The 1st Electronic Conference on Universe 2021

Deep Space Probes

Testing General Relativity vs. Alternative Theories of Gravitation with the SaToR-G experiment

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Summary

- The goals of **SaToR-G**
- The theoretical framework of **SaToR-G**
- The Legacy from **LARASE**
- Conclusions

Satellites **T**ests **o**f **R**elativistic **G**ravity

LAser **RA**nged **S**atellites **E**xperiment

The goals of SaToR-G

- **SaToR-G** (**Sa**tellites **T**ests **o**f **R**elativistic **G**ravity) is a new experiment of the **Astroparticle Physics Experiments Committee** of the Italian **National Institute for Nuclear Physics** (**INFN**) and will expand the activities carried on by the **LA**ser **RA**nged **S**atellites **E**xperiment (**LARASE**, 2013-2019), investigating possible experimental signatures of deviation from **General Relativity** (**GR**)

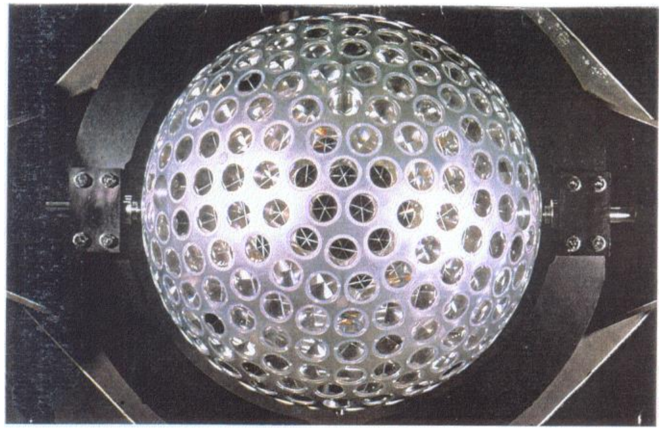
- Similarly to **LARASE**, **SaToR-G** is dedicated to measurements of the gravitational interaction in the **Weak-Field and Slow-Motion** (**WFSM**) limit of **GR** by means of laser tracking to geodetic passive satellites orbiting around the Earth

$$\frac{GM_{\oplus}}{rc^2} \ll 1 \qquad \frac{GJ_{\oplus}}{c^3 r^2} \ll 1 \qquad \frac{v}{c} \ll 1$$

The goals of SaToR-G

- **SaToR-G** exploits the improvement of the dynamic models of the two **LAGEOS** and **LARES** satellites performed by **LARASE**. These satellites still represent the main proof-masses of the new experiment

LAGEOS



LAGEOS II

LARES

The goals of SaToR-G

- **SaToR-G** focuses on verifying the gravitational interaction in the **WFSM** limit beyond the predictions of **GR**, looking for possible effects connected with <u>new physics</u>, and foreseen by different <u>alternative theories of gravitation</u>
- **Alternative theories**:
  - <u>Metric</u> theories as **GR**
  - <u>Non-metric</u> theories

The goals of SaToR-G

- **SaToR-G** main goals
  - From the analysis of the orbits of the satellites it is possible to obtain a whole series of gravitational measurements with consequent constraints on different theories of gravitation.
  - Among the main measures we can consider:
    - Constraints on **long-range interactions** parameterized by a **Yukawa-like potential**
    - **PPN** parameters and their combinations: β, γ, $\alpha 1$, $\alpha 2$
    - Relativistic precessions and non-linearity of the gravitational interaction
    - **EEP** and **Nordtvedt effect**
    - …

The goals of SaToR-G

- **SaToR-G** main goals
  - Looking at the effects on the orbits of artificial satellites allows us to test **GR** vs. other **metric theories**, in their deeper aspects, related to the **<u>curvature of spacetime</u>**, **<u>motion on geodesics</u>** and **<u>field equations</u>**
  - The goal is to provide **precise** and **accurate** measurements, in the sense of a **robust** and **reliable** evaluation of the **<u>systematics errors</u>**, in order to obtain significant constraints for the different theories

The theoretical framework of SaToR-G

At the foundation of **GR** and of metric theories of gravitation is **Einstein Equivalence Principle**

Weak Equivalence Principle (WEP)

- two different bodies fall with the same acceleration: Universality of the Free Fall (**UFF**)
- the inertial mass is proportional to the gravitational (passive) mass
- the trajectory of a freely falling "test" body is independent of its internal structure and composition
- in every local and non-rotating falling frame, the trajectory of a freely falling test body is a straight line, in agreement with special relativity

Einstein Equivalence Principle (EEP)

- **WEP**
- **Local Lorentz Invariance** (**LLI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed
- **Local Position Invariance** (**LPI**)
  - ❑ The outcome of any local non-gravitational experiment is independent of where and when in the Universe it is performed

The theoretical framework of SaToR-G

Metric theories

- **GR** is a metric theory of gravity and all metric theories obey the **EEP**
- Indeed, the experimental results supporting the **EEP** supports the conclusion that the only theories of gravity that have a hope of being viable are metric theories, or possibly theories that are metric apart from very weak or short-range non-metric couplings (as in string theory):

1. **there exists a symmetric metric**
2. **tests masses follow geodesics of the metric**
3. **in Local Lorentz Frames, the non-gravitational laws of physics are those of Special Relativity**

$$g_{\alpha\beta} = g_{\beta\alpha}$$

$$det\left(g_{\alpha\beta}\right) \neq 0$$

$$ds^2 = g_{\alpha\beta} dx^\alpha \, dx^\beta$$

$$G_{\alpha\beta} = 8\pi \frac{G}{c^4} T_{\alpha\beta}$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta}$$

