Consiglio Nazionale delle Ricerche

ISTITUTO DI ELABORAZIONE DELLA INFORMAZIONE

PISA

A CONNECTION ASSIGNMENT YELDING EASILY DIAGNOSABLE SYSTEMS

Piero Maestrini

Nota Interna B79-21

Settembre 1979

A CONNECTION ASSIGNMENT YELDING EASILY DIAGNOSABLE SYSTEMS

Piero Maestrini

Istituto di Elaborazione della Informazione - Pisa, Italy

1. Introduction

This paper reconsiders some relevant problems in the diagnostic model of Preparata, Metze and Chien [1]. This model holds for systems which are partitioned into a number of units. Each unit is supposed to possess computational resources sufficient to enable it to test one or more of the remaining units. Each test is supposed to be complete for the class of relevant faults in the tested unit and each test outcome is supposed to be binary. The test outcome is fully significant if the testing unit is fault-free and completely unreliable if the testing unit is faulty.

The diagnostic model consists of a directed graph $G=(V,A)$, where each vertex $v_i \in V$ corresponds to a unit of the system and there exists one arc (v_i,v_j) from vertex v_i to vertex v_j if and only if the unit represented by v_i tests the unit represented by v_j. Following to application of the test set each arc is labeled with the corresponding test outcome $s(v_i,v_j)$, where $s(v_i,v_j)=0$ if the test passes and $s(v_i,v_j)=1$ if the test fails. The set s of arc labelings resulting from an application of the test set is called a syndrome.

If all units which are faulty at the time of test execution are identified by a process of syndrome decoding, one-step diagnosis is said to occur. If the syndrome decoding enables identification of at least one faulty unit, the diagnosis is called sequential or with repair. For any given system both one-step and sequential diagnosis are possible for all syndromes provided that the number of faulty units does not exceed critical values, called the one-step diagnosability t_0 and sequential diagnosability t_r; such values are integers with $t_r \geq t_0 \geq 0$. The reader is referred to [1, 2,3,4,5,6,7] for more details and results on sequential and one-step diagnosis of digital system in the model of Preparata, as well as in a number of different models.

One problem of interest in system diagnosis consists in evaluating the complexity of self-diagnosable systems. A reasonable measure of the complexity of a diagnostic system consisting of n units is the number of arcs in the diagnostic graph. The problem of determining optimal sequentially diagnosable systems in the model of Preparata, Metze and Chien is still unsolved, although optimal diagnosable systems have been found for any n when $t_r=t_{r\,min}=\lceil 2\sqrt{n}\rceil-3$ [6] or $t_r=t_{r\,max}=\lfloor (n-1)/2\rfloor$ (1). An upper bound to

(1) $\lceil x \rceil$ denotes the smallest integer not less than x; similarly $\lfloor x \rfloor$ is the greatest integer not greater than x.

the complexity of optimal sequentially diagnosable systems is also known for any permissible t_r [3]. A tighter bound will be established in this paper, by analysing a class of diagnostic graphs to be introduced in Section 3.

Additional interest resides in evaluating the complexity of syndrome decoding, that is the computational effort required to identify the faulty units, or at least one of them, when the syndrome is given. It has been proved [9] that syndrome decoding belongs to the class of NP-complete problems, both in one-step and sequential diagnosis, although there may exist classes of diagnosable systems (e.g. the simple circuit in the case of sequential diagnosis) for which the process of syndrome decoding is computationally efficient. It will be shown that the class of sequentially diagnosable systems introduced in this paper is easily diagnosable, since the complexity of syndrome decoding is O(n).

