it is invoked just once. This means that the starting columns, provided by genetic algorithm during the initialization phase, already contain the columns of the optimal solution. Indeed, a single invocation after the initialization means that the GA failed and the exact subproblem certified that an optimal solution was already reached, otherwise GA would have been invoked again in the following iteration. On the other scenarios, the average number of invocations slowly increases up to 3 (in the scenario with 200 sensors and 90 targets). It can be noted that, for any chosen value of the number of sensors, the number of targets have little influence on the required number of invocations. For instance, for $m = 100$ the difference between the minimum and maximum number of invocations, as n varies, is equal to 0.2. This difference increases to 0.3 for $m = 150$, and to 0.4 for $m = 200$. In general, for a given m value, higher Inv values can be observed on instances with the lowest number of targets, i.e. $n = 30$. This can be explained by the fact that on these instances a lower number of sensors is likely to be required in feasible covers, and therefore a higher number of such covers may exist; this is also confirmed by the fact that these scenarios have higher lifetime solution values.

The GA is however always invoked very few times, since on all scenarios it returns a significant number of attractive covers. On average, the number of attractive covers found in each GA iteration is equal to 12.96, with a peak of 24.5, which brings the columns needed to reach an optimal solution to be quickly added to the master problem. In particular, on the largest instances with 200 sensors the average number of returned columns is above 24, that is, almost 50% of the chromosomes in the final population are attractive covers for the restricted master problem.

The most interesting results are, however, those related to the number of failures, which measure the effectiveness of the genetic algorithm. Remarkably, on all the scenarios provided by [12] the number of GA failures is equal to 1, meaning that we need to solve the exact subproblem only once for each instance, in order to certify the optimality of the current incumbent solution.

In order to verify the competitiveness of our approach with respect to those proposed in the literature, the computational times of GCG and those of the Exact, Heur and Mixed algorithms described in [12] are reported in Table 3.

The first three columns show the characteristics of the scenarios, as already mentioned for Table 2. The subsequent four columns report the computational time required by the four algorithms. The last three columns report the percentage CPU time gap among GCG and the other three algorithms. Each gap is computed as $100 \times (Alg - \text{GCG})/Alg$, where $Alg \in \{\text{Exact, Heur, Mixed}\}$ and GCG, Alg refer to the computational time of the related procedure. Finally, the last line of the table reports the average values for the last seven columns. Note that when the CPU time gap between two algorithms is lower than 1 second, we do not report the percentage gap because we consider it to be negligible.

As previously mentioned, the results of the Time column for GCG show that it is able to find the optimal solution in less than 6 seconds on average, regardless of the considered scenario. Therefore, the increment in terms of CPU time, as the size of the considered scenario grows, is bounded to few seconds. The situation appears to be completely different for the other three algorithms. Indeed, they are much slower, and their computational times are significantly affected by the scenarios characteristics. In more detail, by checking the overall average values reported in the last line, it is clear that GCG is faster than Exact by three orders of magnitude, with a gap that is always

m	n	Lifetime	Time				GCG GAP		
			Exact	Heur	Mixed	GCG	vs Exact	vs Heur	vs Mixed
50	30	3.80	0.25	0.30	0.12	0.21			
	60	3.00	1.03	0.53	0.52	0.31			
	90	2.80	2.95	0.82	1.55	0.40	86.42%		74.15%
	120	2.70	8.40	1.20	4.03	0.51	93.87%		87.22%
100	30	8.70	3.29	2.97	1.03	0.44	86.75%	85.32%	
	60	7.20	26.53	4.25	8.41	0.65	97.55%	84.71%	92.28%
	90	6.90	243.95	6.82	74.19	1.11	99.55%	83.77%	98.51%
	120	6.70	749.46	9.70	220.64	1.57	99.79%	83.79%	99.29%
150	30	14.70	17.17	14.51	4.94	0.80	95.37%	94.52%	83.89%
	60	12.30	315.66	22.21	48.96	1.41	99.55%	93.65%	97.12%
	90	11.80	2365.65	30.61	525.21	2.40	99.90%	92.17%	99.54%
	120	11.30	9249.81	48.15	1987.04	3.38	99.96%	92.98%	99.83%
200	30	19.60	38.80	34.85	9.50	1.24	96.80%	96.44%	86.93%
	60	17.30	750.40	56.34	126.39	2.39	99.68%	95.75%	98.11%
	90	16.60	8229.53	132.46	1297.82	4.10	99.95%	96.91%	99.68%
	120	15.50	28942.49	105.87	4393.04	5.14	99.98%	95.15%	99.88%
AVG			**3184.09**	**29.47**	**543.96**	**1.63**	**96.79%**	**91.26%**	**93.57%**

