Review summary

Evasion of tipping in complex systems through spatial pattern formation.

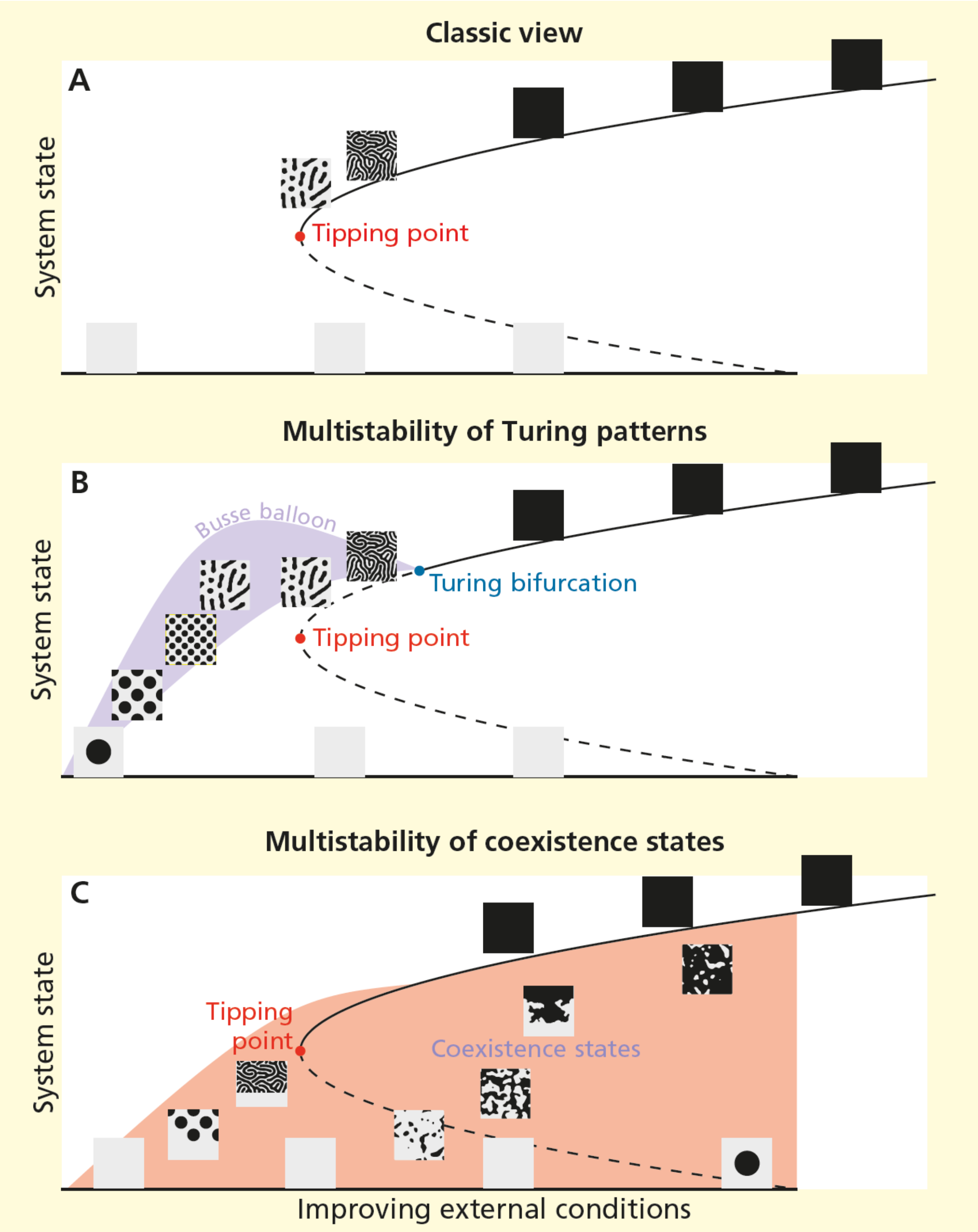
Max Rietkerk, Robbin Bastiaansen, Swarnendu Banerjee, Johan van de Koppel, Mara Baudena and Arjen Doelman

BACKGROUND: In the current Anthropocene, there is a need to better understand the catastrophic effects that climate and land use change may have on ecosystems, Earth system components and the whole system Earth. The concept of critical transitions, or tipping, contributes to this understanding. Tipping occurs in a system when it is forced outside the basin of attraction of the original equilibrium, resulting in a critical transition to an alternative, often less desirable, stable state. In this context, the search for early-warning signals for such imminent critical transitions is ongoing. In particular, spatial self-organization in ecosystems, such as the spontaneous formation of regular vegetation patterns, so-called Turing patterns, was thought to be a prominent early-warning signal.

ADVANCES: However, recent findings indicate that such spatial self-organization should not necessarily be interpreted as an early-warning signal for critical transitions. Instead, spatial self-organization can cause ecosystems to evade tipping points, and can thereby be a signal of resilience. These findings are based on recent mathematical analyses of spatial models and on novel observations of real ecosystems. Both revealed multistability, meaning that many different spatial patterns can co-occur under the same environmental conditions, and each of these patterns can stay stable for a wide range of conditions. This enables complex system states to persist beyond tipping points through spatial self-organization. Moreover, if a complex system with tipping properties experiences a perturbation, subsequent change of the system does not necessarily lead to tipping of the complete system. Instead, the change can stay localized, because the system allows for alternative states to coexist in space, thus called coexistence states. These spatial patterns can also persist beyond tipping points with worsening conditions through this alternative

pathway. We refer to both Turing patterns and coexistence states as spatial pattern formation. Evasion of tipping through these various pathways of spatial pattern formation may be relevant for many ecosystems and Earth system components that are hitherto interpreted as prone to tipping, including for the Earth as a whole.

OUTLOOK: To further study how complex systems evade tipping through spatial pattern formation, savanna ecosystems can be considered as a concrete archetypal example, because of the alternative states and spatial patterns observed for them. Moreover, universal conditions for evading tipping points in both ecosystems and Earth system components can be derived by mathematical analyses. Scenarios can be revealed by which Turing patterns with small amplitude can grow and form large-scale localized interacting structures, thereby aiding complex systems to evade tipping. The effects that global change has on the spatial boundaries between coexistence states should be studied, and the impacts of restrictions of spatial domain, localized and non-local homogenizing effects by humans should be revealed. This approach will advance our understanding and predictions of critical transitions in nature and reveal how these may be avoided or reversed.



Evasion of tipping points. We illustrate the response of complex systems to changes in external conditions (i.e., a bifurcation diagram). Solid lines denote stable non-patterned equilibria, and dashed lines unstable equilibria. Homogeneous dark grey squares depict high density of the system state variable and homogeneous light grey squares illustrate low density. (**A**) Classic view.

