The B-E Distribution Function as a Formula for the Mass Differences inside the SU(3) Multiplets

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Abstract

A formula for the mass differences inside the SU(3) multiplets is given which can be derived from the conjecture that the increase of mass for the strange-content hadrons might originate from the statistical properties of a nonrelativistic system of noninteracting bosons, instead of being caused by a larger mass of the strange quark. This formula coincides with the Bose-Einstein distribution law for energy.

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1. - Introduction.

The statistical approach to the hadron modeling is based on the assumption that a gas of particles may be associated with the hadrons.[1,2,3] These models find a precedent in the application of statistical concepts to the cloud of pions surrounding a nucleon in the problem of the pion production in nucleon-nucleon collisions.[4] Here, we will make use of statistical concepts to interprete some regularities which fit the mass differences inside the SU(3) multiplets. A very preliminary form of these regularities has been first reported in a note of reference[5]. Unlike the Gell-Mann/Okubo formula, which deals with linear masses for baryons and with square masses for mesons,[6] our regularities will concern linear masses both for baryons and mesons. For each multiplet, the value of mass of only one nonstrange-content hadron will be required "to predict" the masses of certain other members. Specifically, we will give a formula for the masses which is consistent with the conjecture that the mass differences between hadrons of a same multiplet are determinable from the statistical thermodynamical properties of a nonrelativistic system of noninteracting bosons, without resorting, as usual, to a larger mass for the strange quark.

2. - The mass formula.

Let us first consider the four ground-state hadron multiplets together with the multiplets which belong to the Regge sequences started by them, i.e. the sequences of multiplets whose fundamental states, that is the states with zero strangeness and nonzero isospin, belong to the same Regge trajectory.[7] We mean the $J^P = 1/2^+$, $5/2^+$ baryon octets, the $J^P = 3/2^+$, $7/2^+$ baryon decuplets, and the $J^{PC} = 1^{--}$, 2^{++}, 3^{--} and $J^{PC} = 0^{-+}$, 1^{+-} meson multiplets. But, we will deal neither with the mainly-singlet states nor with those mainly-octet singlet-states having a nonideal mixing.

Let us now define the following statistical system. We associate a region of a certain volume V with each hadron considered. Within this region let there be a certain number of noninteracting bosons of mass M which are in equilibrium at a certain temperature T, with $M >> kT$. Volume V depend on the multiplet concerned according to the relationship

$$V \propto \frac{m'_0}{q^2} \tag{1}$$

where

$$q = \begin{cases} 2 \text{ for mesons} \\ 3 \text{ for baryons} \end{cases} \tag{2}$$

and m'_0 is the mass of the fundamental hadron of this multiplet if either $L = 0$ or $S = 0$, with L and S the total angular momentum and spin, respectively, otherwise it is the mass of the fundamental hadron of the $L = 0$ multiplet in the same Regge sequence. Let the number of bosons inside the volume be proportional to the number σ of strange constituents (quarks and/or antiquarks) contained in the hadron, i.e.

$$N_\sigma \propto \sigma. \tag{3}$$

Their density will be high enough so as to require van der Waals-like corrections to volume V. The temperature will vary as though our bosons were in thermal contact with a heat reservoir made up of an ideal gas of bosons undergoing adiabatic changes, that is

$$V_{\mathfrak{R}} T^{3/2} = A, \tag{4}$$

where A is a constant and $V_{\mathfrak{R}}$ is the volume of the reservoir taken to be

$$V_{\mathfrak{R}} \propto V. \tag{5}$$

Finally, let this system be characterized by a statistical factor (see eq.[8])

$$g = \begin{cases} 3 \text{ if } S \neq 0 \\ 1 \text{ if } S = 0, \end{cases} \tag{6}$$

as though a certain internal degree of freedom was frozen for total spin $S = 0$.

