European Geosciences Union General Assembly 2016

Vienna | Austria | 17–22 April 2016

Poster X2.308 EGU2016-8304

THE LARASE SPIN MODEL OF THE TWO LAGEOS AND LARES SATELLITES

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Introduction

We are developing the LARASE (LAser RAnged Satellites Experiment) project (Lucchesi et al., 2015) to test the gravitational interactions in the field of the Earth (i.e. in the Weak-Field-Slow-Motion limit of General Relativity). In this experiment we use the laser-ranged data of the two LAGEOS (LAser GEOdynamic Satellite) satellites and those of the most recent LARES (LAser RElativity Satellite). The data are made available by ILRS (International Laser Ranging Service) network. The observations of the station-satellite distance are used to precisely reconstruct the satellites' orbits. If the reconstruction of the orbit is accurate enough, it is possible to directly measure the influence of the non-Newtonian gravitational effects due to General Relativity.

To reach such aim, the contribution of all forces acting on the satellite must be carefully evaluated. A special role is played by thermal surface forces due to differential heating of the satellite. These forces depend on the rotational state of the satellite. A careful modelling/observation of the rotational state is therefore necessary to highlight thermal forces that could mask the gravitational effects that we try to measure.

In the past the spin problem had been already faced by Bertotti and Iess (1991), by Farinella et al., (1996) and by Andres et al., (2004) with the LOSSAM model. All these authors considered averaged torques and reached a good agreement with the experimental data. Conversely Habib et al. (1994) and Williams (2002) faced the general problem of not-averaged equations using the Euler equations. However, these works could not provide any prediction for the spin evolution in agreement with observational data.

Therefore, we have built a set of equations capable to model the spin behavior of the satellites by calculating the spin evolution starting from initial conditions. The calculated values of the spin components can be compared with the available measurements. Our model can produce results using not-averaged torques and therefore can be conveniently used when the spin period is close or larger than orbital period. Furthermore, we adopted Euler equations in the body reference frame, making it possible to easily handle the spin evolution when the rotational axis does not coincide with that of symmetry.

Equations of motion

We decided to solve the equation for the spin evolution in the more general case. In our analysis we considered two different reference frames: the Earth-centered inertial reference frame (ECI) and the body reference frame (BF) identified by the principal inertia axes of the satellite (x_B, y_B, z_B). The transformation from BF to the ECI frame is expressed by the three Euler angles (θ, ϕ, ψ). The Euler equations of the satellite take this form:

$$
\begin{aligned}
\ddot{\theta} &= \frac{\cos\psi M_x}{I_x} - \frac{\sin\psi M_y}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\frac{I_z}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\frac{I_y - I_z}{I_x} + \frac{I_x - I_y}{I_x}\left[\dot{\theta}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)\frac{\sin(2\psi)}{2}\frac{\Lambda}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\sin^2\psi\frac{\Lambda}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\left(\frac{I_y - I_z}{I_y} - \sin^2\psi\frac{\Lambda}{I_y}\right)\right] \\
\ddot{\phi} &= \frac{\cos\psi M_y}{I_y\sin\theta} + \frac{\sin\psi M_x}{I_x\sin\theta} + \frac{\dot{\psi}\dot{\theta}}{\sin\theta}\frac{I_z}{I_y} - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x} + \frac{I_x - I_y}{I_y}\left[\frac{\dot{\psi}\dot{\theta}}{\sin\theta}\left(\frac{\Lambda}{I_x}\sin^2\psi - 1\right) - \frac{\Lambda}{I_x}\dot{\phi}\frac{\sin(2\psi)}{2}\left(\cos\theta\dot{\phi} + \dot{\psi}\right) - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x}\cos^2\psi\right] \\
\ddot{\psi} &= \frac{M_z}{I_z} - \frac{\cos(\theta)}{\sin(\theta)}\left(\frac{\cos(\psi)M_y}{I_y} + \frac{\sin(\psi)M_x}{I_x}\right) + \dot{\phi}\dot{\theta}\frac{1}{\sin\theta}\left(\cos^2\theta\frac{I_y - I_z}{I_x} + 1\right) - \dot{\psi}\dot{\theta}\frac{I_z\cos\theta}{I_y\sin\theta} + (I_x - I_y)\left[\dot{\phi}\dot{\theta}\frac{1}{I_z}\frac{1}{\sin\theta}\left(\sin^2\theta\cos(2\psi) + \cos^2\psi\cos^2\theta\frac{\Lambda}{I_xI_y}\right) - \right. \\
&\left. \dot{\theta}^2\frac{\sin(2\psi)}{2I_z} - \dot{\phi}^2\frac{\sin(2\psi)}{2I_z}\left(\cos^2\theta\Lambda\frac{I_z}{I_xI_y} - \sin^2\theta\right) - \dot{\psi}\dot{\theta}\frac{\cos\theta}{I_y\sin(\theta)}\left(\sin^2\psi\frac{\Lambda}{I_x} - 1\right) + \dot{\phi}\dot{\psi}\cos\theta\sin(2\psi)\frac{\Lambda}{2I_xI_y}\right]
\end{aligned}
$$