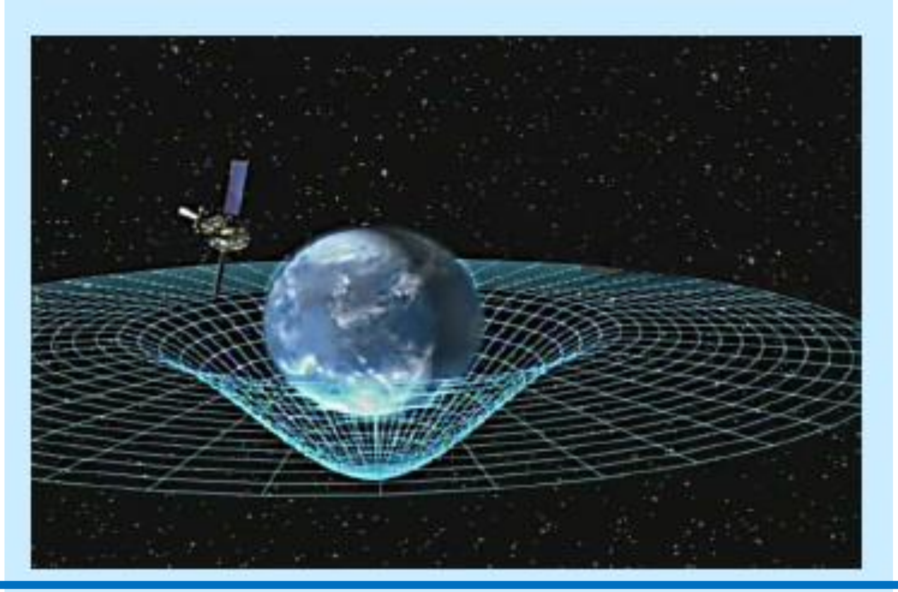
The theoretical framework of SaToR-G

Metric theories

- Metric theories different from **GR** provide additional fields beside the metric tensor $g_{\alpha\beta}$, that act as "new" gravitational fields:
  - **Scalars, Vectors, Tensors,…**
- The role of these gravitational fields is to "mediate" how the matter and the non-gravitational fields generate the gravitational fields and produce the metric

In Metric theories different from GR:

- the spacetime geometry tells mass-energy how to move as in **GR**
- but mass-energy tells spacetime geometry how to curve in a different way from **GR**
- and the metric alone acts back on the mass in agreement with **EEP**



The theoretical framework of SaToR-G

The parametrized post-Newtonian (PPN) formalism

- A way to test a theory of gravitation is at its post-Newtonian limit
- Post-Newtonian formalism or **PPN** formalism details the parameters in which different theories of gravity, under **WFSM** conditions, can differ from Newtonian gravity

C.M. Will Living Rev. Relativity, 17, (2014), 4

$$
\begin{aligned}
g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\
&\quad + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} \\
&\quad + (2\alpha_3 - \alpha_1) w^i V_i + \mathcal{O}(\epsilon^3), \\
g_{0i} &= -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\
&\quad - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}), \\
g_{ij} &= (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).
\end{aligned}
$$

Metric

$$
\begin{aligned}
T^{00} &= \rho(1 + \Pi + v^2 + 2U), \\
T^{0i} &= \rho v^i \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right), \\
T^{ij} &= \rho v^i v^j \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right) + p\delta^{ij}(1 - 2\gamma U).
\end{aligned}
$$

Stress-Energy Tensor

$$
\begin{aligned}
U &= \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
U_{ij} &= \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x', \\
\Phi_W &= \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|}\right) d^3x'\, d^3x'', \\
\mathcal{A} &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x', \\
\Phi_1 &= \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
\Phi_2 &= \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
\Phi_3 &= \int \frac{\rho' \Pi'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
\Phi_4 &= \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
V_i &= \int \frac{\rho' v'_i}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \\
W_i &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x'.
\end{aligned}
$$

Metric Potentials

The theoretical framework of SaToR-G

The parametrized post-Newtonian (PPN) formalism

C.M. Will Living Rev. Relativity, 17, (2014), 4

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Theory	Arbitrary functions or constants	Cosmic matching parameters	PPN parameters γ	β	ξ	α_1	α_2
General relativity	none	none	1	1	0	0	0
Scalar–tensor							
Brans–Dicke	$\omega_{\rm BD}$	ϕ_0	$\frac{1+\omega_{\rm BD}}{2+\omega_{\rm BD}}$	1	0	0	0
General, $f(R)$	$A(\varphi)$, $V(\varphi)$	φ_0	$\frac{1+\omega}{2+\omega}$	$1+\frac{\lambda}{4+2\omega}$	0	0	0
Vector–tensor							
Unconstrained	$\omega, c_1, c_2, c_3, c_4$	u	γ'	β'	0	α_1'	α_2'
Einstein-Æther	c_1, c_2, c_3, c_4	none	1	1	0	α_1'	α_2'
Khronometric	$\alpha_k, \beta_k, \lambda_k$	none	1	1	0	α_1'	α_2'
Tensor–Vector–Scalar	k, c_1, c_2, c_3, c_4	ϕ_0	1	1	0	α_1'	α_2'

The theoretical framework of SaToR-G

The Dicke framework

- Testing for the values of the **PPN** parameters represents a powerful tool to discriminate among different theories of gravitation
- Anyway, within the **SaToR-G** strategy to test a theory of gravity, we are also interested in recovering the more general approach from which the **PPN** formalism itself, in its current version (papers by **Nordtved** and **Will**), was basically born
- In fact, we will try as much as possible to test the different theories in the theoretical/experimental framework conceived by **R.H. Dicke** around the mid-60s
- The main idea at the basis of this framework is to build up a set of experiments to be as unbiased as possible both from classical Newtonian physics and from Einstein's **GR**

Nordtvedt, K. *Equivalence Principle for Massive Bodies*. II. Theory. Phys. Rev. 1968, 169, 1017–1025
Will, C.M. *Theoretical Frameworks for Testing Relativistic Gravity. II. Parametrized Post-Newtonian Hydrodynamics, and the Nordtvedt Effect*. Astrophys. J. 1971, 163, 611–628
Will, C.M.; Nordtvedt, J.K. *Conservation Laws and Preferred Frames in Relativistic Gravity. I. Preferred-Frame Theories and an Extended PPN Formalism*. Astrophys. J. 1972, 177, 757–774

The theoretical framework of SaToR-G

Dicke, R.H. The Theoretical Significance of Experimental Relativity; Blackie and Son Ltd.: London and Glasgow, 1964

The Dicke framework

1. Spacetime is a 4-dimensional differentiable manifold, with each point in the manifold corresponding to a physical event. The manifold need not a priori have either a metric or an affine connection

   - **The hope is that experiment will force us to conclude that it has both**

1. The equations of gravity and the mathematical entities in them are to be expressed in a form that is independent of the particular coordinates used, i.e., in covariant form

Dicke imposes two constraints:

1. Gravity must be associated with one or more fields of tensor character: scalars, vectors and tensors of various ranks
2. The dynamical equations that govern gravity must be derivable from an invariant action principle

The theoretical framework of SaToR-G

The Dicke framework

From Dicke's Framework, theorists have been able to formulate a set of criteria that any theory of gravitation should satisfy if it is to be viable:

1. It must be complete
2. It must be self-consistent
3. It must be relativistic
4. It must have the correct Newtonian limit

THE ASTROPHYSICAL JOURNAL, 163:595–610, 1971 February 1

THEORETICAL FRAMEWORKS FOR TESTING RELATIVISTIC GRAVITY. I. FOUNDATIONS*

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ABSTRACT

This is the first in a series of theoretical papers which will discuss the experimental foundations of general relativity This paper reviews, modifies, and compares two very different theoretical frameworks, within which one devises and analyzes tests of gravity. The *Dicke framework* assumes almost nothing about the nature of gravity; and it uses a variety of experiments to delineate the gross features of the gravitational interaction. Two of its tentative conclusions (the presence of a metric, and the "gravitational response equation," $\nabla \cdot T = 0$, for stressed matter) become the postulates of the *Parametrized Post-Newtonian framework*. The PPN framework encompasses most, if not all, of the theories of gravity that are currently compatible with experiment. Future papers in this series will develop the PPN framework in detail, and will use it to analyze a variety of relativistic gravitational effects that should be detectable in the solar system during the coming decade.

