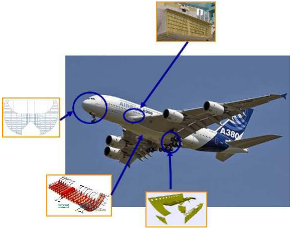
Innovative Multilevel Techniques for Structural Optimization

Margherita Porcelli

ISTI - Area della Ricerca CNR Pisa



Joint work with

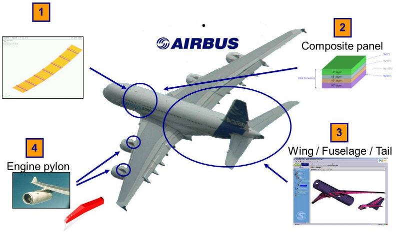
Benoît Colson, LMS Samtech, Liége, Belgium

Philippe L. Toint, FUNDP - University of Namur, Belgium

The LARGO (LARGe-scale Optimization problems) project

- Objective of LARGO: design new numerical methods for solving very large constrained optimization problems (10^4 vars, 10^6 constrs)
- Aircraft optimization: problems arising in the design phase of an aeronautical structure.

Minimize the mass of a fuselage where the design variables are subjected to static mechanic criteria e.g. buckling, strain (Reserve Factors).

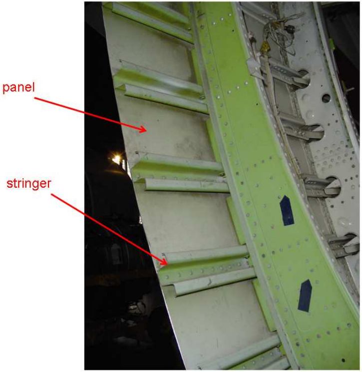


The main tasks of LARGO

- **Task 1** Study of the aeronautical optimization problem addressed by LMS SAMTECH and its structure;
- **Task 2** Design of a numerical algorithm suitable for the solution of the problem (multilevel approach);
- **Task 3** Implementation and numerical validation of the proposed algorithm:
  - Tests on academic problems and real models;
  - Comparison with existing commercial software.

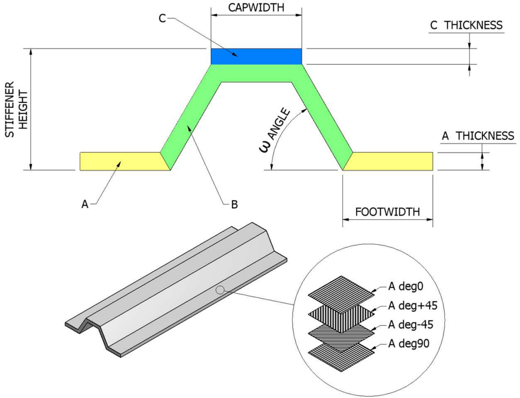
The fuselage structure

- The elementary parts of a fuselage are called super stiffeners: composite stiffened panels consisted of a stringer and two half panels.



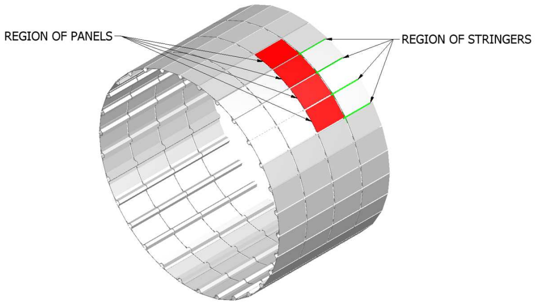
Design Variables (DVs)

- Local geometry parameters: e.g. panel thickness, stringer cross-section dimensions, stringer height.
- Composite laminate variables: e.g. skin laminate percentages.



Hierarchical structure of the problem: regions

- Panels and stringers may be grouped into regions of panels and regions of stringers.
- Members of the same region share the same design variables.



The optimization problem

- The aeronautical problem: minimize the mass of a fuselage where the design variables are subjected to static mechanic criteria (RFs).