Table 3: Comparison of GCG, Exact, Heur and Mixed algorithms on the benchmark instances proposed in [12]. Lifetime values expressed in time units, time values expressed in seconds.

greater than 86%. For the scenario containing the largest instances (that is, 200 sensors and 120 targets) the Exact algorithm spends more than 8 hours to find the optimal solution, while GCG requires less than 6 seconds. The Mixed algorithm results faster than the Exact algorithm; however, when compared to GCG it appears to be slower by two orders of magnitude. Furthermore, the performance gap between these two algorithm is always greater than 83%. Finally, it is remarkable to note that GCG results to be about 20 times faster than the heuristic approach Heur, with a percentage gap which is always greater than 74%.

It has to be highlighted that this comparison cannot be completely accurate since the algorithms proposed in [12] were run on a different hardware and the mathematical models were solved using GLPK. However, since the running time gap can be quantified in orders of magnitude, we believe that the comparison still provides solid evidence about the effectiveness of our approach.

On the one hand, the results obtained by GCG confirm our expectations on the effectiveness and efficiency of our GA algorithm and, on the other hand, they prove that a column generation approach, paired with a fast and effective method to generate new columns, results to be a very suitable approach for lifetime problems on sensor networks.

We now present the results of GCG when used to solve the Group 2 set of benchmark instances proposed in [13] for the α-coverage problem. This is the hardest set of instances proposed in this work, and therefore we considered these results to be more relevant and interesting. Nevertheless, we also tested our approach on the Group 1 dataset, and the related tables are contained in the Appendix. As will be shown, GCG performs well on all these instances as well.

The Group 2 instances contain 100 targets, while the number of sensors is not fixed *a priori*, but is rather computed by making sure that each target is covered by at least 3 sensors. The instances are further divided in two subgroups composed of 30 instances each, named *Scattering*

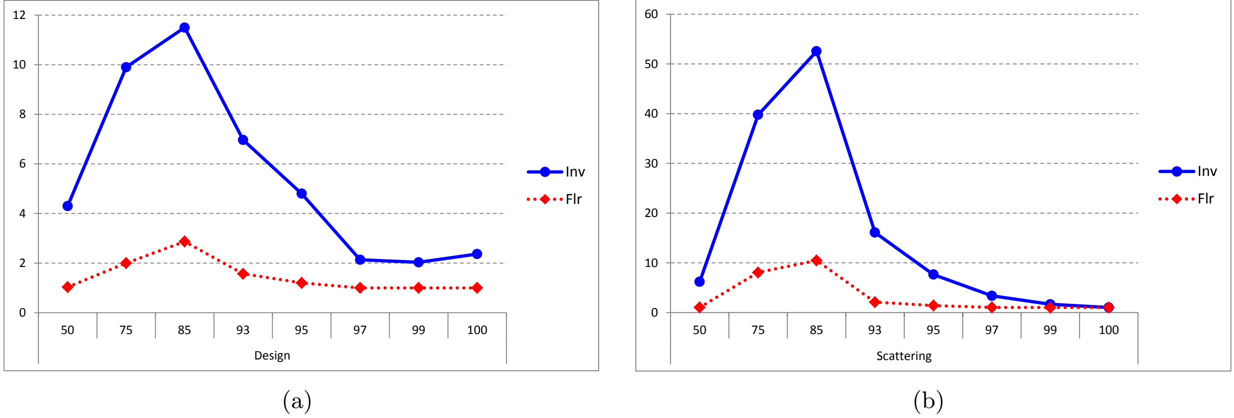
Inst. Subgroup	n	T_α	Lifetime	Time	Inv	Col	Flr
Design	100	50	8.32	0.41	4.30	19.20	1.03
		75	5.42	0.77	9.90	13.63	2.00
		85	4.50	0.90	11.50	12.27	2.87
		93	3.65	0.52	6.97	14.03	1.57
		95	3.34	0.39	4.80	11.87	1.20
		97	3.04	0.26	2.13	7.43	1.00
		99	3.00	0.27	2.03	11.90	1.00
		100	3.00	0.29	2.37	13.63	1.00
Scattering	100	50	20.50	1.19	6.20	23.30	1.03
		75	13.36	9.14	39.77	10.03	8.07
		85	10.57	9.12	52.57	8.13	10.47
		93	7.73	2.35	16.10	17.77	2.10
		95	6.64	1.22	7.63	20.27	1.40
		97	5.37	0.74	3.37	17.00	1.03
		99	3.83	0.56	1.67	7.17	1.00
		100	3.00	0.48	1.00	0.00	1.00
Avg					**10.77**	**12.98**	**2.36**

