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Article

Modeling Robotic Thinking and Creativity: A Classic–Quantum Dialogue

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Abstract: The human mind can be thought of as a black box, where the external inputs are elaborated in an unknown way and lead to external outputs. D'Ariano and Faggin schematized thinking and consciousness through quantum state dynamics. The complexity of mental states can be formalized through the entanglement of the so-called *qualia states*. Thus, the interaction between the mind and the external world can be formalized as an interplay between classical and quantum-state dynamics. Since quantum computing is more and more often being applied to robots, and robots constitute a benchmark to test schematic models of behavior, we propose a case study with a robotic dance, where the thinking and moving mechanisms are modeled according to quantum–classic decision making. In our research, to model the elaboration of multi-sensory stimuli and the following decision making in terms of movement response, we adopt the D'Ariano–Faggin formalism and propose a case study with improvised dance based on a collection of poses, whose combination is presented in response to external and periodic multi-sensory stimuli. We model the dancer's inner state and reaction to classic stimuli through a quantum circuit. We present our preliminary results, discussing further lines of development.

Keywords: quantum computing; qualia; robotic dancer; quantum–classic dynamics

MSC: 81-04; 81V99; 94D99

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1. Introduction

Understanding nature has fascinated and challenged people throughout the whole of history. Understanding the human mind is even more challenging, because the object of study coincides with the tool used to study it, and the reality of thoughts, emotions, abstraction, and creativity overcomes the physical reality of neural structures. Nevertheless, we can try to build theoretical and computational models to imitate some characteristic behaviors and reactions to external stimuli given as inputs. These models lead to gestures and actions as outputs back to the external world. Thus, even if we do not exactly know what is going on in the human mind (as we do not really know for non-human animals), we can build models of the interaction of the mind with the external world. Several studies on quantum computing applied to robotics are appearing [1–4], including references to swarm robotics [5].

In our research, we are interested in building robotic models of human behavior, whose decision-making systems are based on a quantum–classical approach.

D'Ariano and Faggin [6] distinguished between the actual mental state of a person (who knows what they are thinking) and their probable mind-state configuration as predicted by another person. The first state is called *ontic*, while the second one is called *epistemic*. The latter is related to the (external) knowledge of the problem. D'Ariano–Faggin refer to the concept of *qualia*, a finite set of ontic states, whose superposition leads to complex mental configurations. The idea of state superposition is borrowed from quantum mechanics. According to their view, the qualia can be considered as quantum states, representing vectors in Hilbert space. The combination of qualia gives more complex qualia. According to this approach, thoughts and emotions are combinations of qualia, and the evolution of qualia over time can be modeled via quantum time operators. D'Ariano–Faggin propose a quantum formalism, where the operators are organized in monoidal categories from category theory [7]. A *monoidal category* A, also called *tensor category*, is a category equipped with a bifunctor $\otimes : A \times A \rightarrow A$. Associativity and right and left identity properties are verified.

In our research, we adopt their formalism, extending it to an interplay between external, periodic, and multi-sensory classic stimuli as inputs, quantum-mind response, and classic actions as outputs. We model the mental elaboration of the stimuli through a quantum circuit. Even if we can establish the basic responses to given simple, not-composed stimuli, the overall mental response to complex stimuli is not precisely predictable. Thus, the inherently probabilistic nature of the epistemology of quantum mechanics comes in handy.

As a benchmark of these quantum-based cognitive approaches, robots can be used. In particular, the model of qualia can be applied to the modeling of robotic responses to given external stimuli, also multi-sensory ones. These responses are elaborated inside the robot, and thus, we will refer to them as *inner-robotic* responses. However, we do not limit our analysis to the modeling of inner-robotic responses to an external multi-sensory stimulus. Since we treat the robot as an inner quantum system interacting with the environment, it is an open quantum system. Thus, this research may be related with open quantum systems [8–12]. We also model the action of the robot on the environment, the response of the environment, and the reaction of the robot to such a response. The back-and-forth information exchange between the robot and the external environment is seen as a dialogue between classical physics and quantum physics. Quantum physics is used as a paradigm to model the inner activity of the robot, reacting to the external stimuli and preparing its actions on the environment. The actions happen in the external world, the environment, and they are formalized via classic operators. The response of the environment is also classical. In the form of a new set of multi-stimuli, they enter the inner world of the robot, and thus, they are "received" as quantum stimuli. If the robot is receiving stimuli from other robots, their information is comprised inside the classic reality. Each other robot is in fact seen as part of the (classic) environment because our robot can only observe the external exchange of stimuli and actions. Each robot can be considered as a *monad* in the sense of Leibniz (the term "monad" is also used in category theory), constituting a closed, inaccessible reality. The "inner working consciousness" of each robot can be modeled via a quantum circuit, inspired by the quantum approach with qualia, and the language used by Faggin [6]. Each robot is seen as a black box by the other robots: they can only observe their actions. The same happens with living beings: we can infer what a person is thinking according to their action in response to stimuli, but we cannot access their inner thinking. Figure 1 provides a visual synthesis of the proposed idea.

The development of a simple model for human thinking and interaction with the environment can be tested on a robot. To perform a test, we choose the scenario of a dancing robot [13]. The inner-robotic response to external stimuli can lead, through a decision-making system and physical action, to a response in the external world. From the external point of view, the robot can perform actions in response to the stimuli. The set of possible actions can be chosen to create a *choreography*. Because of quantum superpositions of inner states, their intrinsic probabilistic nature, the result of measures can also lead to a combination of dance positions Figures 2 and 3). Thus, we can define the resulting movements of the robots as an *improvised dance*. In our study, the reference to a specific framework, that is, an improvised choreography based on multi-sensory external stimuli, constitutes a toy model of a more general approach toward the formalism of quantum mind/classic context interaction.