The legacy from LARASE

1. Measurements in the field of Gravitation

- A precise and accurate measurement of the **GR** advance of the **argument of pericenter** of **LAGEOS II**
  - **With constraints on alternative theories of gravity**
- A precise and accurate measurement of the **GR Lense-Thirring** precession on the orbits of the two **LAGEOS** and **LARES** satellites

2. Non-Gravitational Perturbations (NGPs) modelling

- **Internal structure** of the satellites
  - **Mass** and **moments of inertia**
- **Attitude** of the satellites
  - **Spin evolution**
- **NGPs** on the satellites
  - **Thermal thrust forces** on the two **LAGEOS** satellites
  - **Neutral drag force** on the **LARES** satellite

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We will focus only on point 1

The legacy from LARASE

- **Measurements in the field of Gravitation**: LAGEOS II pericenter **GR** advance

The expected **GR** precession vs. **Classical** precession

$$\langle \dot{\omega}_{Schw} \rangle_{sec} = \frac{3}{c^2 a^{5/2}} \frac{GM_\oplus^{3/2}}{(1-e^2)}$$

$$\langle \dot{\omega}_{LT} \rangle_{sec} = -\frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$

$$U = -\frac{GM_\oplus}{r} \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(\frac{R_\oplus}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)\left(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda\right),$$

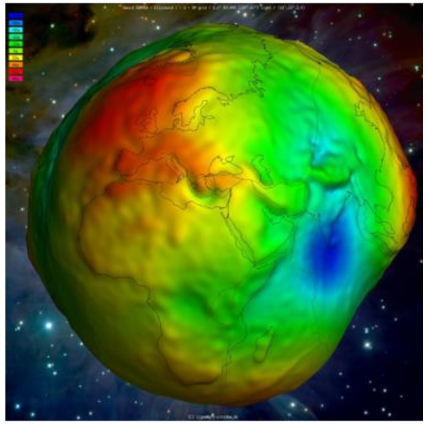
$$\langle \dot{\omega}_{class} \rangle_{sec} = \frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{1}{(1-e^2)^2}\left\{\cos i + \left(1 - \frac{3}{2}\sin^2 i\right)\right\}[-\sqrt{5}\bar{C}_{20}] + \cdots$$

PHYSICAL REVIEW D **89**, 082002 (2014)

TABLE I. Rate (mas/yr) and orbital shift (over 14 days) of the different types of secular relativistic precession on the arguments of pericenter of LAGEOS II and LAGEOS, and their sum (1 mas/yr = 1 milli-arc second per year).

	Precession	Rate (mas/yr)	Shift (m)
	$\Delta\dot{\omega}^{\mathrm{Schw}}$	3351.95	7.61
LAGEOS II	$\Delta\dot{\omega}^{\mathrm{LT}}$	−57.00	-1.29×10^{-1}
	Total	3294.95	7.48
	$\Delta\dot{\omega}^{\mathrm{Schw}}$	3278.77	7.44
LAGEOS	$\Delta\dot{\omega}^{\mathrm{LT}}$	32.00	0.72×10^{-1}
	Total	3310.77	7.51

$$\langle \dot{\omega}_{class} \rangle_{sec} = \begin{cases} -2.8 \times 10^8 \; mas/yr & LAGEOS \\ 5.7 \times 10^8 \; mas/yr & LAGEOS\ II \end{cases}$$



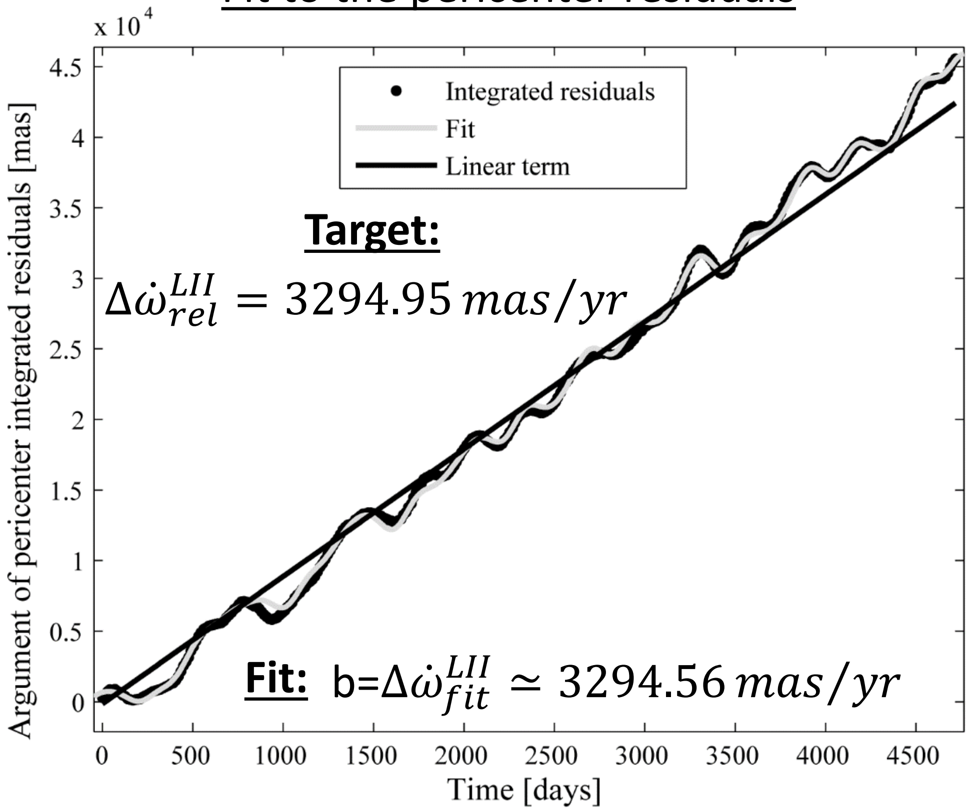
$\dot{\omega}_{GR} \cong 3300 \; mas/yr$ **The GR precession is about 5 orders of magnitude smaller!**

The legacy from LARASE

- **Measurements in the field of Gravitation**: **LAGEOS II** pericenter **GR** advance

Post data reduction analysis: 13-yr analysis of the **LAGEOS II** orbit (FIT)