Minimization problem with bound constraints and general nonlinear inequality constraints.

$$\begin{array}{ll} \min_x & M(x) \\ \text{subject to} & RF(x) \geq 1 \\ & l \leq x \leq u, \end{array}$$

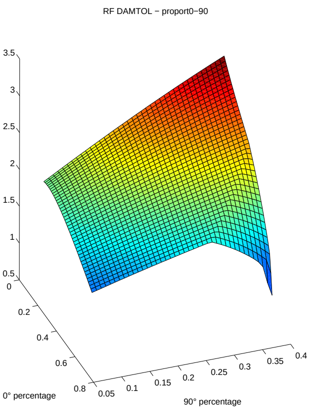
$M : \mathbb{R}^n \rightarrow \mathbb{R}$
$RF : \mathbb{R}^n \rightarrow \mathbb{R}^m$ *expensive black-box*

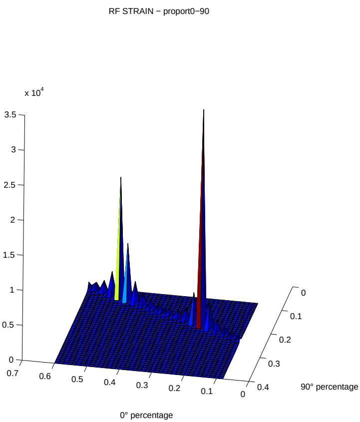
The Reserve Factor evaluations

- The constraint function values are the result of a simulation process
  - skill tools: “in house black-box” codes
  - rapid sizing: smooth approximation (interpolation techniques) of the skill tools (faster)
- The derivatives (Jacobian) are approximated by finite differences.
- The RFs are locally defined on each element of the structure.
- No information on the regularity or convexity of the RFs.

Study the local geometry of the functions and the structure of the Jacobian.

The Reserve Factors: results of the parametric study





Classical approach: decomposition methods

- Individual components of the problems are optimized separately without considering the entire hierarchy, see e.g. [Sobieszczanski-Sobieski et al., 1987].
- Pitfall: computation of optimal solutions with respect to individual components but the combination of such components yields to nonoptimal structures (convergence?).

New approach: multilevel optimization

- The problem is optimized at the global level exploiting at the same time its multilevel structure.
  - Bound constraints:
    Recursive Multilevel Trust Region method (RMTR) [Gratton, Sartenaer, Toint, 2008, Gratton, Mouffe, Toint, Weber-Mendonça, 2008, ...]
  - General constraints:
    Augmented Lagrangian merit function + line-search [Nash, 2010]
    SQP trust-region for PDE constrained optimization [Ziems, Ulbrich, 2011]
    Globalization strategy for elliptic optimal control pbs [Borzì, Kunish, 2005]

FUNDP NAMUR

The RMTR method for bound-constrained problems

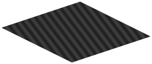
The problem

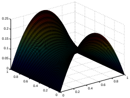
$$\min_{x \in \mathcal{F}} f(x)$$

- $f : \mathbb{R}^n \to \mathbb{R}$ nonlinear, in $\mathcal{C}^2$ and bounded below
- No convexity assumption.
- Let g and H denote the gradient and the Hessian (or an appoximation) of f.
- $\mathcal{F} = \{x \in \mathbb{R}^n : l \leq x \leq u\}$ (possible bound constraints).

Interesting case: the problem is the result of the discretization of some infinite-dimensional problem on a fine grid (n large).

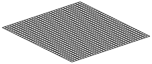
Hierarchy of problem description $(n_r > n_{r-1} > \dots n_1)$

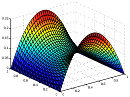




Finest Level $f_r : \mathbb{R}^{n_r} \rightarrow \mathbb{R}$

Restriction $\downarrow R_r$ — $P_r \uparrow$ Prolongation



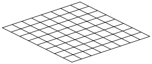


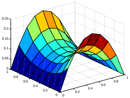
Fine Level $f_{r-1} : \mathbb{R}^{n_{r-1}} \rightarrow \mathbb{R}$

Restriction $\downarrow R_{r-1}$ — $P_{r-1} \uparrow$ Prolongation

$\vdots$

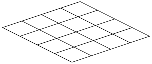
Restriction $\downarrow R_2$ — $P_2 \uparrow$ Prolongation