Table 4: Results obtained by the GCG algorithm on the Group 2 benchmark instances proposed in [13]. Lifetime values expressed in time units, time values expressed in seconds.

and *Design*, respectively. In the Scattering group, sensors are randomly added until the desired coverage level is reached, while in the Design group, sensors are added only when needed to reach it. For the T_α parameter, 8 different values ranging from 50 to 100 have been chosen in [13]. The parameters of these instances are resumed in Table 1; for a more detailed description, see [13].

In Table 4, the results of GCG on the Scattering and Design subgroups are reported. Each line in the table contains average values for the 30 instances of the related scenario. The first two columns specify the subgroup and the number of targets. The column T_α reports the number of targets that must be covered, while the columns Lifetime and Time contain the solution value in time units and the CPU time in seconds, respectively. Finally, the last 3 columns report for GA the same information which we already mentioned regarding Table 2, and the last line contain average values for these columns. In Figure 5 we graphically represent the number of GA invocations and failures, separately for the Design and Scattering scenarios. On the x-axis, the T_α values are reported for these instances.

The results show that the number of invocations is on average 10.77, while the average number of returned columns is 12.98 and the number of average failures is 2.36. In more detail, on the *Design* scenarios a noticeable peak of GA invocations equal to 11.50 can be noticed for the case $T_\alpha = 85$, which also corresponds to the peak of failures, equal to 2.87. The average number of columns returned for each iteration is greater than 10 in all cases except one, in the case $T_\alpha = 97$. The *Scattering* instances result to be harder to solve, with a peak of GA invocations and failures corresponding to 52.57 and 10.47, respectively (again in the case $T_\alpha = 85$). This can be explained considering the additional number of sensors, and therefore the higher amount of feasible covers



(a) (b)

Figure 5: Number of invocations and failures of GA within the GCG algorithm on the Group 2 scenarios (Design on the left, Scattering on the right) proposed in [13]

which exists in such instances. The average number of invocations quickly decreases for scenarios in which T_α increases, and the problem therefore approaches the classical MLP. Indeed, it varies between 2.13 and 2.37 for the Design instances and between 1 and 3.37 for the Scattering ones when $T_\alpha \geq 97$. Accordingly, GA only fails once for each instance with $T_\alpha \geq 97$ for the Design subgroup and with $T_\alpha \geq 99$ for the Scattering one.

While these results may appear to be less impressive than the ones presented in Table 2 and Figure 4, the values in the Time column show that GCG is still very fast. Indeed, the algorithm finds the optimal solution in less than 1 second on average in all scenarios for the Design instances, and always in less than 10 seconds on average for the Scattering ones.

In Table 5, a performance comparison between the GCG and GR algorithms is performed. As mentioned above, we do not evaluate gaps when both procedures report a computational time which is below 1 second. On the Design scenarios, the GR algorithm is able to find all the optimal solutions within its considered 1-hour time limit. However, it is clear that GCG is generally much faster, with a percentage gap that is greater than 81% on the first 6 scenarios and a CPU time that is always lower than a second. More interesting are the results for the Scattering scenarios, where some instances are not solved within the time limit by the GR algorithm. Indeed, it reaches the time limit for 2 instances of the scenario with $T_\alpha = 85$ and 3 instances of the scenario with $T_\alpha = 93$. The solution values of these scenarios are marked into the table with the symbols "**" and "*" to highlight that these values are averages evaluated only on the subset of instances which were solved to completion. All the instances of these two scenarios are instead solved to optimality by GCG, with average computational times equal to 9.12 and 2.35 seconds, respectively.

The values in the GAP column show that GCG is at least 63% faster than GR, with a peak equal to 96.99% and an average equal to 88.08%. The values reported in the last line show that GCG is faster than GR by an order of magnitude, having an average CPU time that is lower than 2 seconds, while the one of GR is equal to 38.65 seconds. These results certify that GCG is the fastest and most effective algorithm, being able to solve all scenarios within 10 seconds on average.