Fit to the pericenter residuals



$$\boxed{\Delta\omega^{FIT} = a + b\cdot t + c\left(t - t_0\right)^2 + \sum_{i=1}^{n} D_i \sin\left(\frac{2\cdot\pi}{P_i}\cdot t + \Phi_i\right)}$$

We obtained b ≅ 3294.6 mas/yr, very close to the prediction of **GR**

The discrepancy is just **0.01%**

From a sensitivity analysis, with constraints on some of the parameters that enter into the least squares fit, we obtained an upper bound of **0.2%**

$$\Delta\dot{\omega} = \Delta\dot{\omega}_{GP} + \Delta\dot{\omega}_{NGP} + \varepsilon\cdot\Delta\dot{\omega}_{GR}$$

$$\varepsilon = 1 - (0.12 \pm 2.10)\cdot 10^{-3} \pm 2.5\cdot 10^{-2}$$

The legacy from LARASE

- **Measurements in the field of Gravitation**: LAGEOS II pericenter **GR** advance

Error budget of the unmodelled pericenter rate measurement

DAVID M. LUCCHESI AND ROBERTO PERON PHYSICAL REVIEW D **89**, 082002 (2014)

TABLE XVII. Error budget of the LAGEOS II pericenter general relativity shift. Top: summary of the errors from the data reduction and the *a posteriori* best fit (see Sections VI and VII). Middle: summary of the systematic errors from the gravitational perturbations (see Section VIII). Bottom: summary of the systematic errors from the nongravitational perturbations (see Section IX).

Statistical errors		
Residuals	Mean	Standard deviation
Range	9.67 cm	3.88 cm
Pericenter	4.57 mas	1.87 mas
Adjusted $\mathcal{R}_a^2$	0.998	
Reduced χ_ν^2 test	0.14	
	$\epsilon_\omega^{sta} - 1 = (-0.12 \pm 2.10) \times 10^{-3}$	
Systematic errors: gravitational perturbations		
Error source	Error value (% $\Delta\dot{\omega}_{II}^{rel}$)	Total not correlated (% $\Delta\dot{\omega}_{II}^{rel}$)
Even zonal harmonics	2.45	
Odd zonal harmonics	4.10×10^{-2}	
Tides (solid + ocean)	2.48×10^{-2}	2.46
Secular trends (ℓ = even)	3.30×10^{-2}	
Seasonal-like effects	0.24	
Systematic errors: nongravitational perturbations		
Error source	Error value (% $\Delta\dot{\omega}_{II}^{rel}$)	Total not correlated (% $\Delta\dot{\omega}_{II}^{rel}$)
Direct solar radiation	0.50	
Earth's albedo	0.39	
Thermal thrusts	0.09	0.64
Drag (neutral + charged)	negligible	
Total not correlated		2.54
	$\epsilon_\omega^{sys} - 1 = \pm 2.54 \times 10^{-2}$	

The legacy from LARASE

- **Measurements in the field of Gravitation**: LAGEOS II pericenter **GR** advance

Summary of the constraints obtained

TABLE XVIII. Summary of the results obtained in the present work; together with the measurement error budget, the constraints on fundamental physics are listed and compared with the literature.

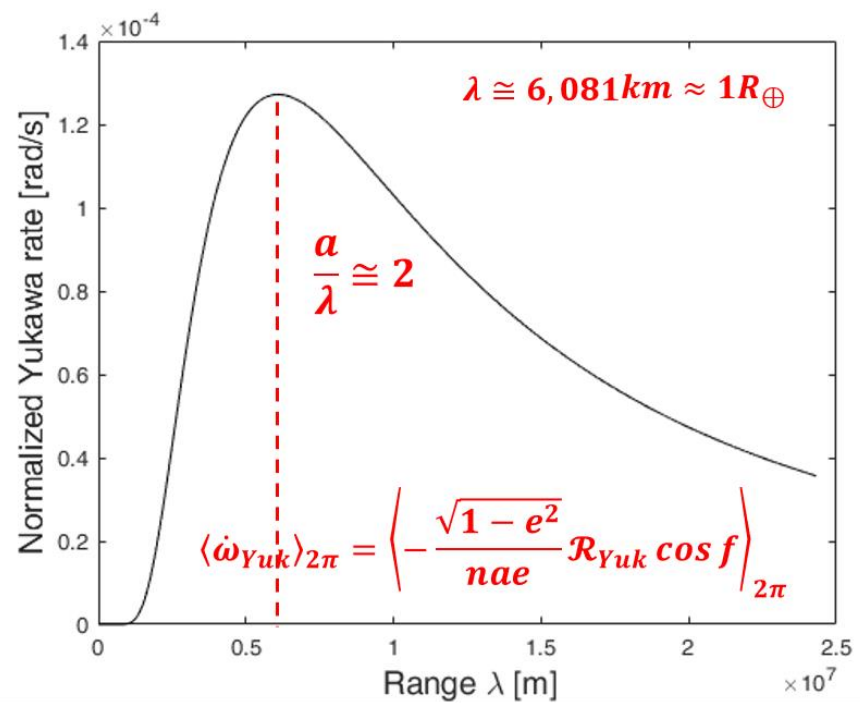
Parameter	Values and uncertainties (this study)	Uncertainties (literature)	Remarks
$\epsilon_\omega - 1$	$-1.2\times10^{-4}\pm2.10\times10^{-3}\pm2.54\times10^{-2}$	...	Error budget of the perigee precession measurement in the field of the Earth
$\frac{\lvert 2+2\gamma-\beta\rvert}{3} - 1$	$-1.2\times10^{-4}\pm2.10\times10^{-3}\pm2.54\times10^{-2}$	$\pm(1.0\times10^{-3})\pm(2\times10^{-2})$[a]	Constraint on the combination of PPN parameters
$\lvert\alpha\rvert$	$\lesssim\lvert 0.5\pm8.0\pm101\rvert\times10^{-12}$	$\pm1\times10^{-8}$[b]	Constraint on a possible (Yukawa-like) NLRI
$\mathcal{C}_{\oplus \mathrm{LAGEOSII}}$	$\leq(0.003\ \mathrm{km})^4\pm(0.036\ \mathrm{km})^4\pm(0.092\ \mathrm{km})^4$	$\pm(0.16\ \mathrm{km})^4$[c]; $\pm(0.087\ \mathrm{km})^4$[d]	Constraint on a possible NSGT
$\lvert 2t_2+t_3\rvert$	$\lesssim3.5\times10^{-4}\pm6.2\times10^{-3}\pm7.49\times10^{-2}$	3×10^{-3}[e]	Constraint on torsion

[a]From the preliminary estimate of the systematic errors of [166] for the perihelion precession of Mercury.
[b]From [167] with Lunar-LAGEOS *GM* measurements.
[c]From [5] and based on a partial estimate for the systematic errors.
[d]From [7] and based on the analysis of the systematic errors only.
[e]From [168] with no estimate for the systematic errors.