The classic view is that spatial self-organization can be interpreted as an early-warning signal for tipping points towards an alternative stable state; here illustrated as the emergence of Turing patterns before the tipping point. (**B**) Multistability of Turing patterns. Recent model analysis revealed multistability of Turing patterns in Busse balloons, supported by satellite observations of real ecosystems. A Busse balloon is the region in parameter space in mathematical models where multistability of patterned equilibria occurs. Here, spatial self-organization through Turing instability arises in parameter regions before the tipping point at the Turing bifurcation, persisting beyond the tipping point, thereby constituting a pathway evading tipping through spatial pattern formation. (**C**) Multistability of coexistence states. Evading tipping can also be due to multistability of coexistence states. Following perturbation, the spatial system allows for alternative stable states in space, or coexistence states, thereby evading tipping of the complete system. These spatial patterns originate in the bistability region before the tipping point; the evolving spatial patterns can also persist beyond the tipping point with worsening external conditions, thereby constituting an alternative pathway evading tipping points.

Evasion of tipping in complex systems through spatial pattern formation.

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The concept of tipping points and critical transitions helps understanding the catastrophic effects that global change may have on ecosystems, Earth system components and the whole system Earth. The search for early-warning indicators is ongoing, and spatial self-organization has been interpreted as one of such signals. Here we review how spatial self-organization can aid complex systems to evade tipping points, and can therefore be a signal of resilience instead. Evading tipping through various pathways of spatial pattern formation may be relevant for many ecosystems and Earth system components that hitherto are identified as tipping-prone, including for the entire system Earth. We propose

a systematic analysis that may reveal the broad range of conditions for which tipping is evaded, and resilience emerges.

The concept of critical transitions, or tipping points, contributes to the understanding of planetary changes in the Anthropocene (*1-3*). This idea entails that ecosystems or Earth system components undergoing global change can typically persist and stay in a similar equilibrium state. The impending danger, however, is that this is only the case until a tipping point is reached, at which this (often desired) stable state disappears and the system undergoes a critical or catastrophic transition towards an alternative equilibrium (*1*, *2*, *4*). The latter is a state that will also prevail if the external change goes back to its original value, thus displaying hysteresis (*1*, *5*). Classic examples of ecosystems thought to exhibit critical transitions with tipping points between alternative stable states are: clear lakes becoming turbid because of nutrient overloading (*6*), barren deserts replacing vegetated areas in dry savannas, or drylands in general, due to drought or overgrazing (*5*, *7*), and savannas replacing tropical forests because of deforestation associated with fire, possibly combined with less rainfall (*8-10*).

Mechanisms that cause such tipping dynamics are positive feedbacks: i.e., processes that amplify change imposed on complex systems. In the above mentioned ecosystems, positive feedbacks are: increased turbidity in lakes leading to less macrophyte plants, which hinders nutrient uptake thus leading to even more turbidity (*6*); less vegetation resulting in less water infiltration into the soil in dry savannas, which in turn leads to even lesser vegetation (*5*, 7); fewer forest trees in tropical forests because of deforestation and fires allowing more grass growth, fueling more fires and preventing forest tree establishment, thus leading to even fewer forest trees (*10-12*). At tipping points, positive reinforcing feedbacks overwhelm the negative balancing feedback processes that maintain the desired state of ecosystems, often leading to the catastrophic loss of ecosystem services to humans.

There are many possibilities of how positive feedbacks may overtake negative feedbacks and tipping can be triggered. Tipping occurs as a result of the loss of resilience, which is interpreted as the amount of environmental change or the strength of perturbation a system can withstand before it tips to another basin of attraction (*13*). Three of the most prominent possibilities have been classified as follows (*14*): bifurcation induced tipping (B-tipping) happens when a parameter shift (e.g. change in environmental conditions) reduces the basin of attraction of the original stable state to zero; noise-induced tipping (N-tipping) occurs as a perturbation of the system state, e.g. due to environmental noise or disturbance, knocks the system outside the basin of attraction of the original state; rate-induced tipping (R-tipping) arises when an environmental (parameter) change is faster than the restorative attraction to the original state. The restorative attraction to the original state, or the speed after which an equilibrium state is restored, is called engineering resilience, or stability, in the ecological literature (*13*).

This idea of tipping or critical transitions is not only important for ecosystems at local scale, but also for many regional-scale Earth system components (*15*, *16*). Two examples are: tipping of the Arctic ice sheets because of warming, inducing changes of surface albedo, leading to more warming (*17-19*), and tipping of the Atlantic Ocean circulation induced by changes in surface water fluxes (*20-22*). The same notion of tipping points also underlies the concept of hazardous planetary boundaries at the global scale (*23*), and has similarly been applied to the Earth's biosphere as a whole, as a response to climate and land use changes (*3*, *24*). So, this concept is relevant for all spatial scales ranging from ecosystems to the entire complex system Earth.

For spatially extended ecosystems, such as drylands, savannas and peatlands, it has been highlighted that critical transitions are associated with the formation of self-organized spatial patterns of vegetation (*2*, *25*). In these systems, as environmental conditions worsen, a uniform coverage becomes unstable to non-uniform (spatial) disturbances, due to the spatial processes, leading to the formation of regular spatial patterns. Such spatial destabilization of a uniform state is called a Turing instability or Turing bifurcation, after Alan Turing who first studied this in reaction-diffusion systems (*26*). After a Turing instability, so-called Turing patterns emerge, that can have various spatial forms depending on environmental conditions, and this has previously been interpreted as preceding a tipping point to an alternative ecosystem state (*25*). Most notably in drylands, the following vegetation patterns are observed, here listed in the order with which they appear with worsening environmental conditions, such as increasing drought or grazing: bare gaps in homogeneous vegetation cover; labyrinthine or striped vegetation cover; spotty vegetation in homogeneous bare soil (*25*, *27-30*).

The mechanistic base of Turing instability is that the positive feedback mentioned earlier is scale-dependent in spatially extended systems: the positive feedback dampens and is subsequently replaced by a negative feedback further away in space, generating scale-dependent feedbacks, due to spatial processes (*31*). The crucial spatial effects of these processes leading to scale-dependent feedbacks are typically neglected when assuming that systems are homogeneous. In drylands, for example, the scale-dependent feedback relates to increased infiltration of water into the soil and larger soil water uptake at places where vegetation is growing. This generates surface and soil water flows towards the vegetation, at the cost of available water further away (*28*, *29*), ultimately resulting in spatial patterns from concentrated vegetation at some places and bare soil in the other.