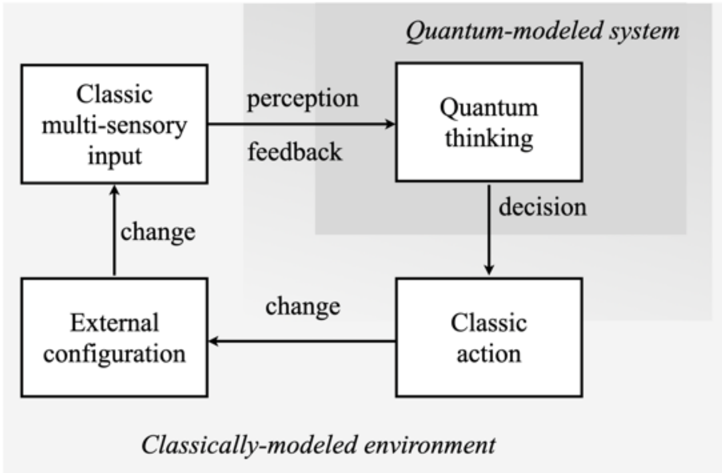

Figure 1. A schematic representation of classic–quantum information exchange.

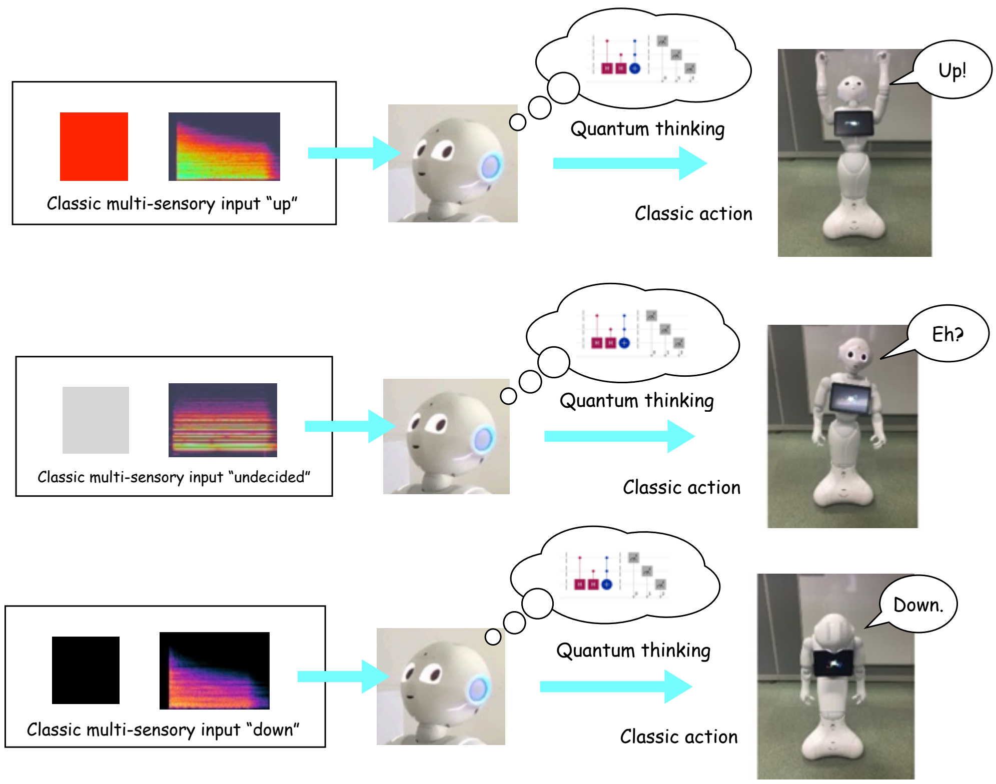

Figure 2. A pictorial representation of classic incoming multisensory stimuli, quantum thinking of the robot, and classical action as the response. The spectrograms refers to audio stimuli used in a color-timbre experiment [14], and the "response" poses of the robot Pepper are taken from [15]. The stimulus *red* is associated with a *forte* of orchestra with brass in evidence; the stimulus *grey* is associated with a softer orchestration with *pianissimo* winds and violins, and stimulus *black* is associated with a low-register *fortissimo* piano cluster.