The legacy from LARASE

- **Measurements in the field of Gravitation**: LAGEOS II pericenter **GR** advance

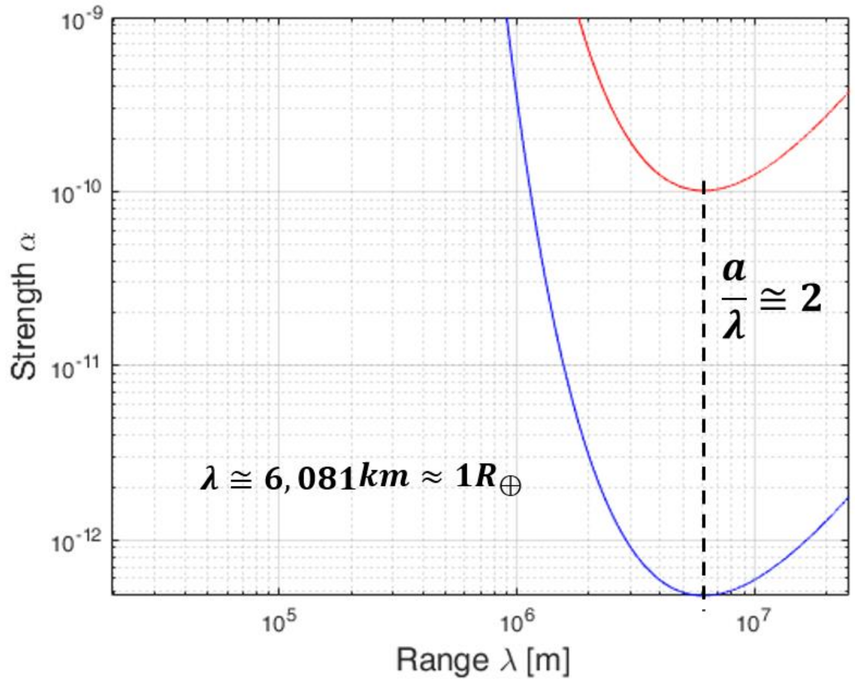
Violation of 1/r^2 law: Yukawa-like potential



$$\boxed{\left\langle \frac{d\omega}{dt} \right\rangle_{2\pi}^{Peak} \cong 1.27394 \cdot 10^{-4} \cdot \alpha \, rad/s}$$

$$\vec{\mathcal{R}}_{Yuk} = -\alpha \frac{G_\infty M_\oplus}{r^2}\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \hat{r}$$

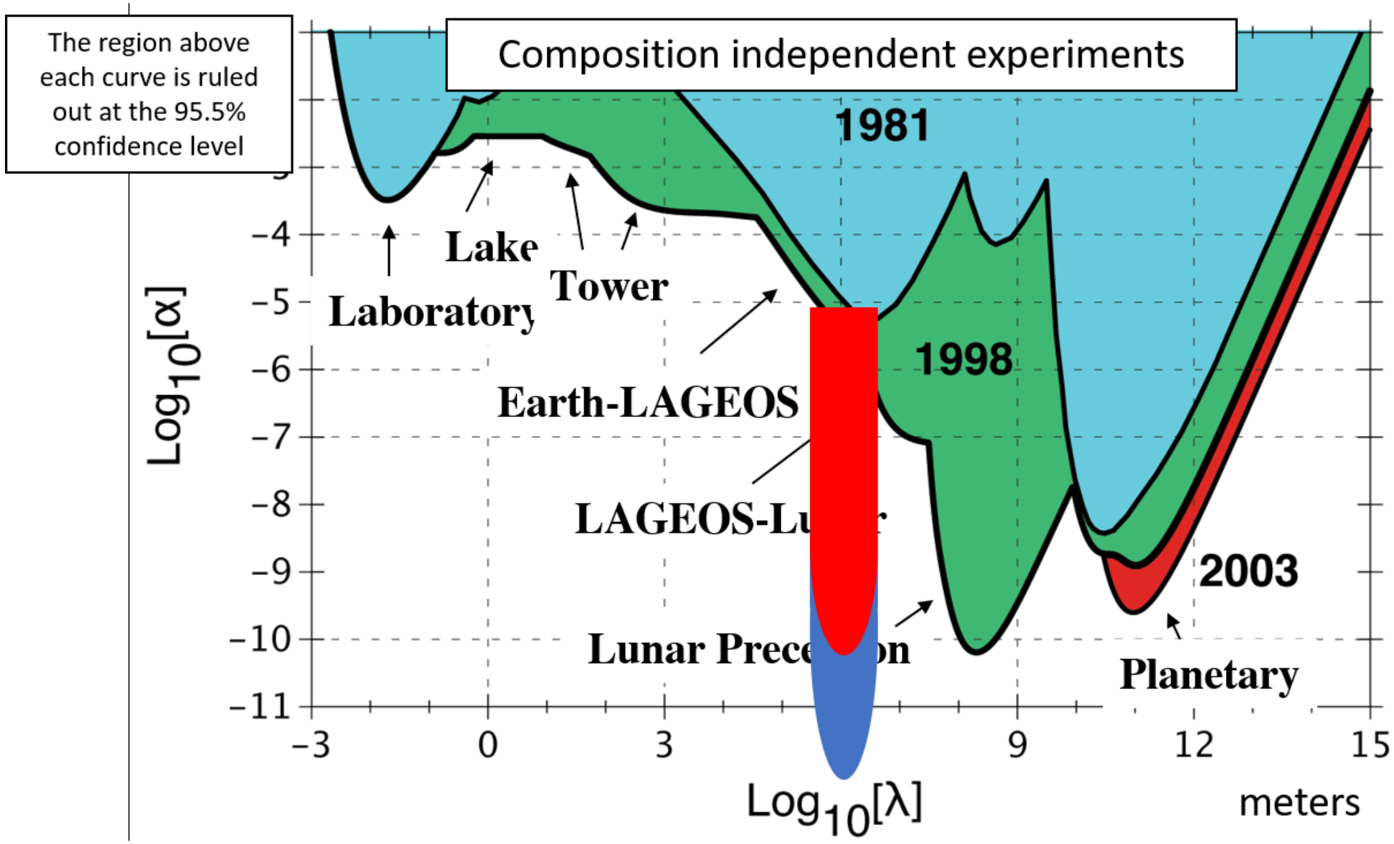
$$|\alpha| \cong \left|(0.5 \pm 8) \cdot 10^{-12} \pm 101 \cdot 10^{-12}\right|$$



The legacy from LARASE

- **Measurements in the field of Gravitation**: **LAGEOS II** pericenter **GR** advance