2. Sequential diagnosability

Let $G=(V,A)$ be the diagnostic graph of a given system and s be a syndrome. Subset $F\subseteq V$ is called a consistent fault pattern of s if $s(v_i,v_j)=1$ for all $(v_i,v_j)\in A$ such that $v_i\in V-F$, $v_j\in F$, and $s(v_i,v_j)=0$ for all $(v_i,v_j)\in A$ such that $v_i\in V-F$, $v_j\in V-F$. Consider the set $\mathcal{F}_s$ of consistent fault patterns of s, such that $|F|\leq t$ for each $F\in\mathcal{F}_s$. If $\bigcap_{F\in\mathcal{F}_s} F\neq\emptyset$, then any unit in $\bigcap_{F\in\mathcal{F}_s} F$ will be diagnosed as faulty under the hypothesis that the number of units does not exceed t. The greatest integer t_r such that this property holds for any syndrome s is the sequential diagnosability of the given graph [3].

Assuming, without any loss of generality [4], that the diagnostic graph is strongly connected, the following ideas provide a constructive approach to determining the sequential diagnosability, or at least a lower bound to this parameter. Given a syndrome s and any $v\in V$, let $g_0^s(v)$ and $g_1^s(v)$ denote the minimal cardinalities of consistent fault patterns F_0 and F_1 of s such that $v\notin F_0$ and $v\in F_1$, respectively. Under the hypothesis that the number of faults does not exceed $x_s(v)=\max(g_0^s(v),g_1^s(v))-1$, v is unambigously recognized to be faulty if $g_1^s(v)\leq x_s(v)$ and otherwise fault-free. In turn, identifying v as fault-free enables identification of at least one faulty unit under the hypothesis that the diagnostic graph is strongly connected, since any path beginning at v and consisting of h arcs labeled with 0 $(h\geq 0)$ followed by an arc (v_i,v_j) labeled with 1, will identify v_j as faulty.

Consider $t_r(s) = \max_{v\in V}(x_s(v))$ and let S be the set of all syndromes: the integer $t_r = \min_{s\in S}(t_r(s))$ is the sequential diagnosability. Further, assume that $g_i^s(v)$, $i \in \{0,1\}$, $v \in V$, is known for all $s \in S$: then $t'_r = g_i(v)-1 = \min_{s \in S}(g_i^s(v))-1$ is a lower bound to sequential diagnosability t_r. Tighter bounds may be determined by considering the index $x_s(v)$ above defined for each $v \in V'$, where $V' \subset V$ and letting $t'_r(s) = \max_{v \in V'}(x_s(v))$ and $t'_r = \min_{s \in S}(t'_r(s))$.

For example, consider a diagnostic graph $G=(V,A)$ consisting of a simple circuit of n vertices (fig. 1) and, for any syndrome s, the unique [1] partition of the set V into sequences $(v_{i1}, v_{i2}, \ldots, v_{ik})$, $k \geq 2$, such that $s(v_{i(k-1)}, v_{ik})=1$ and $s(v_{ip}, v_{iq})=0$ for $1 < q < k$. Assuming that syndrome s partitions V into ν sequences and λ be the number of vertices in a longest sequence, let v_f be that last vertex in a sequence of length λ. It is easily seen that $g_0^s(v_f) = \nu + \lambda - 2$ and $g_1^s(v_f) = \nu$. The integer $t'_r = \lceil 2\sqrt{n} - 3 \rceil$ obtained by minimizing $g^s(v_f)-1 = \max(g_0^s(v_f), g_1^s(v_f))-1$ over the set of permissible pairs (ν,λ) (that is, over the set S of syndromes), is a lower bound to sequential diagnosability of the simple circuit of n vertices. However, since $\lceil 2\sqrt{n} \rceil - 3$ is the upper bound to sequential diagnosability of the simple circuit [6], t'_r actually coincides with the sequential diagnosability t_r.

The same technique was used in [10] to investigate the diagnostic capabilities of a class of rosaces, consisting of k circuits of length 3 and one simple circuit of length m, where a vertex v_0 is common to all circuits. It was proved that sequential diagnosability of rosaces in this class spans all values between the lower and the upper bound and the number of arcs is close to the lower bound, although no graph in this class (except for the simple circuit) is an optimal connection assignment.