Evading tipping points through Turing patterns

The regular spatial patterns resulting from Turing instability were until now understood as early-warning signals for tipping points or critical transitions towards an alternative state (B-tipping) in various ecosystems (*2*, *25*). However, recent mathematical analyses, combined with novel satellite observations of real ecosystems, reveal a drastic alternative view (*32-34*). These analyses exposed the existence of pattern-driven multistability described by Busse balloons, after F.H. Busse (*35*, *32*, *33*). The Busse balloon (Fig. 1) indicates a region of the parameter space in mathematical models for which a large range of wavelengths of regular spatial patterns (vegetation in the dry savanna example) are possible and stable. Also, when conditions or parameters in the model are changed, a specific spatial organization can remain stable, meaning the wavelength does not change for a wide range of environmental conditions, until the edge of the Busse balloon is reached. Here, the spatial organization and ecosystem state variable, such as ecosystem productivity, may adjust in a non-critical or non-catastrophic way (Fig. 1). This can be at parameter values for which, in the non-spatial model, the homogeneous vegetation state may still exist or already collapsed to bare soil.

The alternative view mentioned above stems from the fact that earlier analysis of the ecosystem models only considered the stability of uniform states. Thus, such analysis did conceal the many possible responses of spatially patterned states, overlooking the existence of Busse balloons. New studies beyond the conventional analysis (*32*) also considered the stability of patterned states, exposing the Busse balloons, implying qualitatively new model predictions and inspiring novel observations in real ecosystems.

These model predictions, including the existence of multistability and the notion of Busse balloons, have lately been supported by observations in real ecosystems (*33*). Satellite observations showed regular spatial vegetation patterns, which occur in vast areas in different regions in dry savannas in Africa. Indeed, these observations showed that many spatial patterns with different wavelengths co-occur next to each other in one and the same area with similar environmental conditions, and for different areas within the same larger region, supporting the Busse balloon theory. Moreover, the wavelengths of the patterns in specific areas within those regions remained stable in time for decades, despite changes in environmental conditions. Once again, we refer to these two phenomena combined as multistability, which is illustrated by the Busse balloon (Fig. 1) (*32*, *33*). Also, recent model analyses revealed that at the edge of the Busse balloon, ecosystems adjust their spatial organization in such a way that they stay within the Busse balloon. In other words, the dominant variable generating the spatial pattern (ecosystem productivity in our example) does not change drastically, in the way it would with a critical or catastrophic transition, but more gradually instead (*34*). Moreover, in the patterned state, vegetation persists for environmental conditions beyond the tipping point (Fig. 1b). So this demonstrates a case where the system shows spatial pattern formation at the Turing bifurcation before the tipping point is reached, which then extends beyond the tipping point, thus essentially constituting a pathway evading it (Fig. 1).

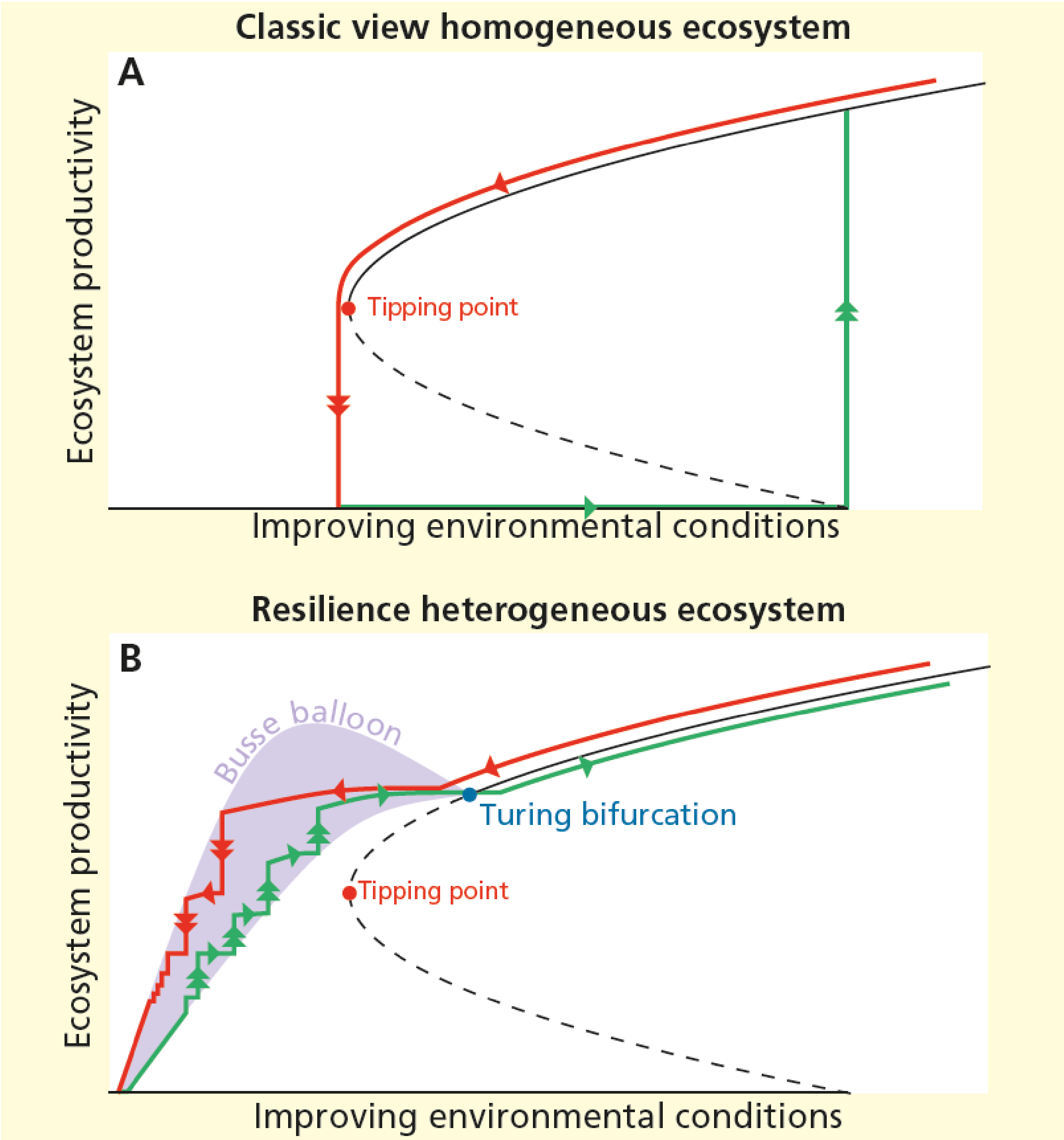

Figure 1. Resilience in homogeneous and heterogeneous ecosystems. Solid lines denote stable non-patterned equilibria and dashed lines unstable equilibria. Double arrows mean ecosystem shifts and single arrows minor ecosystem adjustments. Typical trajectories for ecosystem degradation (red) and recovery (green) are given, showing the narrowed hysteresis loop for heterogeneous, spatially self-organized ecosystems. (**A**) Classic view homogeneous ecosystem. Classically, worsening environmental conditions correspond to a minor adjustment of the ecosystem state or ecosystem productivity, until these drive the system over a tipping point, and a critical or catastrophic transition (B-tipping) occurs. (**B**) Resilience heterogeneous

ecosystem. In multistable spatial ecosystems, every set of environmental conditions beyond the Turing bifurcation allows for multiple stable, spatially self-organized states, indicated by the purple area, named the Busse balloon. Here, instead of one critical transition, multiple smaller ecosystem shifts from one spatially patterned state to another occur, which have minor impact on the function or productivity of the ecosystem as a whole (*32*, *34*).