where I_x, I_y, I_z are the principal moments of inertia, while M_x, M_y, M_z are the components of the total torque acting on the satellite and $\Lambda = I_x + I_y - I_z$.

Torques acting on the satellites

In our analysis we considered four possible torques:

- M_{mag} - from Earth magnetic field
- M_g - from Earth gravitational field
- M_{off} - due to the not coincidence between the center of mass and the geometric center of the satellite
- M_{asy} - due to an asymmetric reflectivity from the two hemispheres of the LAGEOS satellites.

$$M = M_{mag} + M_g + M_{off} + M_{asy}$$

Gravitational torque

The three satellites considered in our analysis are characterized by an imperfect spherical symmetry, therefore the Earth gravitational field produces a torque proportional to their oblateness:

$$
M_{grav} = -3\omega_{\oplus}^2 \begin{bmatrix} I_z - I_y & 0 & 0 \\ 0 & I_x - I_z & 0 \\ 0 & 0 & I_y - I_x \end{bmatrix} \begin{bmatrix} s_y\, s_z \\ s_x\, s_z \\ s_x\, s_y \end{bmatrix}
$$

where $\omega_{\oplus}$ is the Earth angular velocity and s_x, s_y and s_z are the three components of the unit vector from Earth to the satellite.

Magnetic torque

The satellites in their orbits around the Earth, while rotating around a body axis, move through the Geomagnetic field. Therefore, the satellites are polarized by the changing magnetic field, and, consequently a torque is generated:

$$
\begin{aligned}
M^E_{mag} = \; & V\sum_{i=1}^{9}\frac{|\boldsymbol{B_i}|^2}{2|\boldsymbol{\omega_s}|}\left\{A''_i\left[1 + \cos(2\omega_i t + 2\varphi_i)\right] - D'_i\sin(2\omega_i t + 2\varphi_i)\right\}\boldsymbol{\omega_s} + \\
& V\sum_{i=1}^{9}\frac{\boldsymbol{B_i}\cdot\boldsymbol{\omega_s}}{2|\boldsymbol{\omega_s}|^2}\left\{\left[\alpha'(\omega_i) - A'_i\right]\left[1 + \cos(2\omega_i t + 2\varphi_i)\right] - \left[D''_i + \alpha''(\omega_i)\right]\sin(2\omega_i t + 2\varphi_i)\right\}(\boldsymbol{\omega_s}\times\boldsymbol{B_i}) + \\
& V\sum_{i=1}^{9}\frac{\boldsymbol{B_i}\cdot\boldsymbol{\omega_s}}{2|\boldsymbol{\omega_s}|}\left\{-A''_i\left[1 + \cos(2\omega_i t + 2\varphi_i)\right] + D'_i\sin(2\omega_i t + 2\varphi_i)\right\}\boldsymbol{B_i}
\end{aligned}
$$

ω_s is the satellite spin velocity, A_i', A_i'', D_i', and D_i'' are combinations of the complex magnetic polarizability $\alpha(\omega) = \alpha' + i\,\alpha''$ of the satellite calculated at different frequencies, B_i are the components of the magnetic field changing along the orbit with frequencies ω_i and phases φ_i. The largest of the components B_i is that at the double of the satellite's mean motion n, the smallest one is at the rotational Earth angular velocity $\omega_{\oplus}$.

