

On the predictive information criteria for model determination in seismic hazard analysis

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Many statistical tools have been developed for evaluating, understanding, and comparing models, from both frequentist and Bayesian perspectives. In particular, the problem of model selection can be addressed according to whether the primary goal is explanation or, alternatively, prediction. In the former case, the criteria for model selection are defined over the parameter space whose physical interpretation can be difficult; in the latter case, they are defined over the space of the observations, which has a more direct physical meaning.

In the frequentist approaches, model selection is generally based on an asymptotic approximation which may be poor for small data sets (e.g. the F-test, the Kolmogorov-Smirnov test, etc.); moreover, these methods often apply under specific assumptions on models (e.g. models have to be nested in the likelihood ratio test).

In the Bayesian context, among the criteria for explanation, the ratio of the observed marginal densities for two competing models, named Bayes Factor (BF), is commonly used for both model choice and model averaging (Kass and Raftery, *J. Am. Stat. Ass.*, 1995). But BF does not apply to improper priors and, even when the prior is proper, it is not robust to the specification of the prior. These limitations can be extended to two famous penalized likelihood methods as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), since they are proved to be approximations of $-2 \log BF$.

In the perspective that a model is as good as its predictions, the predictive information criteria aim at evaluating the predictive accuracy of Bayesian models or, in other words, at estimating expected out-of-sample prediction error using a bias-correction adjustment of within-sample error (Gelman et al., *Stat. Comput.*, 2014).

In particular, the Watanabe criterion is fully Bayesian because it averages the predictive distribution over the posterior distribution of parameters rather than conditioning on a point estimate, but it is hardly applicable to data which are not independent given parameters (Watanabe, *J. Mach. Learn. Res.*, 2010). A solution is given by Ando and Tsay criterion where the joint density may be decomposed into the product of the conditional densities (Ando and Tsay, *Int. J. Forecast.*, 2010).

The above mentioned criteria are global summary measures of model performance, but more detailed analysis could be required to discover the reasons for poor global performance. In this latter case, a retrospective predictive analysis is performed on each individual observation.

In this study we performed the Bayesian analysis of Italian data sets by four versions of a long-term hazard model known as the stress release model (Vere-Jones, *J. Physics Earth*, 1978; Bebbington and Harte, *Geophys. J. Int.*, 2003; Varini and Rotondi, *Environ. Ecol. Stat.*, 2015). Then we illustrate the results on their performance evaluated by Bayes Factor, predictive information criteria and retrospective predictive analysis.