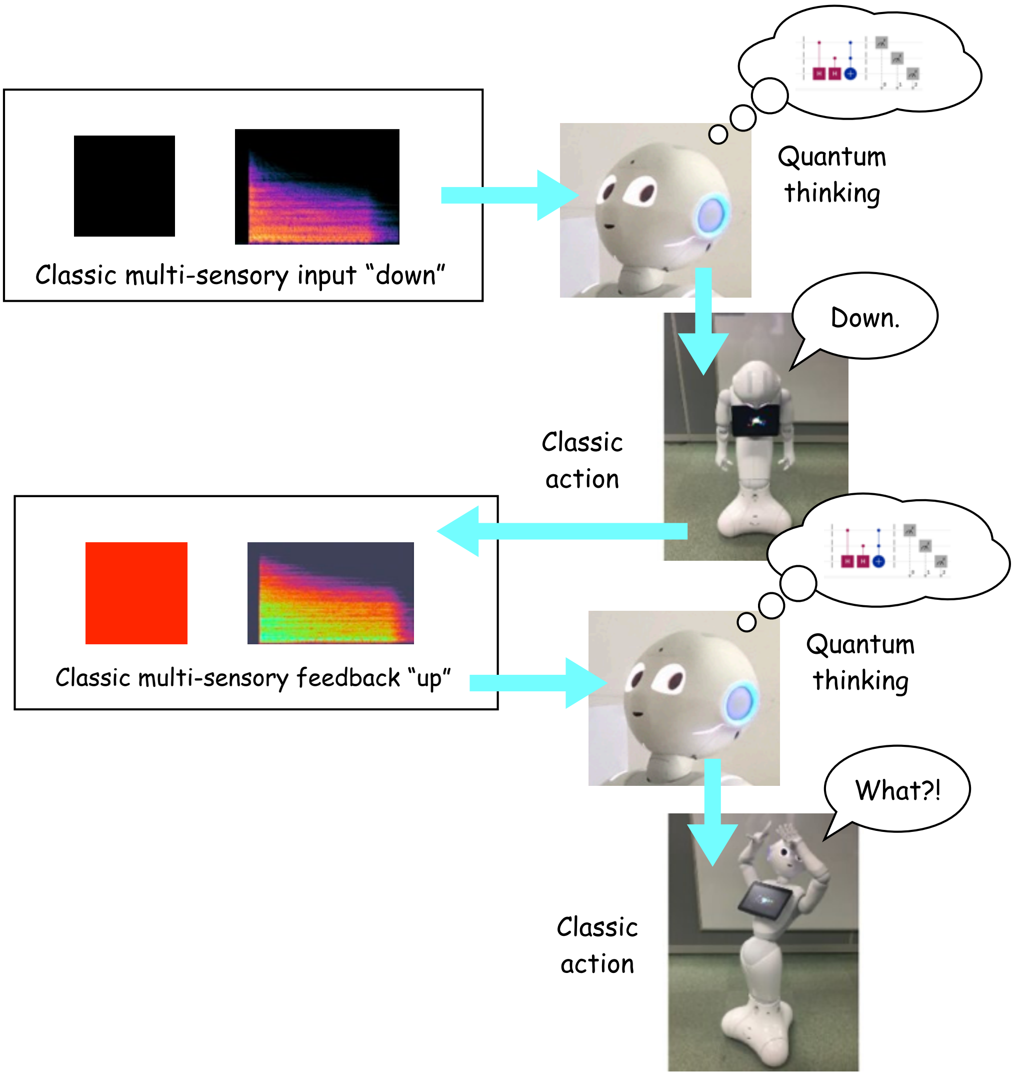

Figure 3. A pictorial representation of the complete cycle of our proposed model, a first external multisensory stimuli, a response from the robot after a step of quantum thinking, feedback from the external environment (which can be independent from the robotic action), and another sequence of quantum thinking and classic action from the robot, which becomes less and less predictable accordingly and more and more mixed. The pictures of the expressive poses of the robot Pepper are taken from [15].

This article is organized as follows. In Section 2, we summarize key ideas of quantum computing applied to robotics, category theory, computational creativity, and robotic modeling of consciousness, the bases of our work. In our research, we shape the multisensory stimuli in terms of color-timbre perceptual associations; in this regard, we provide some some references to their theoretical investigation. In Section 3, we present our first case study, and in Section 4, the second one. In Section 5, we summarize our results and discuss possible research developments.

Here, we propose a hybrid classical–quantum system to model multiple inputs and operate accordingly.

2. Background and Literature Review

The contribution of the present work is to model the mental process and the processing that leads to the decision to perform a particular action. Processing a response to stimuli is based on the ability to understand and interact with the environment. This ability is present in both humans and artificial agents and is referred to as proprioception, that is, the awareness of body in the space and the ability to perceive our own body in terms of states [16]. Advances in robotics have led to increasingly sophisticated and autonomous machines capable of performing complex tasks in variable and unknown environments. However, for robots to be effective and adaptive, sophisticated and reliable proprioception systems must be developed. Over the past decade, we have developed the ability to detect, monitor, and interpret the physical state of one's own body and its environment. To contextualize our research, in this Section, we summarize the background of published literature written by us and by other authors, relevant for this research, in quantum computing applied to robotics, cognition and robotics, abstract mathematics, and computational creativity. We also include a short discussion on the formalization of color-image perceptive associations that will be used in our case studies.

The first application of quantum modeling of the interaction between robots and their environment had been first proposed in a Physics article [2]. Quantum computing is a branch of computer science based on the fundamental laws of quantum mechanics, including state superposition, measurement as the collapse of the wave function, and reversibility of operators [17,18]. Several studies that concern the application of quantum computing to artificial intelligence have appeared [19,20]. Computational resources of quantum computing are investigated also in the domain of robotics [1–4]. The contemporary applications of quantum computing to robotics include swarm robotics [5,21–23], which are instances of distributed intelligence.

In recent years, we have also worked in the area of modeling the behavior of robotic swarms [24]. The main goal of our work has been to model the emergent behavior of the swarm from a global goal to be achieved and the behavior of the individual elements of the swarm. A robot swarm is a concept inspired by nature, for example, ant colonies or bird flocks. Each element of the swarm exhibits what we might call microintelligence; it knows how to perform a few actions in response to stimuli from the outside world. The interaction and communication between the elements of a swarm results in a macrobehavior that emerges from the interactions. The challenge we face is how to model and shape the behavior of individuals to ensure that the emergent behavior is the desired one to achieve the common goal. Emergence is an effect of communication and interaction and therefore cannot be lumped together with a simple stimulus. In line with the work proposed in this paper, we have proposed a modeling of robot behavior in a swarm by combining a matrix representation approach with a quantum approach. The emergent properties are modeled (or simulated) by quantum circuits. The idea is to model a swarm of robots by matrices of matrices. In each matrix, the main diagonal elements represent the robot and its capabilities, while the additional diagonal elements represent the interactions between the robots in one direction or another. In addition, each matrix element is encoded with a number of qubits corresponding to the number of robots in the swarm. For example, in the simplest case, we used four qubits for the interaction between two robots [24]. The result of the quantum simulation led to the specification of the behavioral response of each robot to the input stimuli. Then a reward system was set up to achieve the emergence of the behavior. The concept of emergence is taken out of context in the present paper, but it is important to note that the work carried out so far is in the direction of modeling the responses and thus decision making of robots in response to external and internal stimuli. In the present work, we propose a further step and exploit the concept of separation between classical and quantum approaches already proposed by Faggin [6]. This is a hybrid approach, then, in which we pay attention to the passing of data between the two levels, classical and quantum.

The classical–quantum pair is assumed by D'Ariano–Faggin in terms of memory and retrieval processes and from a higher level of abstraction, that of design, by some scientists who have recently paved the way for quantum software engineering. This topic is definitely new and offers several insights and challenges. The basic principle is that starting from a more or less complex problem to be solved, one must consider that some functionalities can be realized by exploiting the potential of quantum computers and others by those of classical computers. Since there are two different levels that need to be considered in the development of quantum software, the outstanding problem is the integration between the two levels or the two processes, the orchestration between the activities, and the sharing of data. Building on D'Ariano and Faggin's work and on earlier works summarized here, we propose in this paper a hybrid approach to modeling behavior and responses to external stimuli. Figure 2 presents a pictorial representation of our key idea.