Constraints on a long-range force: Yukawa like interaction



$$|\alpha| \cong \left|(\mathbf{0.5} \pm 8)\cdot 10^{-12} \pm \mathbf{101 \cdot 10^{-12}}\right|$$

Previous limits with **LAGEOS's**:

$$\boxed{|\alpha| < 10^{-5} \div 10^{-8}}$$

$$|\boldsymbol{\alpha}| \cong \mathbf{1 \cdot 10^{-10}}$$

$$|\boldsymbol{\alpha}| \cong \mathbf{5 \cdot 10^{-13}}$$

The legacy from LARASE

PHYSICAL REVIEW D **89**, 082002 (2014)

LAGEOS II pericenter general relativistic precession (1993–2005): Error budget and constraints in gravitational physics

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The aim of this paper is to extend, clarify, and deepen the results of our previous work [D. M. Lucchesi and R. Peron, Phys. Rev. Lett. **105**, 231103 (2010)], related to the precise measurement of LAGEOS (LAser GEOdynamics Satellite) II pericenter shift. A 13-year time span of LAGEOS satellites' laser tracking data has been considered, obtaining a very precise orbit and correspondingly residuals time series from which to extract the relevant signals. A thorough description is provided of the data analysis strategy and the dynamical models employed, along with a detailed discussion of the known sources of error in the experiment, both statistical and systematic. From this analysis, a confirmation of the predictions of Einstein's general relativity, as well as strong bounds on alternative theories of gravitation, clearly emerge. In particular, taking conservatively into account the stricter error bound due to systematic effects, general relativity has been confirmed in the Earth's field at the 98% level (meaning the measurement of a suitable combination of β and γ PPN parameters in weak-field conditions). This bound has been used to constrain possible deviations from the inverse-square law parameterized by a Yukawa-like new long range interaction with strength $|\alpha| \lesssim 1 \times 10^{-10}$ at a characteristic range $\lambda \simeq 1$ Earth radius, a possible nonsymmetric gravitation theory with the interaction parameter $\mathcal{C}_{\oplus \mathrm{LAGEOS\ II}} \lesssim (9 \times 10^{-2}\ \mathrm{km})^4$, and a possible spacetime torsion with a characteristic parameter combination $|2t_2 + t_3| \lesssim 7 \times 10^{-2}$. Conversely, if we consider the results obtained from our best fit of the LAGEOS II orbit, the constraints in fundamental physics improve by at least 2 orders of magnitude.

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The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

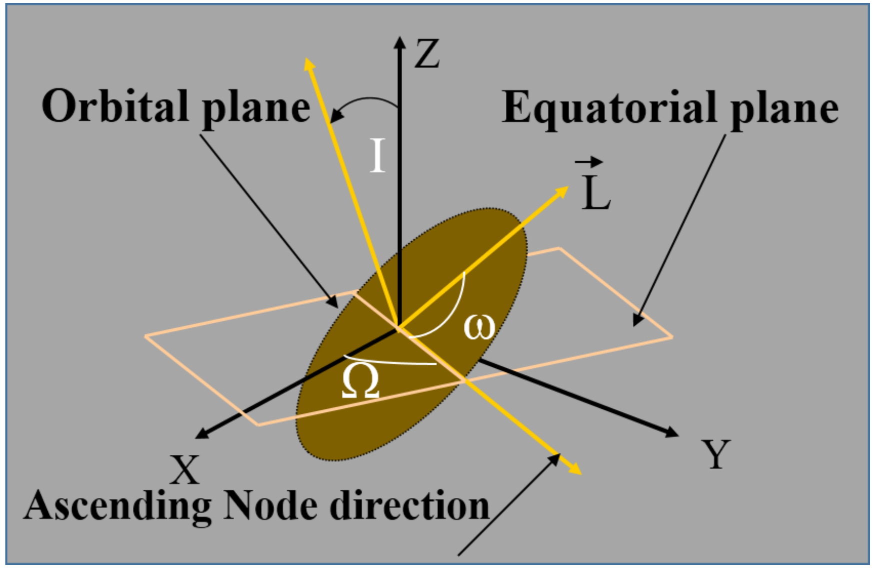
The **Lense-Thirring** effect consists in a precession of the orbit of a satellite around a primary produced by its rotation, i.e. by its angular momentum (mass-currents)

This precession produces a secular effect in two orbital elements:

- **the right ascension of the ascending node Ω**
- **the argument of pericenter ω**

$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \frac{2G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}}$$

$$\left\langle \frac{d\omega}{dt} \right\rangle_{sec} = -\frac{6G}{c^2 a^3} \frac{J_\oplus}{(1-e^2)^{3/2}} \cos i$$



The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

The **Lense-Thirring** precession is very small compared to the classical orbital precessions caused by the deviation from the spherical symmetry for the mass distribution of the Earth, or compared to the same relativistic **Schwarzschild** precession produced by the mass of the primary

$$V(r,\varphi,\lambda) = -\frac{GM_{\oplus}}{r}\left[1+\sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{R_{\oplus}}{r}\right)^{\ell} P_{\ell m}(\sin\varphi)(C_{\ell m}\cos m\lambda + S_{\ell m}\sin m\lambda)\right]$$

$$\langle\dot{\Omega}_{class}\rangle_{sec} = -\frac{3}{2}n\left(\frac{R_{\oplus}}{a}\right)^{2}\frac{\cos i}{(1-e^{2})^{2}}\{-\sqrt{5}\bar{C}_{2,0}\} + \cdots$$

$$\dot{\Omega}^{Obser}_{Lageos} \approx +126\ deg/yr \qquad \dot{\Omega}^{Obser}_{LageosII} \approx -231\ deg/yr$$

$$\dot{\Omega}^{Obser}_{Lares} \approx -624\ deg/yr$$

$$\begin{cases} G \cong 6.670 \cdot 10^{-8} cm^{3}s^{-2}g^{-1} \\ J_{\oplus} \cong 5.861 \cdot 10^{40} cm^{2}gs^{-1} \\ c \cong 2.99792458 \cdot 10^{10}\ cm/s \end{cases}$$

TABLE I. Mean orbital elements of LAGEOS, LAGEOS II and LARES.

Element	Unit	Simbol	LAGEOS	LAGEOS II	LARES
semi-major axis	[km]	a	12 270.00	12 162.07	7 820.31
eccentricity		e	0.004433	0.013798	0.001196
inclination	[deg]	i	109.84	52.66	69.49

TABLE II. Rate in milli-arc-sec per year (mas/yr) for the secular Lense-Thirring precession on the right ascension of the ascending node and on the argument of pericenter of LAGEOS, LAGEOS II and LARES satellites.