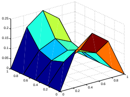




Coarse Level $f_2 : \mathbb{R}^{n_2} \rightarrow \mathbb{R}$

Restriction $\downarrow R_1$ — $P_1 \uparrow$ Prolongation



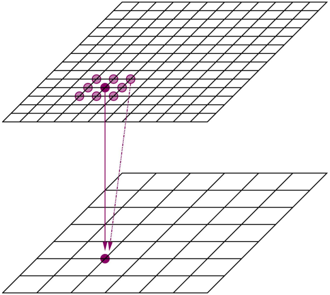


Coarsest Level $f_1 : \mathbb{R}^{n_1} \rightarrow \mathbb{R}$

Grid transfer operators

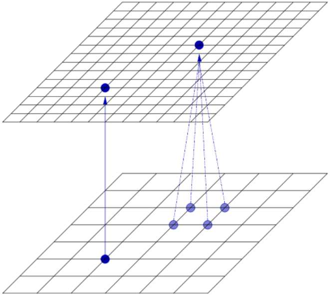
Restriction

$R_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_{i-1}}$



Prolongation

$P_i : \mathbb{R}^{n_{i-1}} \rightarrow \mathbb{R}^{n_i}$



In practice:

- linear interpolation
- cubic interpolation

$$R_i = \sigma_i {P_i}^T$$

The Recursive Multilevel Trust-Region Method (RMTR)

- At each iteration k, compute a trial step s_k by

Smoothing $\longrightarrow$ minimize Taylor's model of f_{up} around x_k within trust-region of radius Δ_k (SCM)

Coarsening $\longrightarrow$

compute g_{up}, H_{up}	trial step s_k
Restriction $\downarrow R$	$P \uparrow$ Prolong.

minimize a coarse model q_{low} around Rx_k within the trust-region of radius Δ_k

- Apply this scheme recursively if several levels

- unconstrained ($\|\cdot\|_2$): [Gratton, Sartenear, Toint, 2008]
- bound-constrained ($\|\cdot\|_\infty$): [Gratton, Mouffe, Toint, Mendonça, 2008]

Models Definition

Taylor model: $m(s) = f_{up} + \langle s, g_{up} \rangle + \frac{1}{2} \langle s, H_{up} s \rangle$

Coarse model:

- Impose first-order coherence via a correction term:

$$g_{low} = R g_{up}$$

- Impose second-order coherence via two correction terms:

$$g_{low} = R g_{up} \quad \text{and} \quad H_{low} = R H_{up} P$$

- Galerkin approximation: $f_{low} = 0$

$$f_{up} \approx q_{low}(s) = f_{low} + \langle s, R g_{up} \rangle + \frac{1}{2} \langle s, R H_{up} P s \rangle$$

$$f_{up} \approx q_{low}(s) = \langle s, R g_{up} \rangle + \frac{1}{2} \langle s, R H_{up} P s \rangle$$

Models Definition

Taylor model: $m(s) = f_{\text{up}} + \langle s, g_{\text{up}} \rangle + \frac{1}{2} \langle s, H_{\text{up}} s \rangle$

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$$f_{\text{up}} \approx q_{\text{low}}(s) = \langle s, R g_{\text{up}} \rangle + \frac{1}{2} \langle s, R H_{\text{up}} P s \rangle$$

Multilevel on Finest scheme

Annihilate oscillatory error level by level (V-cycle):

pre smooth $\longrightarrow$ | Smooth fine e $\downarrow R$ Oscil. coarse e | pre smooth $\longrightarrow$ | (recur) | post smooth $\longrightarrow$ | Smooth coarse e $\uparrow P$ Smaller fine e | post smooth $\longrightarrow$

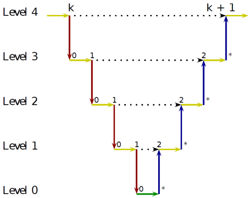
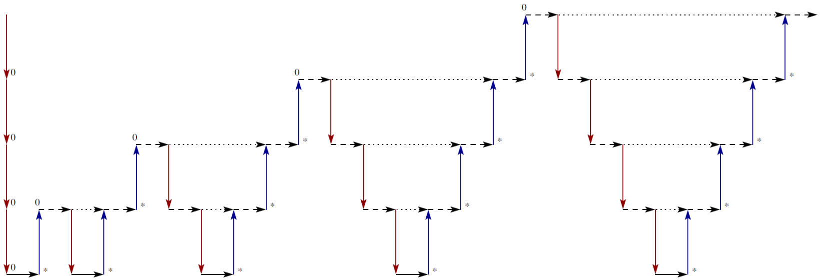


Figure: Multilevel on Finest (MF) scheme

Full Multilevel (FM) scheme



- FM performs a V-cycle scheme to compute the problem solution at each of the increasingly finer grids used in the mesh refinement, i.e. the solution at coarser level is used as a good starting point for the next level.