The model by D'Ariano and Faggin includes the concept of entanglement of *qualia*, which is applied here to the association of multi-sensory stimuli, with their entanglement inside the mind. In our study, the choices of multi-sensory incoming stimuli are supported by recent literature on cross-modal correspondences [25]. In particular, we consider associations between bands of colors and bands of timbres. The extensive description of this topic is beyond the scope of this article. In a nutshell, we consider perceptual similarities of bands of colors (as strong, delicate. . .) with bands of timbres (same: strong, delicate. . .) to roughly consider the stimuli as concordant or discordant. The precise definition of this approach to color–timbre associations has been recently formalized in the framework of category theory [14] and experimentally validated [26]. Specific timbres (colors) are seen as points in a "conceptual space" of timbres (colors), and the transition from one timbre (color) to another one is a path in that space.

Let us first define conceptual spaces and then analyze color-timbre spaces.

Conceptual spaces (CS, see Gärdenfors, [27,28]) have been used extensively to bridge the gap between symbolic and connectionist models of information representation [29,30]. This was originally proposed by Peter Gärdenfors as a way to describe the "geometry of thought". Gärdenfors further developed this idea in his cognitive architecture, which includes an intermediate level called the "geometric conceptual space" [27,28]. This level is located between the linguistic-symbolic level and the associative sub-symbolic level of information representation. A geometric conceptual space is a hybrid representation that combines the strengths of symbolic and connectionist models. This allows for a more nuanced understanding of how concepts are represented in the mind [27]. The conceptual space plays the role of a workspace where low-level and high-level processes can access and exchange information in a bidirectional manner. Gärdenfors' geometric representation of concepts is based on a number of *quality dimensions*, which are used to represent different qualities of objects. For example, brightness, temperature, height, width, and depth are all quality dimensions that could be used to represent objects. Gärdenfors also believes that judgments of *similarity* play a crucial role in cognitive processes. He argues that it is possible to associate the concept of distance with many kinds of quality dimensions. This idea leads to the conjecture that the smaller the distance between the representations of two given objects, the more similar the objects are to each other. In Gärdenfors' model, objects are represented as points in a conceptual space, and concepts are represented as regions within a conceptual space. The shapes of these regions can vary, but regions that correspond to natural kinds or natural properties are typically convex. Convex regions are closely related to the notion of prototype. A prototype is an archetypal representative of a given category of objects. In Gärdenfors' model, the centroid of a convex region is the prototype of the concept that the region represents [27,28].

Now that we defined conceptual spaces, let us consider the parameter spaces of colors and timbres needed to contextualize the multi-sensory stimuli in our study. A space of timbres, as defined in [14], is a space of complex parameters, that is, a space whose points are timbric combinations and whose paths between points are the progressive transitions transforming some colors into other ones. A space of colors is similarly defined. The integration between different incoming sensory stimuli can help a person reconstruct the overall information about their environment, also in case of a partial sensory impairment. This is the core idea of the *supramodal brain hypothesis* by Rosenblum and co-authors [31]: there exists an inner-core of thinking where a mental image is built, which is not depending upon a specific sensory organ and sensory domain, but which collects and elaborates the different triggers to obtain a unitary picture.

Triggers to accomplish an action can also be seen as "commands", or "requirements", a human artist or a robotic simulation of an artist can obey. An instance of a robotic artist can be a dancing robot.

The artist can follow a sort of "inner stimulus", such as the inspiration to create, or a strong emotion to be sublimated into artistic creation. Or he/she can be subject to an external need, such as a commissioned artwork. Thinking of robotic modeling as the creative process, we may schematize the "need" to create as a necessity, an external imposition. Through a logic gate, a robotic musician elaborates a musical reaction. Both the external input, the "need", and the output, i.e., the musical response, are classic. The mechanism for producing the response up to measurement is quantum, a toy model to simulate creative thinking in response to an external stimulus. The interplay between classic physics to model stimuli from the environment and quantum physics to model the inner thinking processes can help us connect these studies with the ideas proposed by D'Ariano and Faggin [6]. A robotic embodiment of artificial intelligence can be used as a benchmark to test models of consciousness and decision making. There is an entire field of research concerning robotic models of consciousness and cognition [32]. The recent paper by Butlin, Long, et al. [33] surveys in detail the relevant mathematical models of consciousness in the literature and their relevance for AI systems. However, they do not consider any model of consciousness based on quantum principles. Instead, this paper provides a framework for studying consciousness by discussing a quantum computational model that may be relevant to human consciousness and conscious AI systems. These models generally do not include quantum computing principles, except for the Orch OR theory [34], which is supposed to be not computable [35,36]. Therefore, our proposed framework is also helpful for studying consciousness based on a quantum-computing model.

Some of the ideas proposed by D'Ariano and Faggin [6] regarding qualia and their role can be related to key notions provided in the conceptual spaces (CS) framework proposed by Gärdenfors [27,28]. This framework has many advantages that were exploited in cognitive robotics and also in computational creativity [30,32]. The described approach to "artistic production" in response to a need, as described above, is an instance of computational creativity.

Computational creativity is a research field where a computational system is built with the aim to show adaptation capabilities, produce meaningful actions, and operate a self evaluation [13]. The adaptation is typically focused on a set of inputs that are relevant for the system performance. For an artificial painter system creating artificial images, the input image is generally another image or an item from the trans-perceptual domain. In the music domain, for instance, in the case of a robotic dance, it can be seen as set of subsequent simple poses and their connecting movements. For such a *dancer robot*, music is the primary input and is used to generate a sequence of pleasant movements that fit well with the perceived song. The performance of the creative system is driven by a domain knowledge, typically learned by a master in a training phase and is assessed through a self-evaluation process providing a measure of the global quality of the output. Examples of a robotic dance, created according to input sound, are presented in [13]. Hidden Markov models, genetic programming, and neural networks are employed to let a robot move with pleasant movements that are compatible with a given song. Recent applications of computational creativity also include quantum computing and sound signal processing [37].