So, a phenomenon earlier considered as an illustrative early-warning signal for imminent critical transitions, now appears to be a sign of resilience instead. In this case, the patterns originate from a Turing instability, leading to multistability of patterns and subsequent gradual change of the system. Similarly, in mussel beds, multistability of patterned states has been found in models and real systems (*36*, *37*). It is noteworthy that the predicted size and number of ecosystem shifts can vary; these depend on the magnitude and rate of environmental change (*32*, *34*).

There are now strong indications that evading tipping points through spatial self-organization may be very common for ecosystems and Earth system components. This is based on earlier work on regular pattern formation in real ecosystems (*31*), combined with the recent new insights outlined here, and a mathematical analysis (Box 1) of a class of models. This is moreover underpinned by other observations: tipping points and alternative stable states (bistability) have been suggested based on simple, non-spatial models, for at least the following ecosystems: drylands or dry savannas (*5*,*7*), savannas (*10*, *11*, *38*), peatlands (*39*), mussel beds (*40*), intertidal mudflats (*41*), barrier islands (*42*) and sea grasses (*43*). Most notably, for all of these model ecosystems, spatial self-organization, or Turing patterns, have been observed in their real counterpart systems, which are always spatially extended (*31*, *44-47*).

Evasion of tipping points may not be restricted to the case of Turing patterns in Busse balloons, but may include more comprehensive spatial pattern formation, as we will outline below in the next section. Apart from spatial pattern formation, there are also other mechanisms via which tipping may be evaded. For example, sufficiently fast reset of the changing parameters as compared to the rate of the changing state variable, may repair the overshooting of a tipping point (that is put the state variable back into the original basin of attraction) in simple Earth system component models (*48*). We expect that both spatial patterns and time delay of the state variable will not only affect B-tipping, but will have a very similar impact on noise-induced (N-) tipping and rate-induced (R-) tipping, that is evasion of (complete) tipping of the system. In this context it is also interesting to note that thresholds related to tipping points are generally rarely detectable from empirical data (*49*).

Box 1. Turing-before-tipping

In mathematical models, (bifurcation, or B-)tipping takes place when an equilibrium state loses its attraction and disappears in response to a parameter, say A, passing through a critical value A^*. At that moment, the system has to shift to an alternative equilibrium state. In the literature (e.g. *10*, *38*, *39*), the models in which tipping occurs typically are spatially homogeneous, i.e. not spatially extended. However, the associated modeled systems are almost without exception spatially extended, and it is thus natural and necessary to model the spatial effects. The main idea underlying the concept of Turing-before-tipping is as follows. For tipping, it is necessary that the (initial) equilibrium state stays stable until it tips. Therefore, it is self-evident to ask the question: Is the homogeneous equilibrium state that precedes tipping also stable against spatial effects? If not, a spatially extended version of the model will not exhibit tipping: spatial patterns emerge before parameter A has reached its critical (tipping) value A^*. In that case, the modeled ecosystem will not tip; instead, it evades the critical transition by forming spatial patterns.

As a general and relatively simple example of how one can determine whether Turing-before-tipping occurs (or not), we consider the dynamics of a biological quantity $b(t)$ in interaction with a (typically limited) resource $r(t)$ and assume that this is described by the model $\frac{db}{dt} = F(b,r)$, and $\frac{dr}{dt} = G(b,r)$, in which $F(b,r)$ and $G(b,r)$ represent various growth, decay and interaction effects that vary with parameter A. For instance, in the (non-dimensionalized) model for vegetation dynamics in drylands of Bastiaansen et al (2018), $b(t)$ is the biomass of the vegetation, $r(t)$ is the available water and $F(b,r) = - Mb + rb^2$, $G(b,r) = A - r - rb^2$, where M models the vegetation mortality rate and A the rainfall. Tipping occurs when, as function of parameter A, two equilibrium states merge into one, the threshold state (b^*,r^*) at $A = A^*$, and subsequently disappear. To guarantee that one of these equilibrium states is stable, and thus observable, until it tips, it is necessary that $\frac{\partial F}{\partial b}(b^*,r^*) + \frac{\partial G}{\partial r}(b^*,r^*) < 0$. To consider the question whether Turing-before-tipping may occur, one thus needs to incorporate spatial effects into the homogeneous model. A simple way to do so is to extend the model for $b(t)$ and $r(t)$ into a system of reaction-diffusion equations for biomass $B(x,t)$ and resource $R(x,t)$: $\frac{dB}{dt} = d_B \Delta B + F(B,R)$, $\frac{dR}{dt} = d_R \Delta R + G(B,R)$, in which Δ models (spatial) diffusion, and d_B and d_R are the diffusion coefficients that govern the diffusive spreading speeds of $B(x,t)$ and $R(x,t)$. It can be obtained that only if $d_R \frac{\partial F}{\partial b}(b^*,r^*) + d_B \frac{\partial G}{\partial r}(b^*,r^*) < 0$ the threshold state (b^*,r^*) is stable against spatial perturbations. In other words, Turing-before-tipping occurs if $\frac{\partial F}{\partial b}(b^*,r^*) + \frac{\partial G}{\partial r}(b^*,r^*) < 0$ and $d_R \frac{\partial F}{\partial b}(b^*,r^*) + d_B \frac{\partial G}{\partial r}(b^*,r^*) > 0$. This condition thus determines whether an ecosystem can evade collapse by forming patterns (or not) and can be checked explicitly in any given model. For instance, in the dryland model of Bastiaansen et al (*33*), these conditions are given by $M < 2$ and $eM > 2$ (at $(b^*,r^*) = (1, M)$ with $A^* = 2M$, $d_R = e$ and $d_B = 1$). In Bastiaansen et al (*33*), the following realistic choices for M and e were made: $M = 0.45$, $e = 500$ (*cf*. *32*). So, Turing-before-tipping takes place, as is also exhibited by the observations reported. Explicit conditions for Turing-before-tipping can also be deduced for multi-component models and/or for models with spatial effects beyond (linear) diffusion.