3. A connection assignment for sequential diagnosis

Consider the class of diagnostic graphs shown in fig. 2, called the LP graphs, consisting of a simple circuit L_m of vertices $v_0, v_1, \ldots v_{m-1}$ and k directed paths from v_0 to v_3, where the path p_i ($1 \leq i \leq k$) has vertices v_{i1}, v_{i2} and arcs (v_0, v_{i1}), (v_{i1}, v_{i2}), (v_{i2}, v_3). Let P_k denote the subgraph whose node set is $(\bigcup_{i=1,k}\{v_{i1}, v_{i2}\}) \cup \{v_0, v_3\}$. An LP-graphs has $n=m+2k$ vertices and $a=m+3k$ arcs. The sequential diagnosability of LP-graphs is known for any k limited to the cases of m=7 and m=10: in both cases $t_r = \lfloor (n-1)/2 \rfloor$ and $a = 3\lceil n/2 \rceil - 5$ [8]. This implies that the sequential diagnosability reaches the upper bound and LP-graphs with m=7 and m=10 are optimal connection assignments for this value of diagnosability. In the following of this section, the technique above described will be used to evaluate a lower bound to the sequential diagnosability of LP-graphs with arbitrary m. In the following section it will be shown that the complexity of syndrome decoding is linear in n, while it is known that the same problem is NP-complete in the general case.

Assume any arc labeling in the path p_i and denote by f_0, f_0', f_1, f_1' the cardinality of a minimal fault pattern in $\{v_{i1}, v_{i2}\}$ which is consistent with the assumption $v_0 \in V-F$ and $v_3 \in V-F$; $v_0 \in F$ and $v_3 \in V-F$; $v_0 \in V-F$ and $v_3 \in F$; $v_0 \in F$ and $v_3 \in F$, respectively. Such numbers are listed in Table 1 for all possible arc labelings $t_j (0 \leq j \leq 7)$ of p_i. A bar in Table 1 means that all fault patterns in $\{v_{i1}, v_{i2}\}$ are inconsistent with the arc labeling under consideration and the assumed state of v_0 and v_3.

Given a syndrome s, let $g_0^s(v_3)$ and $g_1^s(v_3)$ be the indices above defined, under the hypothesis $v_0 \in V-F$. Similarly $\bar{g}_0^s(v_3)$ and $\bar{g}_1^s(v_3)$ denote the same indices under the hypothesis $v_0 \in F$. From Table 1 such indices are easily determined as follows:

$$g_0^s(v_3)=t_2+t_3+t_4+t_6+2t_5+2t_7$$

$$\bar{g}_0^s(v_3)=t_2+t_3+t_6+t_7+2t_1+2t_5$$

$$g_1^s(v_3)=t_2+t_3+t_5+t_7+2t_4+2t_6$$

$$\bar{g}_1^s(v_3)=t_2+t_3+t_6+t_7+2t_0+2t_4$$

By introducing $k = t_0+t_1+t_2+t_3+t_4+t_5+t_6+t_7$, and $x=t_1+t_5-t_0-t_4$, the preceding expressions become:

$$
\begin{aligned}
g_0^s(v_3) &= k+t_5+t_7-t_0=k+x+t_4+t_7 \geq k+x \\
\bar{g}_0^s(v_3) &= k+t_1+t_5+t_0-t_4 = k+x \qquad (1) \\
g_1^s(v_3) &= k+t_4+t_6-t_1=k-x+t_5+t_6 \geq k-x \\
\bar{g}_1^s(v_3) &= k+t_0+t_4-t_1-t_5 = k-x,
\end{aligned}
$$

where $g_0^s(v_3)$ anf $g_1^s(v_3)$ are defined only if $t_1=0$ and $t_0=0$, respectively.