The model of a single robot can be extended to include multiple intelligent, interacting robots, performing art in response to external stimuli. The idea of *need* can be related, in the context of neuroscience, to the action of dopamine, already considered to build conceptual spaces of the brain and mind [38]. More generally, in our research, we can think of the *need* as the reaction of the robot concerning the response of the environment to its action.

To perform tasks in performing arts, a robot needs a form of "awareness" of its body structure and movement. An important point in the management of the robot body is the collection of the information coming from all the components of the robot. A viable solution for the management of multiple inputs is the adoption of a biologically-inspired somatosensory system. It can monitor multiple portion of the robot hardware and let the robot adapt to the state of the global system.

Another pillar of the model proposed in the aforementioned study by Faggin and D'Ariano [6] is category theory, developed in the 1970s to connect different branches and concepts of mathematics and in particular to formalize the concept of *transformations between transformations*. In particular, Faggin and D'Ariano consider monoidal categories [7]. From category theory, we may adopt the idea of relaxation of equality in favor of equivalence up to an isomorphism and the use of monoidal categories. The equivalence concept allows us to consider different sensory stimuli as belonging to the same class, according to some elicited effects [14].

These works can be related to the studies by Faggin through the concept of *qualia*.

Conceptual spaces have also been recently used to represent the balance of neurotransmitters in the brain [39]. In particular, Lövheim [38] proposed a 3D space where the axes represent serotonin, dopamine, and noradrenaline, respectively. Ehresmann and Ramirez also developed a formal model, based on category theory, to relate the physical reality of neurons with the emergence of thought [40].

Summarizing, we can observe a conceptual convergence of hints from quantum computing, computational models of consciousness, the tensor product of Hilbert spaces to formalize the superposition of different parameters for robots and their sound rendition, and cross-modal correspondences. The intersection of these fields can help extend the free-will approach by D'Ariano–Faggin to model the cyclic and non-deterministic interaction of a robot with the environment, as a toy model of human consciousness in a creative and interactive framework.

3. First Case Study

In this Section, we propose our first case study (a toy model), where a robotic dancer receives input from the external world and elaborates on them.

3.1. Theoretical Formalization

We consider here a scenario of a robotic dance seen as a sequence of responses to incoming multisensory stimuli and subsequent responses to the environmental feedback. We model the robotic thinking with a quantum circuit. The results from quantum measurement are mapped into a choice of dancing poses (here, for the sake of simplicity, just seen as a selection of robotic poses). External stimuli and movements of the robots "live" in the classical reality. Thus, with our simple model, we represent an interplay between the classical and quantum world, shaping a toy model of artistic decision-making between an improvising dancer and other artists, sending colors and sounds. Figure 2 presents a pictorial representation of the first part of our model, where the robot receives multisensory stimuli, thinks, and then chooses a response. Figure 3 shows a representation of the complete cycle.

We propose a model of tensor product of conceptual spaces to model robotic cognition and decision making. We embed the robot in a 'classic' environment that can be described by classical physics. The inputs received by the robot are classical. The "thinking mechanism" of the robot is based on quantum mechanics. The measurement of the quantum states is then stored in classical variables, which constitute the classic output.

We present a theoretical formalization and a worked-out simple example, with results obtained from quantum simulators, in particular, the IBM quantum simulator. The robot receives a classic input which is a combination of *qualia* [6]. The reaction mechanism is quantum and thus inherently based on probability amplitudes. To devise the structure of the response, we mainly consider tension–relaxation opposition. We also consider the tensor product of Hilbert spaces to represent stimuli across different conceptual spaces.

In our computational examples, to build a first, simple quantum model, we use the states 0 and 1 and their superposition to indicate an emotion "down" (relaxation) and "up" (tension), respectively. These conditions concern both the inner state of the robot and the characteristics of the given stimuli. In their inspirational study, D'Ariano and Faggin consider the entanglement of sensory stimuli belonging to different sensory domains. Here, we consider a "concordance" versus a "discordance" of the stimuli. When both stimuli are "up", they reinforce each other and trigger a "raising" response of the robot. When they are both "down", they provoke a "lowering" response. If they are mixed, that is, 0, 1 or 1, 0, they can leave the initial, inner state of the robot unchanged, or lead it into a "mixed" state, as per the effect of a Hadamard gate. The precise response of the robot is of course non-deterministic: it is subjected to quantum noise and fluctuation. In addition, at each step of the simulation, the inner state of the robot keeps a memory of its former configurations; thus, the further the simulation proceeds, the less it is predictable by the human researcher.

To devise a precise set of stimuli, we can draw upon research on cross-modal correspondences and the arts, including more formal studies on color-band and timbre-band associations [26]. We can instantiate the stimuli as a color and a musical note with a specific timbre. According to classes of colors and timbres [26], we may consider blue or gray light (to convey a feeling of relaxation and absence of tension), red light (to raise tension and attention), a loud trumpet note (also in this case, to raise tension and attention), and a soft flute sound (to convey a feeling of relaxation and absence of tension). Or black (and in general a very dark color) and a cluster in the lower register of the piano can be associated between them; this has been statistically verified [26], probably because they may express a feeling of deep sadness. Consequently, we would code the visual stimuli as 0 (blue or gray) and 1 (red) and the auditory stimuli as 0 (flute) and 1 (trumpet).