Evading tipping points through coexistence states

The ranges of conditions and mechanisms for which complex systems can evade tipping via spatial pattern formation are not restricted to those leading to Turing patterns in Busse balloons, but they include more comprehensive spatial pattern formation. While non-spatial complex systems can respond to disturbances or perturbations only with a system-wide response that either leads to complete tipping to an alternative stable state (N-tipping) or to full recovery, this is not the case for real systems, which are always spatially extended. In those systems, localized or random disturbances that are omnipresent can lead to spatial pattern formation, in which only part of the spatial domain transitions to the alternative state, and system-wide N-tipping is evaded. Such spatial patterns consist of the coexistence of alternative stable states in space, and are herein referred to as coexistence states. Multistability of different spatial patterns of those coexistence states can occur for given environmental conditions (*50*, *51*). Spatial boundaries, or interfaces, necessarily arise between alternative states in space. These boundaries can range from very simple, in the form of a single front, to a rich patterned structure of alternative stable states. For example, presence and absence of ice are two alternative stable states that can occur in the spatial domain of system Earth; these states coexist on a global scale. Ice only appears at the polar latitudes and no ice occurs in between, with a simple spatial boundary between them, named the ice line or grounding line (*52*, *53*). At smaller scales, such spatial boundaries between ice and no-ice states can consist of much more complex structures of alternative stable states (*54*).

The spatial boundaries separating the coexisting states may themselves become unstable, which might lead to one alternative state invading the other, thereby providing a mechanism for evolving such rich and complex patterned structure of alternative stable states (*51*, *55*). Furthermore, reorganization of spatial patterns may take place due to environmental change, without any abrupt change in system characteristics at global system scale. Thereby the system can evade critical transitions and instead a smoother and much more gradual response can be expected (*50*, *51*). In this manner, the spatial patterns can also persist beyond the tipping point with worsening environmental conditions. This constitutes another essential pathway for ecosystems and Earth system components to evade tipping points through spatial pattern formation and multistability. Most amazingly, such spatial organization could even lead to non-forced reversed transitions in which a state counter-invades or, in other words, the automatic and spontaneous recovery of the original state from the alternative one (*55*).

These coexistence states may also form in ecosystems because of aggregation of organisms into self-organized patchiness (*56*), as was observed in the clustering of mussels in mussel beds (*57*). Here, an interesting parallel was found with the physical theory of phase separation. This theory describes the dynamics of spatial interfaces of phases (i.e., spatial boundaries between coexistence states in our terminology), and explains a rich variety of possibly very slow transient spatial patterns. These include maze or labyrinths patterns, spot patterns and viscous fingering, which may cause very gradual transitions when conditions are changed. Phase separation dynamics leading to coexistence states can be present in ecosystems and Earth system components as well (*58*). For instance, some of the spatial aggregation of organisms and resources can be interpreted as such. Comparisons to phase segregation mechanisms have been made already for other ecosystems besides mussels (*57*), e.g. for the aggregation of vegetation in fingering patterns (*55*).

New theory and procedures need to be developed to distinguish spatial pattern formation from patterns originating from pre-existing heterogeneity, because real systems are generally a mixture of both (*59*). One example of pre-existing heterogeneity that could falsify the mechanism of spatial pattern formation in the specific context of vegetation patterns could be pre-existing drainage patterns governed by topography, explaining tree distribution patterns.

Toward a theory on spatial pattern formation and multistability

The previously described insights highlight how spatial pattern formation in complex systems can lead to evasion of tipping and increased resilience. We have highlighted a few illustrative examples in Box 2 and Box 3. However, some complex systems may still exhibit tipping, even though they are spatially extended. So, when does the classical non-spatial framework of tipping points break down and need to be supplemented by a new theory on spatial pattern formation? For what ecosystems and Earth system components, including complex system Earth as a whole, is this relevant? And for what conditions and at which spatial scales is this the case? Currently, these questions are not fully resolved. In the text below, we provide a synthesis of the current understanding, based on analysis of model and real patterned systems, in which we also explicitly point out missing pieces of knowledge.

Box 2. Spatial pattern formation evading tipping in local-scale ecosystems.

Savanna ecosystems are characterized by the coexistence of trees and grasses. Most homogeneous models indeed reproduce this coexistence. They show that, depending on rainfall, fire intensity and level of herbivory, open savannas (with dispersed trees) may generally switch between multiple alternative homogeneous states (Fig. 2) with: no trees; either barren "desert" or "grassland", closed tree cover of mainly savanna trees; "savanna woodland", or closed tree cover of mainly forest trees; "tropical forest" (e.g. *5*, *10*, *11*, *38*, *64*, *65*, *75*). In other words, neglecting spatial effects, these models predict that savanna ecosystems can exhibit alternative states, and critical transitions with tipping between them.

However, when spatially extended, the models show a rich variety of spatial patterns instead, through the emergence of Turing patterns (*71*, *104*) and due to the coexistence of alternative stable states in space (*51*, *73*) (Fig. 2). The system may, or may not, evade tipping by the appearance of spatial patterns, as analysis (Box 1) indicates that both situations may occur. Whether a tropical forest or savanna collapses or increases its resilience by the multistability associated with spatial pattern formation, depends on local, but system-wide, conditions i.e. the parameters in the model.

Box 3. Multistability evading tipping in regional- and global-scale Earth systems.

Tipping behavior and critical transitions are attributed to many Earth system components (*3*, *15*). However, also for these systems, the framework of tipping points may be too limited, and multistability may play a more important role than previously thought.

As a classic example, we consider here how spatial effects lead to the introduction of coexistence states and thereby change the tipping behavior related to the ice-albedo feedback in an Earth's global climate model (*52*, *53*, *105*, *106*). Changes in the Earth's temperature are directly related to changes in the energy budget, which is computed as incoming solar radiation minus reflected and outgoing ("Planck") radiation. The temperature dependency of the albedo is the ice-albedo feedback: as long as temperatures are low, Earth is covered in ice, which does reflect much of the solar radiation. However, when temperatures rise, the ice melts and consequently less radiation is reflected leading to further temperature increase. If one does not take spatial effects into account, as is commonly done, this feedback mechanism leads to two alternative Earth states in this model: An Earth fully covered in ice ("Snowball Earth"), or an Earth with no ice ("No-ice Earth"). There is a bistable region where both of these states exist, and critical transitions and tipping points between those states occur when one stable state vanishes.

However, in reality, we clearly are not in any of those two Earth states, as ice is present only at the polar regions. That is because spatial effects that play an important role in the real Earth's global energy budget are ignored in such a model. For example, incoming solar radiation is latitude dependent and meridional heat flow forms an integral part of energy distribution. When adding such spatial mechanisms, the models predict also coexistence states of ice and no-ice, in addition to the alternative Snowball and No-ice Earth states. In the coexistence states, ice is present in only part of the Earth: near a pole there is ice, while simultaneously there is no ice at the equator, with a spatial boundary between these alternative states occurring at some intermediate latitude. The presence of these additional states changes the classical tipping properties of the non-spatial system: when a fully (un)covered Earth state disappears, a less critical transition to a system with coexistence states might happen and more gradual transitions are possible.