Let us now consider only one single-particle quantum-state of this system with an energy level equal to the mass m_0 of the fundamental hadron of the same multiplet to which our hadron belongs (note that, for a given hadron, m_0 may or may not coincide with m'_0 of eq. [1]). We will be interested, exclusively, in the average kinetic energy for this only subsystem, i.e. in the amount of energy which is given by its average number of occupation times m_0. It results that this amount of energy well coincides with the increase of hadron mass inside the multiplet. We can, then, write a hadron mass formula by means of the Bose-Einstein distribution function as

$$\Delta m = \frac{m_0}{\exp(\alpha + m_0/kT) - 1} \qquad (7)$$

with Δm equalling the difference of mass between each considered hadron having strange constituents and the fundamental (nonstrange nonzero-isospin) hadron in the same multiplet.

Parameter $\alpha = (-\mu kT)$ is obtained, as usual, from

$$\frac{B\sigma}{g(1 - \sigma b/V)} = \sum_{n=1}^{\infty} \frac{\exp(-n\alpha)}{n^{3/2}} \qquad (8)$$

with

$$B = \frac{h^3 N_1}{V(2\pi MkT)^{3/2}} \quad , \qquad (9)$$

where σb is a term which we introduce as a correction to volume V due to the size of the N_σ bosons.

The values of the various constants to be used in the calculations are

$$A = 5.55 \cdot 10^6 \ \ \mathrm{MeV}^{5/2} \qquad (10)$$

$$B = 1.0325 \qquad (11)$$

$$\frac{m'_0 b}{q^2 V} = 3.0 \ \mathrm{MeV}. \qquad (12)$$

We can, thus, use eq.(7) to predict the mass differences inside our multiplets. The calculated mass values, which are reported in table 1, are in good agreement with the experimental masses, taken from reference[8]. The underlined values are the masses of the fundamental states of the various multiplets which are used as inputs.

Special considerations must be done for the baryon octets, where we have two states, Λ and Σ, with the same number (one) of strange constituents. In this case, in fact, a specification by σ is found to be ambiguous. For the $1/2^+$ octet we have that $N_1 \propto 1$, as in eq.(3), corresponds to the lower (Λ) of the two states, while the other (Σ) would require $N'_1 \propto 1.394$. We find that, for the baryon octets, coincidences occur between states Σ for one of these two values of N, namely the smaller or the larger according to whether the Σ is the lower or the higher of the two states with one strange constituent in the multiplet concerned (thus, the additional coincidence involving the Λ[1116] might be due to some specific characteristic of this lowest-lying Λ). To check this peculiarity of the rule, we, finally, also consider the sufficiently established baryon octets with $L^P = 1^-$, i.e. the $J^P = 1/2^-$ and $J^P = 3/2^-$ multiplets with $S = 1/2$, and the sequence of the $J^P = 1/2^-$, $5/2^-$ multiplets with $S = 3/2$ (and $\Delta J = 2$). We generalize our criterion of choice of m'_0 to the case of the odd-L baryon octets as follows. As it, actually, occurred for the octets with an even L, m'_0 will be consistently taken to be the mass of the nonstrange hadron belonging to the multiplet considered if $J = 1/2$, or belonging to the $J = 1/2$ multiplet with the same spin S if J differs by 2 units, i.e. if $J = 5/2$. In particular, the value of m'_0 for the multiplet of the $J^P = 3/2^-$ N(1520) will have to be calculated by extrapolating its Regge trajectory, parallely to the average slope for baryons in the region below 1520 MeV, up to a fictitious $J = 1/2$ point.

3. - Concluding comments.

We have found a formula for the linear masses of hadrons with strange quarks and/or antiquarks which may follow from the conjecture that the differences of mass inside the SU(3) multiplets might originate from different states of excitation, at different temperatures, of a nonrelativistic gas of noninteracting bosons. Precisely, it is the average kinetic energy of only one single-particle quantum-state of this system, with a specific level of energy, which yields good coincidences with the observed differences of mass. The level of energy of this state equals the value of mass of the fundamental hadron, m_0, in the multiplet concerned. It should be as though a hadron of "base" mass m_0 - supposed to exchange energy with our statistical system - had only one level of energy available which could accommodate just quanta of value m_0 with the same probability as one single-particle state of the boson system with an equal energy. This additional amount of energy, stored in the hadron, would then account for the increase of mass which is related to different contents of strange constituents.