Assume that syndrome s defines ν sequences in the circuit L_m and λ is the maximum length of such sequences. By combining expressions (1) holding for v_3 in P_k and indices $g_0^s(v_3)$ and $g_1^s(v_3)$ of v_3 in the circuit L_m, the indices $g_0^s(v_3)$ and $g_1^s(v_3)$ of vertex v_3 in LP are easily evaluated as follows:

1) v_3 is the last vertex in a sequence of length λ :
$g_0^s(v_3) \geq k+x+\nu+\lambda-2, g_1^s(v_3)=k-x+\nu$;

2) v_3 is the last vertex in a sequence of length $\ell<\lambda$:
$g_0^s(v_3) \geq k+x+\lambda+\nu-2-(\lambda-\ell)$; $g_1^s(v_3)=k-x+\nu$;

3) v_3 is not the last vertex in a sequence: $g_0^s(v_3) \geq k+x+\nu$; $g_1^s(v_3) \geq k-x+\nu$.

Let $t_r'(s)$ be an estimate of the maximum number of faults under which the state of at least one unit can be diagnosed when syndrome s occurs. If $g_0^s(v_3) > t_r'(s) \{g_1^s(v_3) > t_r'(s)\}$, unit v_3 is diagnosed as faulty {fault-free} in the hypothesis of $t_r'(s)$ or less faults, the case where $g_0^s(v_3) > t_r'(s)$ and $g_1^s(v_3) > t_r'(s)$ corresponding to the occurrence of more that $t_r'(s)$ faults. If $g_0^s(v_3) \leq t_r'(s)$, $g_1^s(v_3) \leq t_r'(s)$ the state of unit v_3 cannot be diagnosed; however denoting by v_f the last unit in a sequence of length λ it is easily seen that in this hypothesis $g_0^s(v_f)=k+2\nu-t_r'(s)+\lambda-2$ in case 1 and 3 and $g_0^s(v_f)=k+2\nu-t_r'(s)+\lambda-2+\ell-2 \geq k+2\nu-t_r'(s)+\lambda-2$ in case 2, where the last inequality derives from $\ell \geq 2$. In order v_f to be diagnosed as faulty in the assumption of $t_r'(s)$ or less faults, must be $t_r'(s) \leq g_0(v_f)-1$ and $t_r'(s)$ is determined from the preceding expressions as $t_r'(s) = k+\nu+(\lambda-3)/2$. A lower bound t_r' to sequential diagnosability of LP graphs is thus determined by minimizing $t_r'(s)$ over the set of permissible pairs (ν, λ), resulting from consideration of all syndromes. By the same technique used in [10], it is seen that the lower bound holding for an LP graph consisting of k paths p_i of and one circuit of length m is the following:

$$t_r' = k + \left\lfloor (\sqrt{2m+1} - 1)/2 \right\rfloor + \left\lfloor \left(\left\lceil m / \left\lfloor (\sqrt{2m+1}-1)/2 \right\rfloor \right\rceil -3 \right)/2 \right\rfloor \qquad (2)$$

It should be noted that the bound established by expression (2) is one less than the sequential diagnosability of CP graphs for m=7 and m=10. However it is seen that for all of those syndromes such that $\max(g_0^s(v_3), g_1^s(v_3))-1 < \lfloor (n-1)/2 \rfloor$, there must exist at least one path $(v_0, v_{11}, v_{12}, v_3)$ such that interchanging (v_{i1}, v_{i2}) with (v_1, v_2) yields a syndrome s' such that $\max(g_0^{s'}(v_3), g_1^{s'}(v_3))-1 \geq \lfloor (n-1)/2 \rfloor$.

It is also interesting to observe that although the sequential diagnosability of CP-graphs cannot be determined exactly, the preceding analysis proves that the set of CP-graphs includes sequentially diagnosable systems whose diagnosability spans all values between the lower and the upper bound, and any CP graph with n nodes and diagnosability not less than t'_r places an upper bound to the complexity (i.e., the number of diagnostic connections) of sequentially diagnosable systems.