In addition to this example of coexistence states, there is multistability attributed to Earth system components. In fact, the Busse balloon finds its origin in the study of thermal convection (*35*), which is closely related to turbulence in fluid mechanics, and thus relevant also for the global atmosphere and the ocean circulation of the Earth. However, in the example of the Atlantic meridional overturning circulation mentioned above, when using box models with relatively few boxes, tipping is observed (*107*), while multistability of coexistence states is not detected. We suggest that, despite taking into account spatial processes between the boxes, homogenizing within only a restricted number of boxes severely limits the many emergent spatial responses that the system may unfold. Indeed, increasing the spatial resolution, by adding more boxes, increases the number and nature of stable states (*108*, *109*), and gradually brings back the multistability. In this context, an interesting line of research is to investigate whether such tipping of Earth system components in general (*15*) persists in higher resolution models, such as in the state-of-the-art global climate models (*16*), or is replaced by multistability of coexistence states including more gradual transitions. If the latter is the case, this may be a possible explanation why the full-complexity global climate models seem more stable than the simple or intermediate-complexity ones (*110*).

An archetypical system: Evading savanna tipping

Savannas are defined by the coexistence of grasses and trees, spread over one eighth of the land surface worldwide, and are an important source of livelihood for many (*60*). The expected changes in climate and land use can lead to significant alteration of vegetation characteristics and global savanna distributions (*61*, *62*). Tree cover is highly variable and notoriously difficult to predict; it has been suggested that space could be a main element elucidating this, which is missing in many studies of savannas (*63*). Savannas can be considered as a model ecosystem to derive and validate conditions for evading tipping points because of the following two main reasons. First, observations in models and real ecosystems show multiple alternative states and tipping phenomena. The ecosystem may change amongst a system with no trees (barren desert or grassland), open savanna with grasses and scattered trees, closed savanna woodland covered with savanna trees and shrubs, and closed tropical forest with forest trees (*5*, *8-12*, *38*, *64-68*) (Fig. 2). Savanna trees are more fire tolerant and less shade tolerant than forest trees (*69*). This, together with the flammability and fire resistance of grasses that can easily regrow after fires, is the key of the positive feedback mentioned earlier, generating alternative stable states between open savanna and tropical forests. Second, both types of spatial pattern formation as we outlined, namely Turing patterns and coexistence states, have been observed in real and model systems (*12*, *25*, *68*, *70-76*) (Fig. 2). Therefore, studying savanna ecosystems is an excellent opportunity to obtain results that are of general interest and applicability.

Towards a unified spatial savanna model

To the best of our knowledge, no unified savanna model exists that explains all of the possible ecosystem states including spatial Turing patterns and coexistence states. To develop a unified spatially explicit savanna model, existing savanna models (e.g. *71*, *76*, *77*) could be combined in such a way that the new model consists of (at least) the state variables water, grass biomass, savanna tree biomass, and forest tree biomass. Savanna and forest trees should be distinguished by their different fire responses and shade tolerance (*78*, *79*). Non-spatial models that consider rain, fire and herbivory may aid in the development of such unified framework (*64*, *80*) and studies of savanna-forest transitions may also prove useful (*10*, *81*). The new model could include positive feedbacks between water infiltration into the soil and biomass, and between fire and grass biomass (*11*). Herbivory (grazing and browsing) could be added as an extra negative term (*82*) or, possibly as an extra dynamic equation (*83*). It could then be analyzed to address the following questions along a rainfall gradient (Fig. 2): Under which conditions do sharp or gradual spatial boundaries exist between open savanna on the one hand, and tropical forest, savanna woodland, or desert on the other? How are those boundaries evolving or bifurcating, or moving with respect to (changes in) climate, herbivory and fire? Under which conditions do spatial patterns occur, either Turing patterns or coexistence states? Answers to these questions can be obtained by a combination of simulations, mathematical bifurcation analyses and numerical continuation (*84*, *85*).