⋆

The aeronautical problem

$$\begin{array}{ll} \min_x & M(x) \\ \text{subject to} & RF(x) \geq 1 \\ & l \leq x \leq u, \end{array}$$

where $M(x)$ is the overall mass and $RF(x)$ are the RFs.

Minimization problems with simple bounds and nonlinear equality constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & h(x) = 0 \\ & l \leq x \leq u, \end{array}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R}^n \to \mathbb{R}^m$.
Two reformulation to define equalities h:

- **R-slack** by adding slack variables $s \in \mathbb{R}^m$:
  $h_s(x, s) = 1 - RF(x) + s, s \geq 0$;
- **R-max**^{2}: $h_m(x) = \max(1 - RF(x), 0)^2$.

The bound-constrained Augmented Lagrangian approach

Problem with simple bounds and nonlinear equality constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & h(x) = 0 \\ & l \le x \le u, \end{array}$$

with $f : \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R}^n \to \mathbb{R}^m$ [LANCELOT, 1992].

We use the Augmented Lagrangian function

$$\mathcal{L}_A(x, \lambda; \mu) = f(x) - \lambda^T h(x) + \frac{\mu}{2} h(x)^T h(x)$$

where $\mu > 0$ is the "penalty parameter" and $\lambda \in \mathbb{R}^m$ is an explicit estimate of the Lagrange multipliers $\lambda \in \mathbb{R}^m$.
We solve a sequence of bound-constrained problems:

Given x_k, λ_k and μ_k, find x_{k+1} s.t.

$$x_{k+1} = \operatorname*{argmin}_x \mathcal{L}_A(x, \lambda_k; \mu_k) \text{ s.t. } l \le x \le u.$$

Update λ_k and μ_k based on x_{k+1}.

The Augmented Lagrangian RMTR (ALRMTR) method

Use RMTR to solve the subproblem

$$\min_x \quad \mathcal{L}_A(x, \lambda_k; \mu_k)$$
$$\text{subject to} \quad l \le x \le u.$$

$\Rightarrow$ definition of a multilevel structure for the dual variables $\lambda \in \mathbb{R}^m$

Different implementations depending on the multilevel scheme (FM or MF) and the use of **R-max**^{2} or **R-slack**

1. ALRMTR-FM (Full Multilevel)
2. ALRMTR-MF (Multilevel on Finest):
   - Galerkin approximation: compute values of $\mathcal{L}_A$ only at the finest level;
   - No multilevel structure for dual variables is employed if **R-max**^{2} is used.

Prolongation/Restriction operators for dual variables

- Use the same operators of the primal variables.
- Note that the dual variables associated with continuous constraints are not necessarily continuous (δ-function-like behaviour).
- Approximate the dual variables by a piece-wise linear function may not fully capture the behaviour of λ.
- Idea: smooth the multipliers before applying the Prolongation/Restriction operator.
- The inverse of the Laplacian operator Δ^{-1} may be used as a smoother:

$$\lambda_{up} \xrightarrow{\text{smooth}} \Delta^{-1}\lambda_{up} \xrightarrow{\text{restriction}} R(\Delta^{-1}\lambda_{up}) \longrightarrow \lambda_{low} = \Delta R \Delta^{-1}\lambda_{up}$$

$\star$ [Bank, Gill, Marcia, 2003]

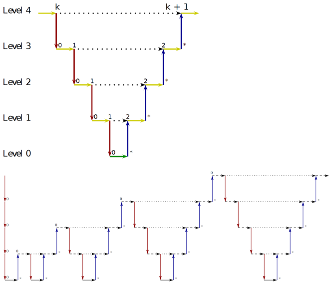
Compared procedures

ALRMTR-AM: All on Finest
Standard Newton trust-region algorithm (PTCG)

ALRMTR-MF: Multilevel on Finest
Algorithm RMTR applied at the finest level

ALRMTR-FM: Full Multilevel
Algorithm RMTR applied successively from coarsest to finest level.

