# Gyrocontinua

## Girocontinui



*Maurizio Brocato*
Istituto di Elaborazione della Informazione
Consiglio Nazionale delle Ricerche
via Santa Maria 46, 56126–Pisa, Italia
brocato@iei.pi.cnr.it

We name gyrocontina a particular example of continua with microstructure, more precisely of Cosserat's continua, interesting in view of applications to control theory.

Actuation of structures' control through gyroscopic effects has recently became viable thanks, especially, to reduction of weight penalties, obtainable using a number of small, fast rotors, and to the possibility of diffuse monitoring and guidance.

Rotors can have their axis pin-fixed to the main structure; movements of the latter will affect their rotatory inertia and be affected by the reactions arising in the pins. Thus, tuning the spin velocity, control of the structure's movements can be achieved.

More complex devices can be imagined, where these reactions do not result from such perfect constraints as the above mentioned pins, but must be described through constitutive assumptions (*e.g.* flexible joints may allow moderate changes of direction of the gyroscopic axis) or, else, can be controlled too.

Here, to furnish a general model, we study the case of a continuum the microstructure of which is a rotating mass with a privileged axis which is not kinematically constrained: both the angular spin about the axis and its precession velocity vary independently. Constraints such as null precession or constant spin will be treated as particular occurrences.

Results are available in the literature to deal with the settled physical problem. Therefore, with reference—*e.g.*—to [1] (§19 and §23), we might assume $\nu$ (the microstructure) be $R \in Orth^+$ (the absolute rotations of gyroscopes), $\dot{\nu}$ be $w = -\frac{1}{2}\mathrm{e}(\dot{R}R^T)$, $\mathcal{A}$ (the infinitesimal generator of rotations of the microstructure) be $\mathrm{e}R$ and draw the consequences.

Alternatively, one may derive the wanted equations of motion calling upon the method of virtual power (*cf. e.g.* [5]).

Let $J$ denote the inertia tensor of a gyroscope, $\alpha_1$ its largest eigenvalue and $\alpha_2$ its only other, double, eigenvalue, $g$—with $|g| = 1$—the present gyroscopic direction (*i.e.* the eigenvector associated with $\alpha_1$), $\omega$ the relative spin velocity about $g$, $p \perp g$—with $|p| = 1$—the present direction of precession of $g$ and $\pi$ the relative precession velocity; let $q := \pi p$. ('Relative' refers here to a frame moving with the material point endowed with that particular gyroscope, whilst 'absolute' refers to Galilean observers).

Let us take $g$, $\omega$, $p$ and $\pi$ as fields on the present configuration of the body, variable from point to point and in time; eigenvalues $\alpha_1$ and $\alpha_2$ are supposedly the same for all points and at all times.

Other symbols are standard within an Eulerian description of motion in the frame of work of continua with microstructure.

With the above listed notations, gyroscopes' absolute spin writes:

$$w = \omega g + \pi g \times p - \frac{1}{2}\,\mathrm{rot} v\,. \tag{1}$$

Internal micro-actions can be optionally defined through a vector $z$, dual to the absolute spin $w$, and a second order tensor $S$, dual to grad$w$, or through variables (the constitutive



...ature of which is easier to under... $\omega$, $s$ to grad$\omega$, $\bar{z}$ to $q$ and $\bar{S}$ to ... Equivalence of the two sets of ...

$$\zeta = z \cdot ...$$
$$\bar{z} = z \times ...$$

In addition to the usual ext... external actions $b_g$ and $f_g$, dual ... in the unit volume and on the ... understood as the torque supplie... The kinetic energy of the gyro... recalling that the eigenvalues of ... Therefore:

$$\dot{\kappa} ...$$

In order the kinetic energy t... a vector that $\dot{\kappa} = b_{Ig} \cdot w$; this de... presentation of the issue), allowi...

$$b_{Ig} = ...$$

(note that $\frac{\partial J}{\partial \tau} w$ is powerless).

Under constraint $\pi = 0$ an... tinuum is much smaller then t... expressions:

$$b_{Ig} \approx \frac{1}{2}\alpha_1 \omega g ...$$

With these notations and t... write:

$$\begin{cases} \mathrm{div}T ... \\ z + ... \\ \mathrm{e}T + ... \\ Tn + ... \\ Sn = ... \end{cases}$$

(rot$_*$ denotes a rotation comp...

$$\begin{cases} \mathrm{div\,sym}T - ... \\ \zeta + \mathrm{div}s + ... \\ \bar{z} + \mathrm{div}\bar{S} + ... \\ [\mathrm{sym}T + \frac{1}{2} ... \\ s \cdot n = f_g \cdot ... \\ \bar{S}n = f_g \times ... \end{cases}$$

(substitution div$S = g\,$div$s$ − ...
The kinematic constraint ... reactions appear there, but f... $\bar{z}$.

feature of which is easier to understand) that are dual to relative degrees of freedom: $\zeta$ dual to $\omega$, $s$ to grad$\omega$, $\bar{z}$ to $q$ and $\bar{S}$ to grad$q$.