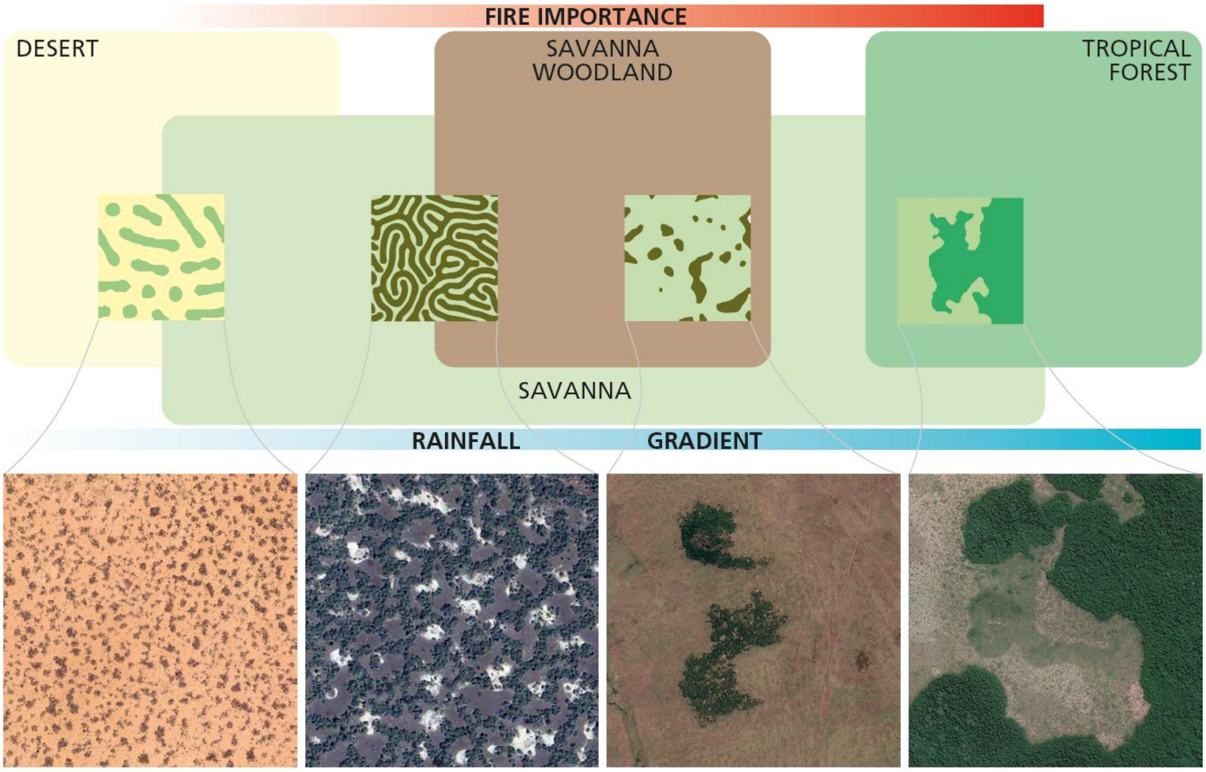

Figure 2. **Savannas along a rainfall gradient.** Open savannas with dispersed trees can be classified into three broad ranges according to annual rainfall (*67*): humid, mesic and dry (blue bar indicating rainfall decreasing from humid to dry, from right to left). The humid range potentially demonstrates bistability with tropical forest. The mesic range shows possible bistability with savanna woodland, and the dry end with barren desert (or grassland, not shown). Fire and herbivory play an important role in savanna dynamics determining varying tree cover given a certain amount of rainfall. The importance of fire (illustrated by the red shade in the fire bar) decreases from humid toward dry ranges, where water availability is the main limiting factor and driving force. We compare the expected spatial structures with symbolic Google Earth image examples, from right to left with decreasing rainfall, namely: coexistence states between forests and humid savannas (Gabon 1^0 16′ 1.06′′ S - 13^0 56′ 18.63′′ E, 2000 x 2000 m.), coexistence states as well as Turing patterns between savanna woodland and mesic open savanna (Kenya 1^0 28′ 8.83′′ S - 34^0 53′ 55.60′′ E, 1200 x 1200 m.; Mali 13^0 03′ 15.09′′ N - 6^0 40′ 50.86′′ W, 800 x 800 m.), and Turing patterns between dry savannas and desert (Sudan 11^0 26′ 47.53′′ N - 27^0 53′ 30.07′′ E, 1500 x 1500 m.). There is no unifying model explaining all savanna states and spatial patterns observed. We suggest that developing and analyzing such a unifying savanna model is needed, by incorporating relevant state variables and parameters, by making the model spatially

explicit, and by a combination of mathematical bifurcation analyses, simulations and numerical continuation methods (*84*, *85*).

General new predictions based on the theory that could be tested with such unified model are as follows (Fig. 2):

- At the humid end of the rainfall gradient, at the tropical forest-savanna boundaries, where fire is generally assumed to drive alternative stable states between tropical forests and open savannas, we expect to discover coexistence states (*cf*. *68*, *86*), evading tipping.
- In the mesic range, where a combination of fire and herbivory drives alternative states between closed savanna woodland and open savanna, we expect to find both coexistence states and Turing patterns (*70*, *73*, *74*), evading tipping.
- At the dry end of the gradient, where dryness drives alternative stable states between open savanna on the one hand, and desert on the other, we suppose that the occurrence of Turing patterns associated with Busse balloons (*33*, *34*) leads to the evasion of tipping points.
- Increased fire intensity will lead to movement of the tropical forest-savanna boundary towards tropical forest (humid savannas replacing forest) (*81*). A decrease in fire or herbivory will lead to savanna woodland invading mesic open savanna, and decreased rainfall will lead to desert replacing dry savanna.

Observations of spatial patterns evading tipping points

Towards the wet end of the rainfall gradient, sharp tropical forest-savanna fronts (*87*) without spatial pattern formation could be interpreted as spatial substitutions of critical transitions (*88*). However, it is not clear for which conditions tropical forest-savanna boundaries are sharp or gradual, whether they are moving and whether patterned structures of alternative stable states (coexistence states) occur. So, the same questions as for the model analyses described above can be addressed through observations. After being tested by model analyses, the predictions mentioned above can be validated, focusing on fire and dryness as main drivers. Spatial signatures of sharp vegetation boundaries (*88*) between tropical forests and open savannas can be explored worldwide from satellite data (*cf*. *87*, *89*). This can be done at multiple spatial resolutions to create time series for multiple years or decades (*90*), to detect vegetation boundaries at both coarser and finer resolutions, necessary to screen boundaries of spatial patterned structures of coexistence states (*cf*. *68*, *86*) and Turing patterns, which could both be easily overlooked by only using coarser spatial resolutions. The same approach can be followed for open savanna-savanna woodland boundaries for the mesic ranges of the rainfall gradient and for dry savanna-desert boundaries at the dry end along the gradient.

Such findings can be related to the main drivers such as dryness and fire as mentioned above through change indicators (*91*, *92*). Metrics of spatial boundaries can also be correlated to annual burned area maps (*93*). This would allow tests of the predictions of how these drivers determine the sharpness of vegetation boundaries, whether patterned structures of coexistence states and Turing patterns occur, and how moving of the boundaries are associated with a change of the main drivers. This is important in the light of expected changes in climate and land use, and it will provide insight in the conditions for which tipping points may be evaded or not.

Universal conditions for spatial pattern formation and multistability

A major part of the mathematical literature on spatial pattern formation focuses on the onset of Turing patterns close to homogeneous equilibria (*94*). However, real ecosystems and Earth system components in their natural state typically are not close to such onset at which the patterns first emerge, but "far from equilibrium" instead, at which the spatial patterns are fully developed. There is only limited mathematical insight in many of the spatial patterns in real systems, unless the governing system shows a sufficient large scale separation, which is the condition that spatial transport of the components of the system plays out on vastly different spatial and temporal scales. A system for which this occurs is called "singularly perturbed" in the mathematical literature. Notably, the spatially extended models and real systems considered here, typically are singularly perturbed, where the singularly perturbed nature stems from observations that pattern formation in ecosystems is indeed driven by counteracting feedback mechanisms on widely different spatial scales (*31*). Therefore, the realistic, far from equilibrium patterns considered here can be studied in mathematical detail by the methods of singular perturbation theory (*95*). Moreover, the most relevant patterns commonly exhibited by ecosystems and Earth system components (including the examples of Box 2 and 3) have the nature of interacting localized structures, such as areas with vegetation bands surrounded by areas with bare soil (*33*), the boundaries mentioned above between open savanna and savanna woodlands (*73*) and the grounding line of glaciers (*54*). Resilience by multistability (*33*, *34*) is directly coupled to the very rich variety of patterns associated to interacting localized structures like localized stripes and spatial boundaries.

Therefore, the formation of these fundamental spatial patterns and multistability provide mechanisms by which spatial ecosystems (Box 2) or Earth system components (Box 3), including the complex Earth system as a whole, may evade tipping in general. Whether this occurs or not, will depend on local (but system-wide) conditions in real systems, or parameter combinations in models. For various classes of systems, there will be universal conditions that lead to evasion of tipping points through spatial pattern formation and multistability, such as for savanna ecosystems. These are classes of ecosystems and Earth system components for which currently tipping points are predicted.

Turing-before-tipping.