BOSS quattro optimization solvers:
GCM, CONLIN, SQP



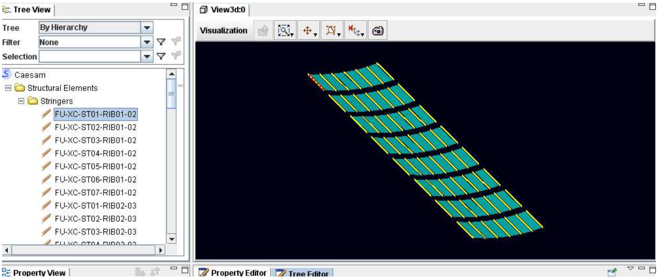
The problems

- Well-known unconstrained problems: the minimum surface problem and the BRATU problem.
- Ad-hoc inequality constraints to mimic RFs: upper bound on the local curvature.

Comments

- All the multilevel implementations converge to the solution (MF and FM more efficient than AF);
- ALRMTR is more robust and more accurate than BOSS quattro.
- ALRMTR -**R** - **max**^{2} is more efficient than ALRMTR-**R** - **slack** (slack structure not fully exploited).
- ALRMTR-**R** - **slack** is more accurate (inequality satisfaction) than ALRMTR-**R** - **max**^{2}.

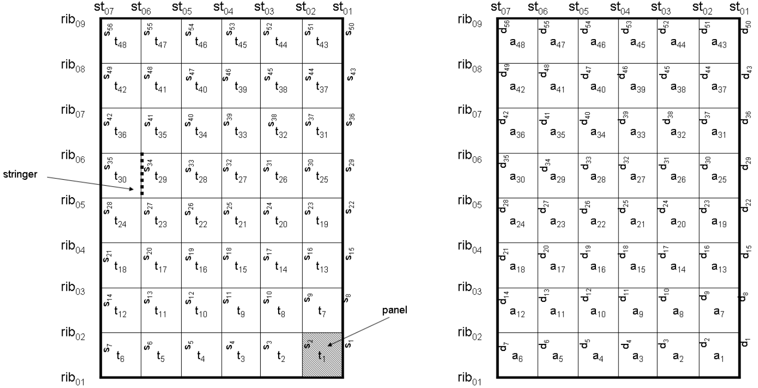
Small test problem: rectangular piece of an aircraft fuselage (RFUSE) (6×8 panels and 7×8 stringers)



- 2 DVs per element: the panel thickness t and the stringer section area s.

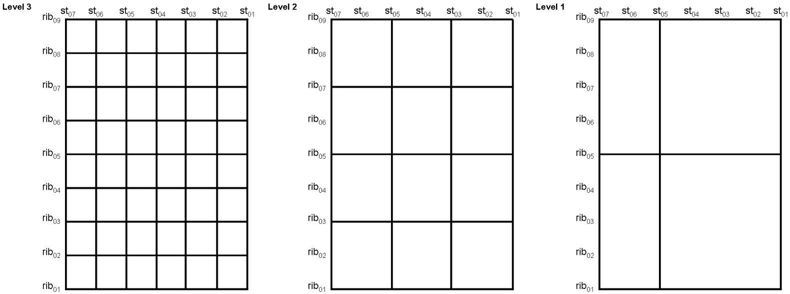
Test case provided by LMS SAMTECH

The industrial problem: RFUSE



- DVs: the panel thickness t_i and the stringer section area s_i.
- Data: the panel area a_i and the stringer length d_i.
- RFs: 3 RFs per calculation points (internal stringer).

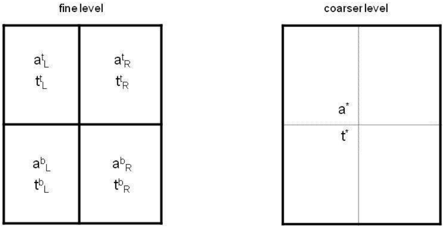
Multilevel structure



level	N_l	N_c	# t	# s	# DVs	# CPs	# RFs
3	8	6	48	56	104	40	120
2	4	3	12	16	28	6	24
1	2	2	4	6	10	2	6

Table: Multilevel dimensions for RFUSE.