Rate in the element	LAGEOS	LAGEOS II	LARES
$\dot{\Omega}_{\mathrm{L\text{-}T}}$	30.67	31.50	118.48
$\dot{\omega}_{\mathrm{L\text{-}T}}$	31.23	−57.31	−334.68

30 mas ≅ 1.8 m in 1-year

The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

1. We considered several models for the background gravitational field of the Earth
   - **This allows to highlight possible systematics errors among the different models**
2. For the first **10/15** even zonal harmonics we considered their explicit time dependency following the monthly solutions from **GRACE** measurements
   - **This has reduced the systematic error of the background gravitational field**
3. Together with the relativistic **Lense-Thirring** precession we estimated also some of the low-degree even zonal harmonics (ℓ = even and m = 0) of the background gravitational field
   - **This allows to estimate the direct correlation between the relativistic Lense-Thirring precession with the coefficients of the gravitational field**

The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

4. The relativistic **Lense-Thirring** precession has been measured both in the residuals of the rates of the combined nodes and in their integration
    - **This is the first time that the measurement has been performed on the rate of the combined observables**
5. The measurement has been obtained both via linear fits and non-linear fits
    - **This is also the first time that a reliable measurement of the Lense-Thirring precession has been obtained by means of a simple linear fit**

The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

The data reduction of the satellites orbit has been done with **GEODYN II** (NASA/GSFC) on a time span of about 6.5 years (2359 days) from **MJD 56023**, i.e. April 6th 2012, and we computed the effects on the orbit elements of **LAGEOS**, **LAGESOS II** and **LARES**:

- **Background gravity model: GGM05S + other fields from GRACE**
- **Arc length of 7 days**
- **No empirical accelerations**
- **Thermal effects (Yarkovsky Schach and Rubincam) not modelled**
- **General relativity modelled with the exception of the Lense-Thirring effect**

1. **EIGEN-GRACE02S (2004)**
2. **GGM05S (2014): official field of the ILRS**
3. **ITU_GRACE16 (2016)**
4. **Tonji-Grace02s (2017)**

Rate (mas/yr)	LAGEOS	LAGEOS II	LARES
$\left\langle \dot{\Omega}_{LT} \right\rangle_{sec}$	30.67	31.50	118.48

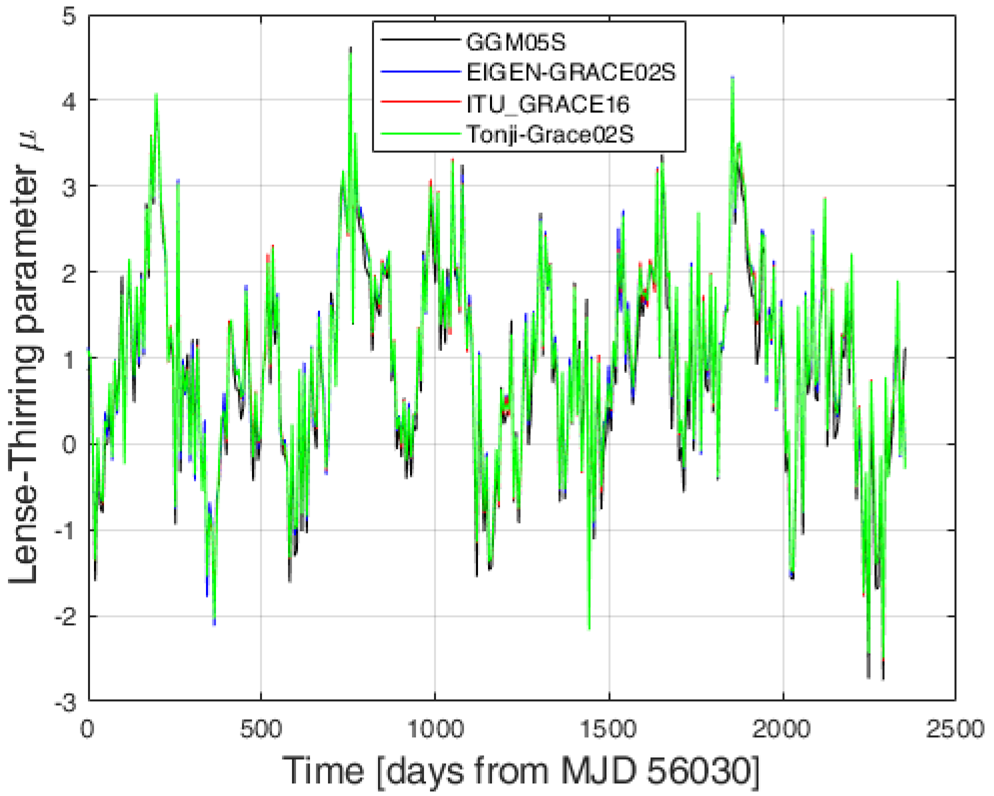
$$\left\langle \frac{d\Omega}{dt} \right\rangle_{sec} = \mu \frac{2G}{c^2 a^3} \frac{J_{\oplus}}{(1-e^2)^{3/2}}$$

$$\begin{cases} \mu = 0 & Netwonian\ gravitation \\ \mu = 1 & Einstein's\ GR \end{cases}$$

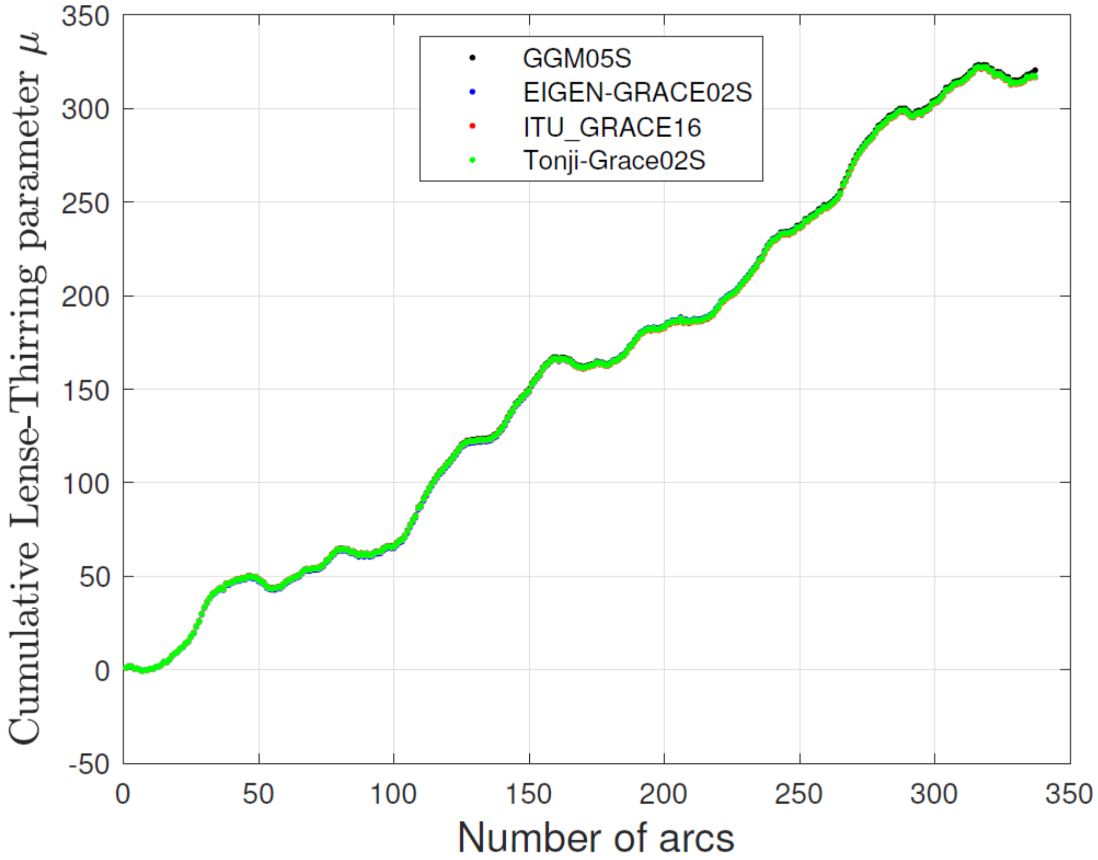
The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