Inst. Subgroup	T_α	GR		GCG		GAP
		Lifetime	Time	Lifetime	Time	
Design	50	8.32	3.20	8.32	0.41	87.28%
	75	5.42	13.94	5.42	0.77	94.46%
	85	4.50	11.46	4.50	0.90	92.15%
	93	3.65	7.03	3.65	0.52	92.56%
	95	3.34	2.68	3.34	0.39	85.41%
	97	3.04	1.43	3.04	0.26	81.61%
	99	3.00	0.59	3.00	0.27	
	100	3.00	0.21	3.00	0.29	
Scattering	50	20.50	11.13	20.50	1.19	89.30%
	75	13.36	216.98	13.36	9.14	95.79%
	85	10.56**	302.91	10.57	9.12	96.99%
	93	7.38*	36.18	7.73	2.35	93.50%
	95	6.64	8.02	6.64	1.22	84.78%
	97	5.37	2.01	5.37	0.74	63.15%
	99	3.83	0.56	3.83	0.56	
	100	3.00	0.05	3.00	0.48	
AVG			**38.65**		**1.79**	**88.08%**

Table 5: Comparison of GCG and GR algorithms on the Group 2 benchmark instances proposed in [13]. Lifetime values expressed in time units, time values expressed in seconds.

6. Conclusion

In this work we addressed the maximum lifetime problem on wireless sensor networks, and more in particular we considered two variants in which either all sensors have to be covered, or a portion of them can be neglected at all times in order to increase the overall network lifetime. We presented an efficient genetic algorithm aimed at producing new covers, which can be embedded within a column generation framework. The obtained algorithm is shown to be highly efficient in terms of requested computational time, and to perform significantly better than the ones proposed in the literature.

Further research will involve the study of more complex problem variants, able to model aspects such as sensor-to-sensor communication.

Acknowledgements

The authors wish to thank K. Deschinkel, who provided the set of benchmark instances proposed in [12].

References

[1] H. Alemdar and C. Ersoy. Wireless sensor networks for healthcare: a survey. *Computer Networks*, 54(15):2688–2710, 2010.

[2] A. Alfieri, A. Bianco, P. Brandimarte, and C. F. Chiasserini. Maximizing system lifetime in wireless sensor networks. *European Journal of Operational Research*, 181(1):390–402, 2007.

[3] C. Blum, M. J. Blesa Aguilera, A. Roli, and M. Sampels, editors. *Hybrid Metaheuristics - An Emerging Approach to Optimization*, volume 114 of *Studies in Computational Intelligence*. Springer-Verlag, Berlin/Heidelberg, 2008.

[4] M. Cardei, M. T. Thai, Y. Li, and W. Wu. Energy-efficient target coverage in wireless sensor networks. In *Proceedings of the 24th conference of the IEEE Communications Society*, volume 3, pages 1976–1984, 2005.

[5] M. Cardei, J. Wu, and M. Lu. Improving network lifetime using sensors with adjustable sensing ranges. *International Journal of Sensor Networks*, 1(1-2):41–49, 2006.

[6] F. Carrabs, R. Cerulli, C. D'Ambrosio, M. Gentili, and A. Raiconi. Maximizing lifetime in wireless sensor networks with multiple sensor families. *Computers & Operations Research*, 60:121–137, 2015.

[7] F. Castaño, E. Bourreau, N. Velasco, A. Rossi, and M. Sevaux. Exact approaches for lifetime maximization in connectivity constrained wireless multi-role sensor networks. *European Journal of Operational Research*, 241(1):28–38, 2015.

[8] F. Castaño, A. Rossi, M. Sevaux, and N. Velasco. A column generation approach to extend lifetime in wireless sensor networks with coverage and connectivity constraints. *Computers & Operations Research*, 52(B):220–230, 2014.

[9] R. Cerulli, R. De Donato, and A. Raiconi. Exact and heuristic methods to maximize network lifetime in wireless sensor networks with adjustable sensing ranges. *European Journal of Operational Research*, 220(1):58–66, 2012.

[10] R. Cerulli, M. Gentili, and A. Raiconi. Maximizing lifetime and handling reliability in wireless sensor networks. *Networks*, 64(4):321–338, 2014.

[11] L. Davis, editor. *Handbook of Genetic Algorithms*. Van Nostrand Reinhold, New York, 1991.

[12] K. Deschinkel. A column generation based heuristic for maximum lifetime coverage in wireless sensor networks. In *SENSORCOMM 11, 5th Int. Conf. on Sensor Technologies and Applications*, volume 4, pages 209 – 214, 2011.

[13] M. Gentili and A. Raiconi. $\alpha-$coverage to extend network lifetime on wireless sensor networks. *Optimization Letters*, 7(1):157–172, 2013.