The problem of determining an upper bound to the complexity of optimal sequentially diagnosable systems with arbitrary $|V|$ and any permissible value of t_r has been considered by Maheshwari and Hakimi [3]. They have established a bound of $n+t_r-1$ for t_r-diagnosable systems whose diagnostic graph has n vertices. Since in the LP graph of Fig. 2 the number of arcs is $a = 3k+m$ and:

$$n=2k+m,\ k=t'_r - \left\lceil(\sqrt{2m+1}-1)/2\right\rceil - \left\lfloor\left(\left\lceil m/\left\lceil(\sqrt{2m+1}-1)/2\right\rceil\right\rceil-3\right)/2\right\rfloor,$$

it is easily seen that the complexity of a system whose diagnosability is no less than t_r is equal to:

$$a=n+t_r-\left\lceil(\sqrt{2m+1}-1)/2\right\rceil-\left\lfloor\left(\left\lceil m/\left\lceil(\sqrt{2m+1}-1)/2\right\rceil\right\rceil-3\right)/2\right\rfloor,$$

which constitutes a tighter bound to optimality.

4. Syndrome decoding

Another problem of interest consists in evaluating the computational effort required to actually diagnose at least one unit, given the syndrome. It has been proved that syndrome decoding in sequential diagnosis is a NP-complete problem [9] and this implies that sequential diagnosis becomes computationally untractable as the size of the problem increases. However efficient decoding algorithms may exist for special classes of diagnostic graphs: this has been actually proved for the simple circuit of Fig. 1 [9] and for a class of rosaces [10]. A similar result holds for the LP graphs introduced in this paper since it is clear that the following decoding algorithm, which derives from the preceding analysis, is $O(n)$ in time. With the notations defined above, the decoding algorithm is as follows:

A1) Determine if v_3 is the last element of a sequence in the circuit of length m and if such sequence has length λ, and compute $g_0^s(v_3)$, $g_1^s(v_3)$ using the corresponding formulae. If $g_0^s(v_0) > t'_r$, v_0 is recognized to be faulty in the hypothesis of at most t'_r faults; if $g_1^s(v_0) > t'_r$, v_0 is diagnosed as fault-free in the same hypothesis. Else:

A2) Denoting by v_f the last vertex in a sequence of length λ in the circuit of length m, v_f is recognized to be faulty in the hypothesis of at most t'_r faults.

References

[1] F.P. Preparata, G. Metze and R.T. Chien, "On the Connection Assignment Problem of Diagnosable Systems". IEEE Trans. Electron. Comput., Vol. EC-16, pp. 848-854, Dec. 1967.

[2] J.D. Russel and C.R. Kime, "System Fault Diagnosis: Closure and Diagnosability with Repair". IEEE Trans. on Comput., Vol. C-24, pp. 1078--1089, Nov.1975.

[3] S.N. Maheshwari and S.L. Hakimi, "On Models for Diagnosable Systems and Probabilistic Fault Diagnosis". IEEE Trans. on Comput., Vol. C-25, pp. 228-236, March 1976.

[4] F. Barsi, F. Grandoni and P. Maestrini, "A Theory of Diagnosability of Digital Systems". IEEE Trans. on Comput., Vol. C-25, pp. 585-593, June 1976.

[5] L. Simoncini and A. Friedman, "Incomplete Fault Coverage in Modular Multiprocessor Systems". Proc. ACM Annual Conference 1978 (Washington D.C., 1978), pp. 210-216.

[6] F.P. Preparata, "Some Results on Sequentially Diagnosable Systems". Proc. Hawaii Int. Conference on System Sciences; Univ. of Hawaii Press, Jan. 1968, pp. 623-626.

[7] S.L. Hakimi and A.T. Amin, "Characterization of Connection Assignments of Diagnosable Systems". IEEE Trans. on Comput., Vol. C-23, pp. 86-87, Jan. 1974.

[8] P. Ciompi and L. Simoncini, "Analysis and Optimal Design of Self-Diagnosable Systems with Repair". To appear on IEEE Trans. on Comput., May 1979.