Preliminary analyses of spatially extended ecosystems, such as dry savannas, modeled by activator-inhibitor type reaction-diffusion equations (*27*, *75*, *76*), show that tipping may, or may not, be preceded by a pattern forming Turing bifurcation (Box 1, Fig. 1). The conditions for which this may, or may not, happen can be explicitly expressed in terms of model parameters (Box 1). However, the Turing bifurcation only gives insight into the onset of spatial patterns, but not into their behavior beyond onset (far from equilibrium) where the patterns become more developed and observable. Moreover, it is the dynamics beyond onset that determine if, and how, these Turing patterns lead to evasion of tipping. An explicit scenario has been unraveled within the literature on dry savannas (*32*, *34*) where Turing patterns evolved beyond the onset in such a way that indeed tipping is evaded in the system. The crucial question now is whether this scenario may also play a role in other types of systems and whether there are alternative scenarios through which small amplitude Turing patterns may evolve into large scale interacting localized structures that enable the system to evade tipping. Due to the singularly perturbed nature of the models, it is possible to study the basic localized structures and their interactions mathematically (*96*-*98*). To make the crucial connection between these localized patterns and those that appear from the Turing bifurcation, a further analysis of the Busse balloon, and especially the nature of its boundary, by a combination of bifurcation analyses, simulations and numerical continuation (*32*, *84*, *85*) is necessary.

Such analyses will also reveal the dynamics of systems preceding bifurcation points and hence lead to better early-warning signs. The classical theory of tipping, predominantly based on non-spatial models, led to the creation of generic early-warning signs before such tipping occurs (*2*, *99*), mainly associated with critical slowing-down of recovery after perturbation. In spatial systems, this idea becomes more complicated, because this may now crucially depend on the nature of the different spatial perturbations; critical slowing-down will only show for perturbations that have a specific spatial structure (*74*), which may be hard or even impossible to detect in spatially averaged data. Simultaneously, the form of the perturbation that does show critical slowing-down can help to determine the type of bifurcation before onset; in other words, whether a system will tip or evade tipping by forming spatial patterns. After the system is exposed to some generic perturbation (that might be e.g. random or localized) before a bifurcation is reached, the system will restore during a transient restoration period, and temporarily either a patterned or a non-patterned state may emerge. The form of this emerging transient state could now be distinctive and the type of bifurcation it precedes could be identified. Leading up to Turing bifurcation, such emerging transient state may be some spatially periodic pattern. In contrast, tipping bifurcation could be preceded by emerging transient states that are either spatially homogeneous or extremely localized. This would make it possible to distinguish between both types of bifurcations from spatial time series before they occur. Of course, one main challenge is the extraction of these spatial perturbations from data, but existing so-called mode decomposition algorithms (e.g. *100*) may be a viable option.

Destabilizations of interfaces between coexistence states.

Isolated interfaces between coexistence states, such as the open savanna-savanna woodland boundary, the tropical forest-savanna boundary (Box 2 and Fig. 2), and the interfaces between ocean and ice (Box 3), are necessary for the multistability by which tipping of the whole spatial system through perturbation of the stable state may be evaded. However, for the spatial coexistence patterns to persist beyond the tipping point with worsening external conditions, an additional mechanism is required and likely. Similarly to homogeneous states that may be destabilized by Turing bifurcations, spatial fronts between coexistence states may typically also bifurcate and may thus be the origin of a multitude of evolving localized spatial patterns. These patterns subsequently may provide the ecosystem or Earth system component with further multistability and with various gradual routes it may follow beyond the tipping point; in other words, with evasion of the tipping point when environmental conditions worsen. These bifurcations can be traced numerically (*51*, *55*). But more importantly, once again, the singular perturbed nature of the models allows for developing a fundamental understanding of the underlying destabilizing mechanisms (*98*, *101*). For instance, conditions can be derived for which an invading front that leaves a homogeneous stable state behind triggers a counter-invasion of an alternative patterned state, which comprises multistability and thus a gradual route, thereby circumventing tipping points.

The impact of domain, localized and non-local homogenizing effects.

Most mathematical studies of spatial pattern formation in spatially extended complex systems take place assuming a highly idealized domain: a sufficiently large open space or volume in which environmental conditions do not change throughout the domain. In real systems, however, such idealized domains do not exist and it is not evident if, and how, results carry over from idealized domains to more realistic ones. For instance, if the spatial domain in which a system can evolve is too small for spatial pattern formation, evasion of tipping cannot work anymore. A prominent example could be alternative stable states and tipping of the spatially confined and shallow lakes (*6*). This implies there is a minimum domain size for complex systems to form spatial patterns and enhance resilience; this minimum size would depend on the spatial scale of the dominant mechanisms and resulting spatial patterns of the specific system under consideration.

Moreover, localized effects, e.g. by human interventions, can have a strong impact on the formation, stability and dynamics of patterns (*102*). Examples are logging in tropical forests and imposed fixing of sand dunes or building dikes in coastal dune systems. The point here is that, albeit localized, such human perturbation may significantly reduce the flexibility and thus resilience of the patterned system as a whole. Similarly, and probably even more importantly, the same can happen if humans homogenize spatial patterns characteristic of pristine systems non-locally, because the mechanisms outlined here enhancing resilience will not function anymore. Examples are large-scale agriculture in terrestrial ecosystems, spatially homogeneous restoration efforts combating desertification, and destructive bottom trawling in marine ecosystems. Therefore, the study of resilience through spatial pattern formation in complex systems should be embedded in a thorough analysis of the impact of spatial restrictions of the domain and the effects of localized and non-local homogenizing, human induced effects. A relevant approach would be a combination of computational and analytical studies to determine the effects of such spatial (in)homogeneities on pattern dynamics and resilience (*103*). This is important for ecosystem restoration and mitigating the effects of land use and climate change.

Conclusions

Here we have shown how spatial self-organization and multistability resulting from Turing patterns may help complex systems evade tipping points and enhance resilience. Additionally, we have outlined that the ranges of conditions and mechanisms for which tipping is evaded are supposed to be much broader than those leading to Turing patterns, owing to more comprehensive spatial pattern formation and multistability, including the occurrence of coexistence states. We highlighted that both types of spatial pattern formation originate before and can persist beyond tipping points, demonstrating various pathways evading tipping, while strongly enhancing the resilience. We emphasized that such spatial pattern formation and multistability have also been observed recently for real ecosystems, and we argued this may be relevant for many ecosystems and Earth system components, including for complex system Earth as a whole. Savannas can be considered an archetypal ecosystem to further investigate this, because of observations of tipping phenomena together with spatial pattern formation. Better understanding of the dynamics of spatial pattern formation in general is needed to determine how these respond to external changes of various magnitudes and rates, and to localized and non-local homogenizing perturbations. Such understanding will help determine which conditions and spatial patterns lead to the evasion of tipping and which do not. We expect that identifying these in the many ecosystems and Earth system components that are supposedly tipping-prone will reveal that some are much more resilient than currently thought.

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