[14] M. Pejanovic Durisic, Z. Tafa, G. Dimic, and V. Milutinovic. A survey of military applications of wireless sensor networks. In *Proceedings of the Mediterranean Conference on Embedded Computing*, pages 196–199, 2012.

[15] A. Raiconi and M. Gentili. Exact and metaheuristic approaches to extend lifetime and maintain connectivity in wireless sensors networks. In J. Pahl, T. Reiners, and S. Voss, editors, *Network Optimization*, volume 6701 of *Lecture Notes in Computer Science*, pages 607–619. Springer, Berlin/Heidelberg, 2011.

[16] P. Rawat, K. D. Singh, H. Chaouchi, and J. M. Bonnin. Wireless sensor networks: a survey on recent developments and potential synergies. *The Journal of Supercomputing*, 68(1):1–48, 2014.

[17] A. Rossi, A. Singh, and M. Sevaux. An exact approach for maximizing the lifetime of sensor networks with adjustable sensing ranges. *Computers & Operations Research*, 39(12):3166–3176, 2012.

[18] A. Rossi, A. Singh, and M. Sevaux. Lifetime maximization in wireless directional sensor network. *European Journal of Operational Research*, 231(1):229–241, 2013.

[19] E-G. Talbi. *Metaheuristics: from design to implementation*. Wiley, 2009.

[20] Q. Zhao and M. Gurusamy. Lifetime maximization for connected target coverage in wireless sensor networks. *IEEE/ACM Transactions on Networking*, 16(6):1378–1391, 2008.

Appendix

Tables 6 and 7 contain the results related to the Group 1 instances proposed in [13]. Each instance in this group contains 15 targets, while the number of sensors for the different scenarios is specified under the m heading in the tables. The T_α parameter varies between 4 different values ranging from 8 to 15. The parameters are also reported in Table 1. Each line in the tables contain averages over 5 different instances with the same characteristics. For a description of the table headings, refer to the description of Tables 4 and 5 in Section 5.

m	T_α	**Lifetime**	**Time**	**Inv**	**Col**	**Flr**
25	8	13.60	0.29	3.00	11.80	1.00
	11	10.40	0.26	3.40	16.40	1.00
	13	6.60	0.19	2.80	17.00	1.00
	15	3.60	0.19	2.00	19.60	1.00
50	8	27.23	0.59	4.60	19.40	1.00
	11	19.40	0.40	4.60	18.80	1.00
	13	13.93	0.33	3.60	21.00	1.00
	15	9.40	0.26	2.40	15.20	1.00
100	8	54.90	1.27	9.20	21.00	1.00
	11	41.49	1.25	11.00	23.20	1.20
	13	30.40	0.87	7.00	26.60	1.00
	15	15.40	0.52	3.00	21.80	1.00
150	8	87.60	2.39	11.80	22.00	1.00
	11	66.98	2.40	15.40	22.80	1.40
	13	51.72	1.97	12.20	27.60	1.00
	15	25.00	0.89	4.00	24.60	1.00
AVG				**6.25**	**20.55**	**1.04**

Table 6: Results obtained by the GCG algorithm on the Group 1 benchmark instances proposed in [13]. Lifetime values expressed in time units, time values expressed in seconds.

Sensors	T_α	**GR**		**GCG**		**GAP**
		Lifetime	**Time**	**Lifetime**	**Time**	
25	8	13.60	0.26	13.60	0.29	
	11	10.40	0.44	10.40	0.26	
	13	6.60	0.11	6.60	0.19	
	15	3.60	0.01	3.60	0.19	
50	8	27.23	1.11	27.23	0.59	
	11	19.40	0.68	19.40	0.40	
	13	13.93	0.39	13.93	0.33	
	15	9.40	0.01	9.40	0.26	
100	8	54.90	5.95	54.90	1.27	78.72%
	11	41.49	8.03	41.49	1.25	84.39%
	13	30.40	2.74	30.40	0.87	68.42%
	15	15.40	0.02	15.40	0.52	
150	8	87.60	15.24	87.60	2.39	84.31%
	11	66.98	13.90	66.98	2.40	82.74%
	13	51.72	9.79	51.72	1.97	79.86%
	15	25.00	0.02	25.00	0.89	
AVG			**3.67**		**0.88**	**79.74%**

Table 7: Comparison of GCG and GR algorithms on the Group 1 benchmark instances proposed in [13]. Lifetime values expressed in time units, time values expressed in seconds.