We note that this tentative point of view, suggested by the form of formula (7), clearly differs from the usual assumption that the differences of mass inside the multiplets are due to a larger mass of the strange quark in comparison with quarks u and d. The cause of these increases of mass has been here transferred to the properties of the environment, namely a gas of bosons, whose thermodynamical state would depend on both the SU(3) multiplet of the hadron considered and the number of strange constituents involved.

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Table Caption

Table 1. "Prediction" of the mass of hadrons with strange constituents from the mass of the nonstrange nonzero-isospin hadron in the same SU(3) multiplet.

$J^{P(C)}$	hadron	kT (MeV)	α	g	σ	m_{calc} (MeV)	m_{exp} (MeV)
1^{--}	$\rho(770)$	–	–	–	0	770	768-769
1^{-}	$K^{*}(892)$	940	1.175	3	1	891	892-896
1^{--}	$\phi(1020)$	940	0.596	"	2	1017	1019
2^{++}	$a_2(1320)$	–	–	–	0	1318	1318
2^{+}	$K_2^{*}(1430)$	940	1.175	3	1	1426	1425-1432
2^{++}	$f_2'(1525)$	940	0.596	"	2	1525	1525 ± 5
3^{--}	$\rho_3(1690)$	–	–	–	0	1688	1691 ± 5
3^{-}	$K_3^{*}(1780)$	940	1.175	3	1	1779	1774 ± 8
3^{--}	$\phi_3(1850)$	940	0.596	"	2	1858	1854 ± 7
0^{-+}	π	–	–	–	0	140	135-140
0^{-}	K	2930	0.284	1	1	496	493-498
1^{+-}	$b_1(1235)$	–	–	–	0	1233	1233 ± 3
1^{+}	$K_1(1400)$	687	0.332	1	1	1400	1402 ± 7
$1/2^{+}$	N	–	–	–	0	938	938-939
"	Λ	1415	1.163	3	1	1118	1116
"	Σ	1415	0.881	"	1	1193	1189-1197
"	Ξ	1415	0.575	"	2	1321	1315-1321
$5/2^{+}$	N(1680)	–	–	–	0	1680	1670-1690
"	$\Sigma(1915)$	1415	0.881	3	1	1923	1900-1935
$\geq 5/2^{?}$	$\Xi(2030)$	1415	0.575	"	2	2028	2025 ± 5

$3/2^+$	Δ(1232)	–	–	–	0	**1234**	1230-1234
"	Σ(1385)	1179	1.170	3	1	1385	1383-1387
"	Ξ(1530)	1179	0.586	"	2	1534	1532-1535
"	Ω	1179	0.297	"	3	1670	1672
$7/2^+$	Δ(1950)	–	–	–	0	**1915**	1910-1960
"	Σ(2030)	1179	1.170	3	1	2040	2025-2040
$3/2^-$	N(1520)	–	–	–	0	**1520**	1510-1530
"	Σ(1670)	1253	1.168	3	1	1675	1665-1685
"	Ξ(1820)	1253	0.583	"	2	1822	1823±5
$1/2^-$	N(1535)	–	–	–	0	**1520**	1520-1560
"	Σ(1620)	1026	1.173	3	1	1635	~1620
$1/2^-$	N(1650)	–	–	–	0	**1650**	1620-1680
"	Σ(1750)	971	1.174	3	1	1749	1730-1800
$5/2^-$	N(1675)	–	–	–	0	**1675**	1660-1690
"	Σ(1775)	971	1.174	3	1	1773	1770-1780

TABLE 1