[9] H. Fujiwara and K. Kinoshita, "On Computational Complexity of System Diagnosis". IEEE Trans. on Comput., Vol. C-27, pp. 379-384, April 1978.

[10] P. Maestrini "A class of functionally distributed systems which are easily diagnosable". Proc. 5th Seminar "Applied Aspects of the Automata Theory". Varna, Bulgaria, 15-19th May 1979, pp. 605-611.

Abstract

The problem of determining nearly optimal connection assignments in functionally distributed systems which are sequentially diagnosable in the model of Preparata is reconsidered. A class of diagnostic graphs yelding any permissible value of sequential diagnosability is introduced and it is shown that the number of diagnostic connections of systems in this class reaches the lower bound whenever this bound is known. Although syndrome decoding in sequential diagnosis is a NP-complete problem, it is proved that efficient diagnostic procedures exist for system in the class under consideration. A syndrome decoding algorithm is presented.

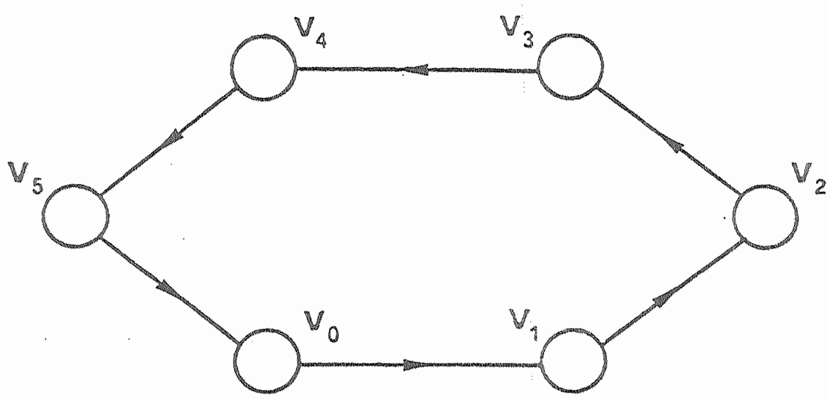

Fig. 1

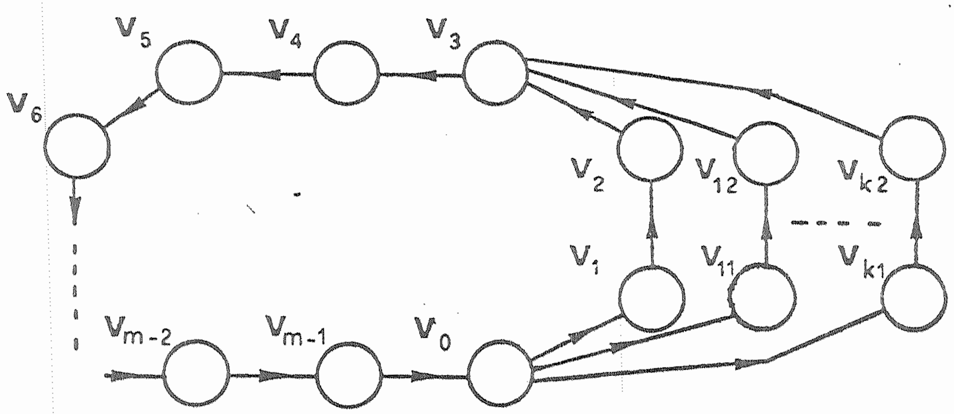

Fig. 2

Type	Arc labels	f_o	f'_o	f_1	f'_1
t_o	0 0 0	0	0	-	2
t_1	0 0 1	-	2	0	0
t_2	0 1 0	1	1	1	1
t_3	0 1 1	1	1	1	1
t_4	1 0 0	1	0	2	2
t_5	1 0 1	2	2	1	0
t_6	1 1 0	1	1	2	1
t_7	1 1 1	2	1	1	1

Table 1