In our case study, we present a toy model with the simulation of a robotic dancer's response to classic stimuli. Adopting D'Ariano–Faggin's notation, we indicate with x_t the visual stimulus, with y_t the auditory stimulus, and with the ω_t the dancer's ontic state. x_t, y_t are classic, while ω_t is quantum. The transformation $\Phi_{F_t}^{(t,x_t,y_t)}$ represents the inner elaboration of the dancer who received the classic, external stimuli. The apex contains information about time t, the first incoming stimulus x_t, and the second incoming stimulus y_t. The pedex indicates the operator F_t that, in [6], represents the *free will*. Here, we are not considering a human but instead a robot. We are modeling the decision-making action of the robot in terms of the output of a quantum circuit, that is, labeling the effect of quantum noise and quantum unpredictability in terms of an effect of "freedom". The output of such a transformation is the dancer's modified ontic state ω_{t+1}. It constitutes one of the inputs of the subsequent transformations, jointly with the new classical external stimuli x_{t+1}, y_{t+1}. The transformation that receives these inputs and gives as output the modified ontic state ω_{t+1}, is $\Phi_{F_{t+1}}^{(t+1,x_{t+1},y_{t+1})}$. In this adopted notation, the pedex indicates the first performed transformation. The apex contains information about the time point and the two incoming stimuli. In fact, in general, we can create a structure with three inputs, two classical and one quantum, and a quantum output. The applied transformation, indicated in the pedex, becomes the apex (the "input transformation") of the operator that represents the following step, and so on. The first transformation can be indicated as ϵ_k. From [6], ϵ_k is defined as follows:

$$\epsilon_k = \sum_{F_t} O_{F_t}^{t,x_t}, \quad (1)$$

that is, as the sum of all ontic states corresponding to all possible outcomes. F_t in [6] is considered as a classical output, because the actions decided by the free will unfold in the classical reality. In our research, this first transformation takes two external inputs but also an "inner input", the initial inner state of the robot, and it contains the effect of quantum indeterminacy labeled as robotic "free will". Thus, its outcome is not entirely determined by the environment.

The pedex of the second transformation, related to a random component, becomes the apex of the second transformation, that is, $\lambda_j^{(k)}$. Similarly, the pedex of this transformation becomes the apex of the following transformation, $\mu_w^{(j)}$. For the sake of simplicity, we are not indicating ancilla qubit and measurement operations here.

Each one of the transformations can be modeled as a quantum circuit. In Section 3.2, we focus on the possible logic gate and its implementation through a quantum circuit.

3.2. Computational Example

In this section, we propose a possible realization of the "quantum core" of our model. We propose a logic gate (Table 1), we build a quantum circuit to implement it (Figure 4), and we run ten tests to analyze the robotic dancer's inner state change for the given inputs (Table 2). We can make predictions on the effects of the classic stimuli, but the results we actually obtain depend on the inner state of the dancer (which we can computationally initialize, but we do not know in a realistic setup), by the "resonance" of the stimuli and by the unpredictability of the quantum process. In the logic gate, we anyway assume that the inner state of the dancer is initialized as 0.

Table 1. Truth table with the incoming stimuli and the expected "external" response from the dancer.

Inputs		Output
Aud.	**Vis.**	**Response**
0	0	0
1	0	0.5
0	1	0.5
1	1	1

Table 2. Results of ten tests with the circuit in Figure 4. The first qubit of the result indicates the state of the dancer after the stimuli.

	Inputs			Results		
	Auditory	**Visual**	**Dancer's Initial State**	**Obtained States**	**Frequency**	**Outcome**
test 1	0	0	0	000	1024	lowered
test 2	1	0	0	001, 101	517, 507	mixed
test 3	0	1	0	110, 010	538, 486	mixed
test 4	1	1	0	111	1024	raised
test 5	1	1	0.5 (H)	111, 011	529, 495	no influence
test 6	0	1	0.5 (H)	010	1024	lowered
test 7	0	0	0.5 (H)	100, 000	521, 503	no influence
test 8	1	0	1	101, 001	513, 511	mixed
test 9	1	1	1	011	1024	lowered
test 10	0.5	0	1	101, **100**, 001	252, **518**, 254	raised (0.7)

The logic gate we propose has 0, 1, and 0.5 as theoretical (classical, not-superposed) outcomes. Thus, we can consider it as a fuzzy logic gate, whose truth table is given in Table 1. Quantum logic might be considered as a particular instance of multi-value logic, similar to fuzzy logic. From a quantum point of view, the given truth table shows, as inputs, only eigenstates of a visual stimulus and an auditory stimulus. The eigenstate 0 of the inner dancer's state is considered throughout. With these inputs, there are four possible classical responses as output. However, for mixed superpositions of the input states, we can obtain less predictable outcomes. Thus, the richness of quantum computing comes in handy. The gate of Table 1 is implemented by the quantum circuit of Figure 4, whose code in Qiskit is proposed in the following lines.

```
OPENQASM 2.0;
include "qelib1.inc";

qreg q[3];
creg c[3];
barrier q[0], q[1], q[2];
ch q[0], q[2];
ch q[1], q[2];
ccx q[0], q[1], q[2];
barrier q[0], q[1], q[2];
measure q[0] -> c[0];
measure q[1] -> c[1];
measure q[2] -> c[2];
```

We have two controlled Hadamard gates (CH) and a Toffoli gate (CCN). The circuit works in the following way. If only auditory or visual stimuli are 1, CCN is not called, and only one of the CH gates is activated. If both auditory and visual stimuli are up, then $(CH)(CH) = \mathbb{I}$, when $\mathbb{I}$ indicates the unitary matrix, and thus, only CCN is activated. The proposed circuit is a realization of ϵ_k. We can assume that, in this example, ϵ_k, $\lambda_j^{(k)}$, and $\mu_w^{(j)}$ have the same structure. Thus, the whole system presents a translational symmetry of the involved operators.

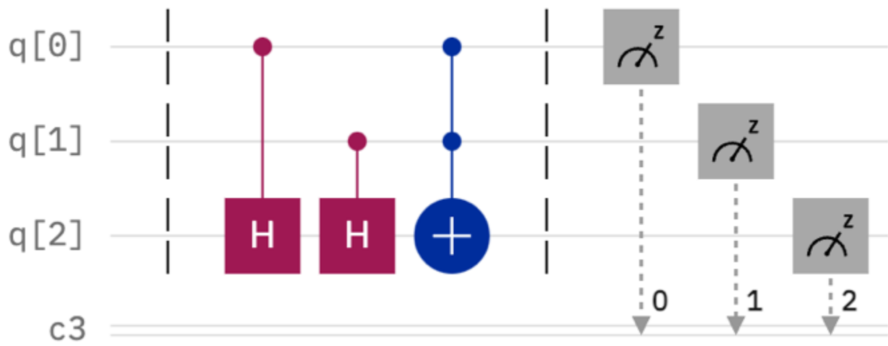

Figure 4. Quantum circuit designed for our tests. Qubits $q(0)$ and $q(1)$ are initialized with the states of the auditory and visual stimuli, respectively, while qubit $q(2)$ is initialized with the (initial) inner state of the dancer.