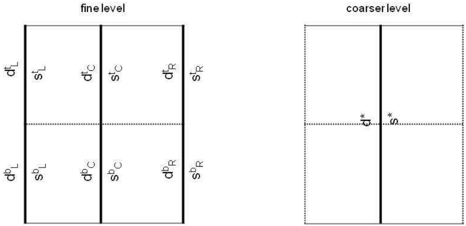
Transfer operator: panel thickness t



$$t^* = \frac{a_R^b t_R^b + a_R^t t_R^t + a_L^b t_L^b + a_L^t t_L^t}{a_R^b + a_R^t + a_L^b + a_L^t}$$

$$a^* = a_b^R + a_t^R + a_b^L + a_t^L$$

Transfer operator: stringer section area s

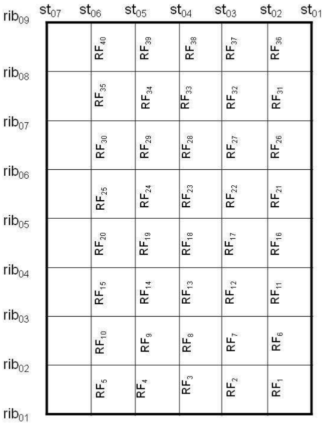


$$s^b = \frac{d_L^b s_L^b/2 + d_C^b s_C^b + d_R^b s_R^b/2}{d_L^b/2 + d_C^b + d_R^b/2}, \quad d^b = \frac{d_L^b/2 + d_C^b + d_R^b/2}{2},$$

$$s^t = \frac{d_L^t s_L^t/2 + d_C^t s_C^t + d_R^t s_R^t/2}{d_L^t/2 + d_C^t + d_R^t/2}, \quad d^t = \frac{d_L^t/2 + d_C^t + d_R^t/2}{2},$$

$$s^* = \frac{d^b s^b + d^t s^t}{d^b + d^t}, \quad d^* = \frac{\left(d_L^b/2 + d_C^b + d_R^b/2\right) + \left(d_L^t/2 + d_C^t + d_R^t/2\right)}{4}$$

Transfer operator: RF constraints



Implementation issues for RFUSE

- **R - max**^{2}: $\nabla \mathcal{L}_A(x,\lambda;\mu) = \nabla f(x) - J_{RF}^T \lambda + \mu J_{RF}^T h(x)$
  R - slack:
  $$\nabla \mathcal{L}_A((x,s),\lambda;\mu) = \begin{pmatrix} \nabla f(x) \\ 0 \end{pmatrix} - \begin{pmatrix} J_{RF}^T \\ I \end{pmatrix} \lambda + \mu \begin{pmatrix} J_{RF}^T \\ I \end{pmatrix} h(x,s)$$
  ∇f and J_{RF} are provided by BOSS
  $\nabla^2 \mathcal{L}_A$ approximated by a diagonal matrix.
- MF scheme and Galerkin model (functions only at finest level).
- Ad hoc linear interpolation operators for DVs and dual variables.
- Problem dimensions (3 levels):
  R - max^2: $n_3 = 104, n_2 = 28, n_1 = 10,$
  R - slack: $n_3 = 224, n_2 = 52, n_1 = 16,$
  $m_3 = 120, m_2 = 54, m_1 = 6.$
- Initial point:
  - **R - max**^{2}: $x_0 = 0$
  - **R - slack**: $x_0 = 0, s_0 = 0$ and $x_0 = 0, s_0 = s_f = RF(x_0) - 1.$

Results

	f^*	$\|h^*\|_\infty$	$\#v^*$	$\max v^*$	τ^*	$\#a^*$
BOSS-GCM	27.55		18	2.3E-05		64
MF-**R** - **max**^{2} $x^* = l$	25.25	6.5E-02	49	3.6E-01	0.0E+00	104
MF-**R** - **slack** $s_0 = s_f$	39.41	3.4E-01	26	3.3E-01	8.7E-02	48
MF-**R** - **slack** $x_0 = x^*_{gcm}, s_0 = s_f$	26.32	3.3E-01	40	3.3E-01	8.5E-02	64

$$\#v^* = \#RF^* < 1,\ \max v^* = \max(1 - RF^*),\ \#a^* = \#x^* = \{l, u\}$$

⋆

Conclusion

Fuselage problem

- BOSS quattro is better than ALRMTR-MF (constraint violations count, number of iterations)
- ALRMTR-**R** - **slack** is slightly better than ALRMTR-**R** - **max**^{2} in terms of number of constraint violation $\max v^*$.

the practical use of multilevel techniques in aircraft optimization deserves further research...

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Thanks for your attention