Equivalence of the two sets of variables occurs under conditions:

$$\begin{aligned} \zeta &= z \cdot g - S \cdot \mathrm{grad} g \quad ; \quad s = S^T g \\ \bar{z} &= z \times g + \mathrm{e}[(\mathrm{grad} g) S] \quad ; \quad \bar{S} = (\mathrm{e} g) S \, . \end{aligned} \tag{2}$$

In addition to the usual external body and surface forces, we need introducing global external actions $b_g$ and $f_g$, dual to $w$ and applied respectively to the gyroscopes embedded in the unit volume and on the unit boundary's surface. Their projections along $g$ can be understood as the torque supplied to the gyroscopes.

The kinetic energy of the gyroscopes is $\kappa := \frac{1}{2} w \cdot J w$; its time derivative must be computed recalling that the eigenvalues of $J$, $\alpha_1$ and $\alpha_2$, are fixed, while $g$ and $p$ rotate with spin $w$. Therefore:

$$\dot{\kappa} = w \cdot J \dot{w} + \frac{1}{2} w \cdot [(\mathrm{grad} J) v] w . \tag{3}$$

In order the kinetic energy theorem be verified, the inertia $b_{Ig}$ of gyroscopes must be such a vector that $\dot{\kappa} = b_{Ig} \cdot w$; this defines it up to a component normal to $w$ (*cf.* [2] for a general presentation of the issue), allowing the special assumption:

$$b_{Ig} = \frac{\partial J w}{\partial \tau} + [\mathrm{grad}(J w)] v - \frac{1}{2} [(\mathrm{grad} J) v] w \tag{4}$$

(note that $\frac{\partial J}{\partial \tau} w$ is powerless).

Under constraint $\pi = 0$ and $\frac{\partial \omega}{\partial \tau} = 0$, when—as in ordinary cases—the spin of the continuum is much smaller then that of the gyroscopes, use can be made of the approximate expressions:

$$b_{Ig} \approx \frac{1}{2} \alpha_1 \omega g \times \mathrm{rot} v + [\mathrm{grad}(\alpha_1 \omega g)] v - \frac{1}{2} [(\mathrm{grad} J) v] \omega g \, . \tag{5}$$

With these notations and the usual continuity requirements, the local equations of motion write:

$$\left\{ \begin{array}{ll} \left. \begin{array}{l} \mathrm{div} T + \frac{1}{2} \mathrm{rot} b_g + b = \rho \dot{v} \\ z + \mathrm{div} S + b_g = b_{Ig} \\ \mathrm{e} T + z = 0 \end{array} \right\} & x \in V_\tau \\ \left. \begin{array}{l} T n + \frac{1}{2} b_g \times n = f - \frac{1}{2} \mathrm{rot}_* \, \mathrm{div} S \\ S n = f_g \end{array} \right\} & x \in \partial V_\tau \end{array} \right. \tag{6}$$

($\mathrm{rot}_*$ denotes a rotation computed on the surface) or, equivalently if (2) holds:

$$\left\{ \begin{array}{ll} \left. \begin{array}{l} \mathrm{div} \, \mathrm{sym} T - \frac{1}{2} \mathrm{rot} \, \mathrm{div} S + \frac{1}{2} \mathrm{rot} b_{Ig} + b = \rho \dot{v} \\ \zeta + \mathrm{div} s + b_g \cdot g = b_{Ig} \cdot g \\ \bar{z} + \mathrm{div} \bar{S} + b_g \times g = b_{Ig} \times g \end{array} \right\} & x \in V_\tau \\ \left. \begin{array}{l} [\mathrm{sym} T + \frac{1}{2} \mathrm{e} \, \mathrm{div} S - \frac{1}{2} \mathrm{e} b_{Ig}] n = f - \frac{1}{2} \mathrm{rot}_* \, \mathrm{div} S \\ s \cdot n = f_g \cdot g \\ \bar{S} n = f_g \times g \end{array} \right\} & x \in \partial V_\tau \end{array} \right. \tag{7}$$

(substitution $\mathrm{div} S = g \, \mathrm{div} s - g \times \mathrm{div} \bar{S}$ has been omitted to shorten the equations).