Table 2 shows the results we obtained across ten tests. The first qubit of the measure's result indicates the state of the dancer after the visual and auditory stimuli. When the outcome indicates "raised", the dancer is reaching an "excited state", and the choreography is more energetic than the initial state. The initial state of the robot does not depend on the environment. It is some "inner state", like "personal thinking" that is set independently from the environment at the beginning of the simulation; however, during the simulation, it is influenced by the external received stimuli. Thus, in Tables 2 and 3, the dancer's initial state is set independently from the other parameters.

When the outcome indicates "lowered", the energy of the choreography is decreasing with respect to the initial dancer's state.

In Test 6, the stimuli were not "resonating" together, and the state "down" of the dancer prevailed.

In Test 8, the state of the dancer is "lowered" from 1 to 0.5, because the stimuli were not "resonating".

In Test 9, the outcome is an unexpected 0. If this is correct, we can interpret it as an excess of sensory stimulation to an already "up" inner state, which provokes a diminution of the activity as the response.

Test 10 presents a more interesting superposition of three possible outcomes, with 2/3 probability of getting 1, that is, keeping the initial inner state of the dancer, and 1/3 probability of lowering it to 0. Thus, we may consider the outcome as 0.7, which is a new result, with respect to 0, 0.5, or 1.

Summarizing, if we limit our analysis to the ten considered cases, the measurements should be mapped to four different positions of the dancer, corresponding to 0, 0.5, 0.7, and 1, respectively.

The proposed tests correspond to the responses to one set of stimuli. The same code can be run again to obtain the responses to the second set of stimuli. The measurement can be performed on the ancilla (not included here for the sake of simplicity), while the input of the "second inner transformation" is as follows: visual stimulus 2, auditory stimulus 2, and modified inner state of the dancer, which is the output of the presented circuit.

Table 3. Continuation of Table 2 with the robotic action after quantum measurement, possible elicited response of the environment, and a new sequence of robotic thinking. The measurement of the inner state of the robot is performed on an ancilla qubit to leave the original system unchanged. When the first set of stimuli had no influence on the robot, the inner state remained the one at the corresponding line of Table 2. This information constitutes one of the inputs of the new quantum circuit, jointly with the world's feedback. The last one can be arbitrarily tuned by the researcher. Here, we show some examples of the response. The new robotic results are obtained via further quantum measurements.

Measure's Outcome	Robot's Action	Robot's Inner State	World's Feedback		New Results	
			Visual	Auditory	Results	Frequencies
lowered	depressed	0	1	1	110, 010	(479, 545)
mixed	random	0.5	1	1	110, **011**, 010	(256, 509, 259)
mixed	random	0.5	0	0	101, 001, 000	(241, 256, 527)
raised	excited	1	1	0	111	1024
no influence	former: mixed	0.5	0	1	101, 100, 001	(247, 509, 268)
lowered	depressed	0	1	1	110, 010	(503, 521)
no influence	former: mixed	0.5	0	0	101, 001, 000	(267, 245, 512)
no influence	former: excited	1	1	1	011	1024
lowered	depressed	0	0	1	100	1024
raised	excited (0.7)	0.7	1	0	111, 110, 010	(662, 182, 180)

4. Second Case Study

In this second case study, the robotic dancer does not only "listen to music", but it also performs actions in response to its perception and inner thinking (Table 3). Its actions, in the form of dancing movements to reach different poses, are visible from the outside. In this sense, we say that they "modify the environment" in terms of visual and auditory landscape, acting on the surrounding. The robotic dancer takes *decisions* and produces an *action* through the *actuators* (Figure 5). From a physiological point of view, we can think of a motor neuron sending an impulse to a muscle fiber.

Thus, we have a process quantum $\rightarrow$ classical, because the external world can be described classically. The response from the world is feedback that constitutes a new input for the robot, and thus, we have the information-exchange classical $\rightarrow$ quantum. The robot receives information as *sensory input* and elaborates a *perception*. From a physiological point of view, we can think of a sensory neuron receiving information from visual, tactile, and auditory receptors.

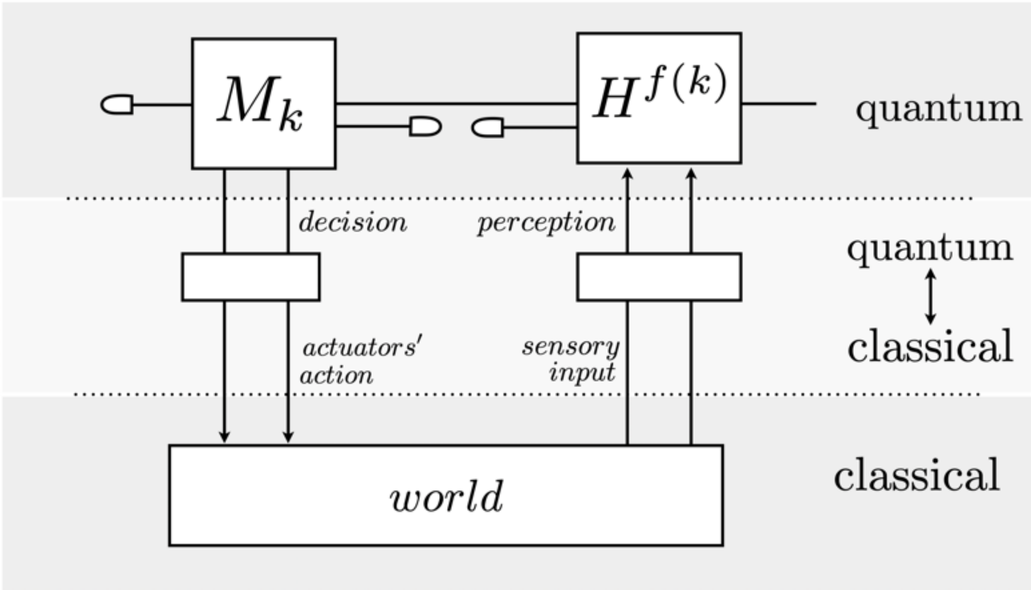

Figure 5. Scheme of robotic actions associated with the outcome of quantum measurements, environmental (world) feedback in the form of new incoming multi-sensory stimuli for the robot, robotic elaboration of the stimuli as inner perception. The scheme can be reiterated, leading to less and less predictable behavior of the robot. For this reason, this toy model may approximate the creative behavior of an improvising dancer. Here, the notation is still derived by [6]. At the inner-robotic level (quantum domain), the quantum operator M_k is followed by $H^{f(k)}$, where the apex function $f(k)$ depends on the pedex of the first operator, transformed through the function f, which takes into account the feedback provided by the world. This is the very general scheme which can be instantiated as shown in Table 3.