Results for μ from the linear system



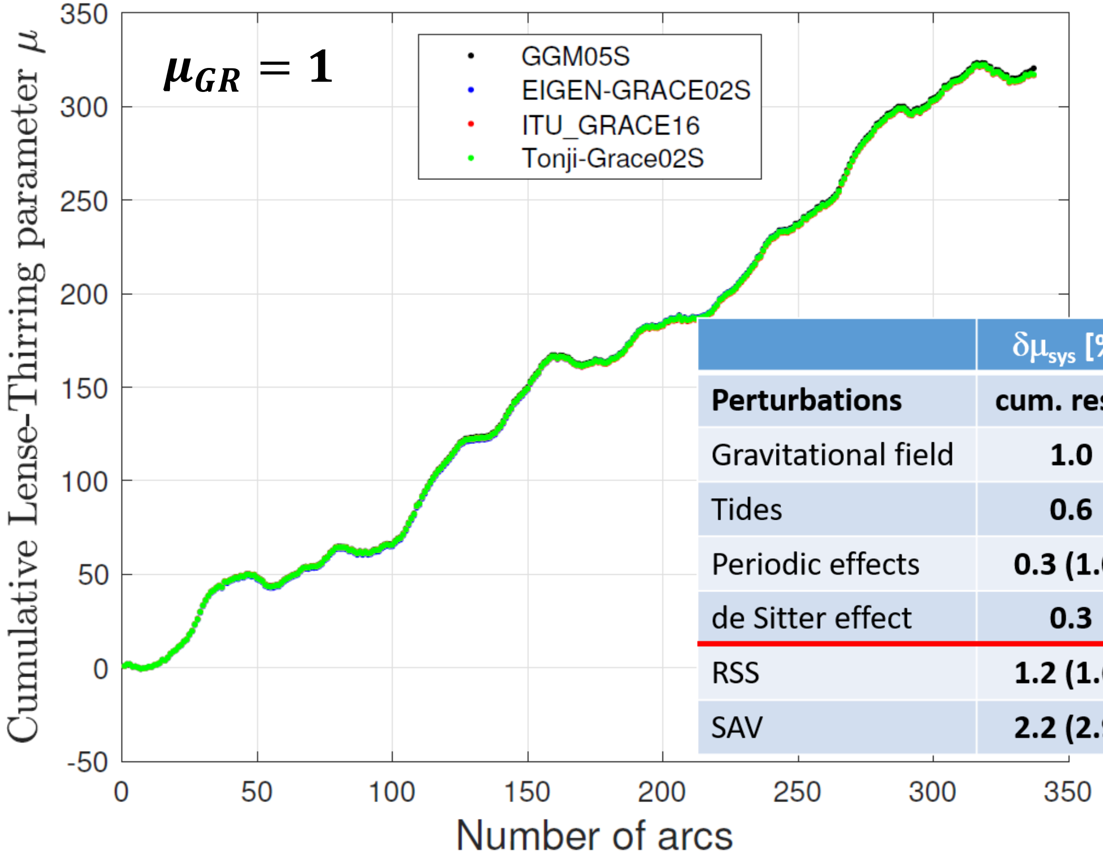
Cumulative sum for μ



The legacy from LARASE

- **Measurements in the field of Gravitation**: The **Lense-Thirring** precession

Lense-Thirring effect measurement



	$\delta\mu_{sys}$ [%]
Perturbations	**cum. resid.**
Gravitational field	**1.0**
Tides	**0.6**
Periodic effects	**0.3 (1.0)**
de Sitter effect	**0.3**
RSS	**1.2 (1.6)**
SAV	**2.2 (2.9)**

$$\begin{cases} \mu = 0 & \textit{Netwonian gravitation} \\ \mu = 1 & \textit{Einstein's GR} \end{cases}$$

Errors @ 95% CL

Model	$\mu \pm \delta\mu$	$\mu - 1$
GGM05S	1.0053 ± 0.0074	$+0.0053$
EIGEN-GRACE02S	1.0002 ± 0.0074	$+0.0002$
ITU_GRACE16	0.9996 ± 0.0074	-0.0004
Tonji-Grace02s	1.0008 ± 0.0074	$+0.0008$

$$\boxed{\mu_{meas} - 1 = 1.5 \times 10^{-3} \pm 7.4 \times 10^{-3} \pm 16 \times 10^{-3}}$$

Estimation of the systematic errors

The legacy from LARASE

universe

MDPI

Article

A 1% Measurement of the Gravitomagnetic Field of the Earth with Laser-Tracked Satellites

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Conclusions

- The **SaToR-G** goals were briefly introduced with the theoretical framework within which our work will be developed

- **SaToR-G** will expand the activities already started with the **LARASE** experiment

- **LARASE** has achieved significant results in the development of new models for the **Non-Gravitational Perturbations** for laser-ranged satellites and in the **measurement** of **relativistic precessions** and in **constraints** to some **alternative theories of gravitation**

- With **SaToR-G**, a series of activities were initiated with the aim of setting up new kinds of measurements in the field of gravitation with Earth-bound laser-ranged satellites. These activities will be based on a **theoretical/experimental** framework not ''simply'' described by **PPN** parameters, but also as close as possible to the original **framework** proposed by Professor **R.H. Dicke**

Many thanks for your kind attention

SaToR-G

Satellites **T**ests **o**f **R**elativistic **G**ravity