The kinematic constraint $\pi = 0$ leave all of equations (7) 'pure', *i.e.* no traces of unknown reactions appear there, but for the third of them which allows evaluating the reactive part of $\bar{z}$.

Furthermore, adding the constraint $\omega$ =constant, the first of (7) is still pure wh[illegible] second and the third give the reactive part of $z$ which, unlike the previous case, has [illegible] non zero component along $g$.

To offer a simple example of application we look at a linear elastic homogeneous [illegible] terial with shear modulus $\mu$, neglect body forces $b$ and micro-stress $s$, consider the [illegible] mentioned constraints and take the approximate expression (5) of inertia forces. We [illegible] $g = [1\ 0\ 0]$ everywhere (with $\omega \geq 0$) and look for solutions like isochoric waves of the [illegible] $u = \bar{u}(x_1)\exp(\pm\iota\sigma\tau)$, where $u$ is a small displacement.

The indefinite equations of motion reduce to:

$$\begin{cases} \bar{u}_{1,11} + \frac{\rho\sigma^2}{\mu}\bar{u}_1 = 0 \\ \bar{u}_{2,11} \pm \frac{\iota\sigma\alpha_1\omega}{4\mu}\bar{u}_{3,11} + \frac{\rho\sigma^2}{\mu}\bar{u}_2 = 0 \\ \bar{u}_{3,11} \mp \frac{\iota\sigma\alpha_1\omega}{4\mu}\bar{u}_{2,11} + \frac{\rho\sigma^2}{\mu}\bar{u}_3 = 0\,, \end{cases} \tag{8}$$

showing that waves propagating with circular frequency $\sigma$ have fixed wavelength $\lambda = \sigma^{-1}\sqrt{\mu/\rho}$ along the $x_1$ direction, while along the other two directions the wavelength depends on the choice of $\omega$ and can thus be controlled.

If our system is such that a characteristic length $\bar{\lambda}$ fixes, through given boundary conditions, a numerable set of wavelengths $\lambda_\iota$ (with $\iota \in \mathbb{N}$) and thus a corresponding set of frequencies $\sigma_\iota = \iota\pi/\bar{\lambda}$, we may find explicitly the dependence of the controlled modes on the spin velocity $\omega$:

$$\lambda_{\kappa\iota} = \lambda_\iota\sqrt{1 + \frac{\iota\pi\alpha_1\omega}{4\mu}} \tag{9}$$

(notice that the shift of transverse wavelengths is larger for the higher modes, *cf.* [3] and [4] for similar results). This, for instance, is the case of a layer of thickness $2\bar{\lambda}$ along the $x_1$ direction, with free end condition at the two bounding planes giving $\lambda_\iota = \bar{\lambda}/\iota\pi$.

REFERENCES


1. G. Capriz, Continua with microstructure, Springer–Verlag, New York, 1989.
2. G. Capriz and P. Giovine, On microstructural inertia, Math. Mod. Meth. Appl. Sci., vol. 7, no. 2, 1997, 211–216.
3. G. M. T. D'Eleuterio and P. C. Hughes, Dynamics of Gyroelastic Continua, J. App. Mech., vol. 51, 1984, 415–422.
4. G. M. T. D'Eleuterio, On the Theory of Gyroelasticity, J. App. Mech., vol. 55, 1984, 488–489.
5. P. Germain, The method of virtual power in continuum mechanics. Part 2: Microstructure, SIAM J. Appl. Math., vol. 25, no. 3, 1973, 556–575.






# Shuffle and agi

# Scompiglio ed ag


Dipartir

via


Granular materials partake under different circumstances, that they represent a new stat the search for an overall conti confines inquiry to fast flows, t be described by an adequate of such appealing terms as gr the appropriateness of the bo quantities involve kinetic entit being unpredictable on princi when exploiting vague analogie for the fact that, in generic gr many orders of magnitude gre behaved observer. Thus one n continuum of properties inferr laboratory; questions of objec difficulties would be to evalua traditional way in continuum a way that cannot be pursued in the analysis of vibrations earth, of the motion of a soap to a place occupied by some occurs, the movement in the motion of the structure; hence is easy to find: the set of ba momentum can be interpreted which an objective measure o satisfactory portrayal of 'ther

Again, the many differen reconsideration of the ties of ensembles of granules having thus including easily, for inst

Finally, when one pursue fundamental tenet that it b element. Then test domains central rôle; the question how boundary effects must be ad valid either in the interior o might take precedence over

However, whichever char emerge and it is attractive