The overall mechanism can, in fact, be considered as an exchange of information between the system (the "mind") and the environment, a simplification frequently carried out in theoretical physics. Concerning operators, at the inner-robotic level (quantum domain), the operator M_k is followed not by H^k, with the simple dependence on k, the pedex of the former operator. Instead, it is followed by $H^{f(k)}$, where the dependence from k is present through the function f, taking into account the feedback provided by the world. The overall mechanism, visually schematized in Figure 5, is general. It can be instantiated as proposed in Table 3, showing the possible associations between the results of quantum measurements and the classical actions undertaken by the robot, through its actuators. The actions elicit a response (feedback) from the world, received by the robot in the form of new sensory inputs. The robotic perception is elaborated as new quantum inputs and stimulates a new sequence of "thinking". The overall mechanism is reiterated, and it constitutes a dialogue between the quantum domain (inner system of the considered robot) and the classic domain (everything external to the considered robot). The mechanism is visually depicted in Figure 5. The association between the measure's outcome and the action of the robot is up to the researcher. It should be respectful of cross-modal correspondences [25] and color–timbre–motion associations [26] to carry an emotional meaning for humans. In Table 3, we consider four possible states as follows:

- *Former* when the initial behavior of the robot is unchanged, and then, the inner state of the robot corresponds to the former one;
- *Random* when the response is unpredictable due to the contrast of incoming stimuli;
- *Excited* when the tension/activity level is raised;
- *Depressed* when the activity level is lowered.

In order to shape a concrete coding implementation, also the *random* action should be set up by the human researcher. When 'lowering' is applied to 0, we can introduce a phase, to be embodied as the opposite action. Or the operation of lowering an already-low state can let the robot remain in that state or change reaction. In our experiments, we encountered an equivalent issue in the first case study, where the 'raising' trigger was applied to the initial state 1. We interpret the 'down' result as an excess of sensory stimulation to an already "up" inner state, which provokes a diminution of the activity as the response. The result of test 8 in Table 3 confirms that, in the case of an already 'excited' state, an 'up' new incoming stimulus has the effect of raising the energy level of the robot. Further research can make the whole process more precise, considering an automatic output from the first set of measurements toward the inputs to the second one, building a more fine-grained state superposition.

These states can constitute the basis of a choreography. The actions undertaken after a second round of environmental responses, robotic thinking, and quantum measurements can be defined as state superposition of the fundamental four responses. Thus, the idea of quantum superposition appears also in the choice of (classical) actions. At each iteration of the whole mechanism, the behavior of the robot becomes less and less predictable by the human researcher, constituting a toy imitation of the *creative behavior* of an improvising human dancer. Interestingly, each robotic dancer is a monad for the other ones. Direct communication between quantum minds is impossible; the communication does only happen between their classical outputs, which become the classical inputs of another mind. This could also constitute a model for human communication: we do not know what is going on in the mind of other people; we can just infer their patterns of thought from their words and behavior, that is, their *actions* in the auditory–visual–gestural environments. This is also a connection with the philosophy by Leibniz, where each person is a *monad*, a closed entity, to the others.

5. Discussion and Conclusions

In this article, we started from D'Ariano–Faggin's [6] exploration of thinking mechanisms through the formalism of quantum mechanics. We adopted their notation as operators in Hilbert spaces, and the distinction between quantum formalism for inner mind processes versus classical formalism for external actions and stimuli. We extended D'Ariano–Faggin's study including actions undertaken according to quantum measurements' outcome, the feedback provided by the environment in the form of new multi-sensory stimuli, and the further set of inner responses from the quantum mind. We instantiated such a general and abstract approach building a robotic choreography, as a toy model for the human behavior of an improvising dancer. We considered basic incoming multi-sensory stimuli, which could be considered as "resonating" or contrasting, according to cross-modal correspondences [25] and the supra-modal brain hypothesis [31]. The incoming stimuli can trigger some changes in the initial inner robotic state. The induced changes can be measured and mapped to actions to be performed via robotic actuators. The actions are observed from the outside of the robotic mind, being an element of classical reality. The external "world" provides feedback in the form of a new set of stimuli. They are perceived by the robot and elaborated in its "mind", eliciting a new set of measurements and corresponding actions. The overall structure of robot-world dialogue is a quantum–classic dialogue; it can be reiterated, and at each step of the process, the resulting robotic behavior is less and less predictable. We draw a comparison with Leibniz's theory of *monad*: each mind is a closed being; it is not possible to know what is going on in it from the outside. Consequently, each person is a closed entity, and perfect and complete communication is impossible. In our toy model, the world cannot know what is precisely going on inside the robotic mind; it is only possible to observe the robotic actions. Similarly, in the human dimension, we cannot know other people's thoughts. We can only try to infer information from their patterns of behavior and response to our own behavioral outcomes.

Future research could involve more parameters and the use of a real quantum computer. As further developments of the proposed research, we can set up a concrete experiment to test the proposed model with a robot in response to a set of external stimuli and environmental feedback. Our research can help connect robotic modeling of the mind with recent studies in the field of neuroscience, including studies on dance and gestural similarities [41].

Ultimately, art can stimulate mathematical modeling and thinking in physics, and physics can provide new creative insights and shed light on the complex mechanisms inside the arts.

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