$$
\begin{aligned}
\omega_1 = \omega_2 &= \omega_{\oplus} - 2n \\
\omega_3 = \omega_4 &= \omega_{\oplus} + 2n \\
\omega_5 = \omega_6 &= 2n \\
\omega_7 = \omega_8 &= \omega_{\oplus} \\
\omega_9 &= 0
\end{aligned}
$$

$$
\varphi_i = \begin{cases} -\frac{\pi}{2} & \text{for i=2,4,6,8} \\ 0 & \text{for i=1,3,5,7,9} \end{cases}
$$

$$
A'_i = \frac{\alpha'(\omega_s - \omega_i) + \alpha'(\omega_s + \omega_i)}{2} \qquad D'_i = \frac{\alpha'(\omega_s - \omega_i) - \alpha'(\omega_s + \omega_i)}{2}
$$

$$
A''_i = \frac{\alpha''(\omega_s - \omega_i) + \alpha''(\omega_s + \omega_i)}{2} \qquad D''_i = \frac{\alpha''(\omega_s - \omega_i) - \alpha''(\omega_s + \omega_i)}{2}
$$

Geometrical offset radiation torque

If the geometric center of the satellite, where the resultant of surface forces is applied, does not coincide with the satellite's center of mass but it is at a distance h, a torque arises on the satellite (Vokrouhlicky, 1996):

$$
M_{off} = \nu\pi\rho^2\frac{\Phi}{c}C_R\left[h_x\,(\hat{x}_b \times \hat{s}_{\odot}) + h_z\,(\hat{z}_b \times \hat{s}_{\odot})\right]
$$

where h_x and h_z are the two components of h along the x and y axes, C_R is the satellite's surface reflectivity, ρ is the satellite radius, c is the speed of the light, Φ is the solar constant, $s_{\odot}$ is the unity vector from the satellite to the Sun, ν is the shadow function that measures the percentage of solar flux that reaches the satellite due to eclipses produced by the Earth (ν =1 when there is no eclipse and ν = 0 when the satellite is in eclipse).

Asymmetric radiation torque

An asymmetry in the value of the reflectivity coefficient C_R on the LAGEOS surface was hypothesized to explain the along-track residuals (Scharroo, 1991). In our analysis we consider the effect of this asymmetry on the spin dynamics. The correspondent torque acting on the satellite is given by (Vokrouhlicky, 1996):

$$
M_{ar} = \nu\,\frac{2}{3}\rho^3\,\frac{\Phi}{c}\,\Delta_R\,(\hat{z}_b \times \hat{s}_{\odot})\,|\hat{z}_b \times \hat{s}_{\odot}|
$$

Δ_R is the difference between the reflectivities of the two hemispheres of the satellite, ρ is the satellite radius, c is the speed of light, Φ is the solar constant, $s_{\odot}$ is the unit vector from the satellite to the Sun, ν is the shadow function that measures the percentage of solar flux that reaches the satellite due to eclipses produced by the Earth (ν =1 when there is no eclipse and ν = 0 when the satellite is in eclipse).

Conclusions

The result obtained will be used within the LARASE project devoted to measure General Relativity effects. Indeed, a reliable spin model is needed in order to model with precision the surface thermal forces that could hide the effects we aim to measure due to their long-term perturbations on the right ascension of the ascending node and argument of pericenter of the satellite's orbit.

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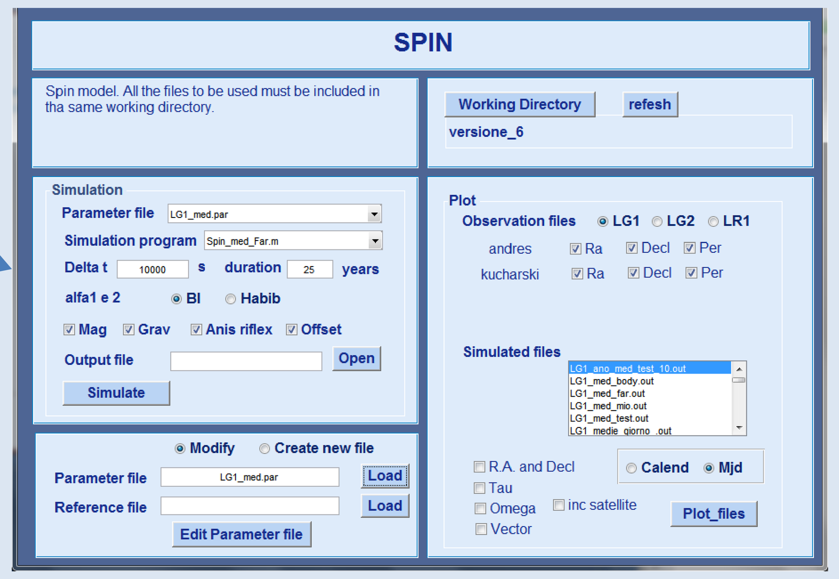
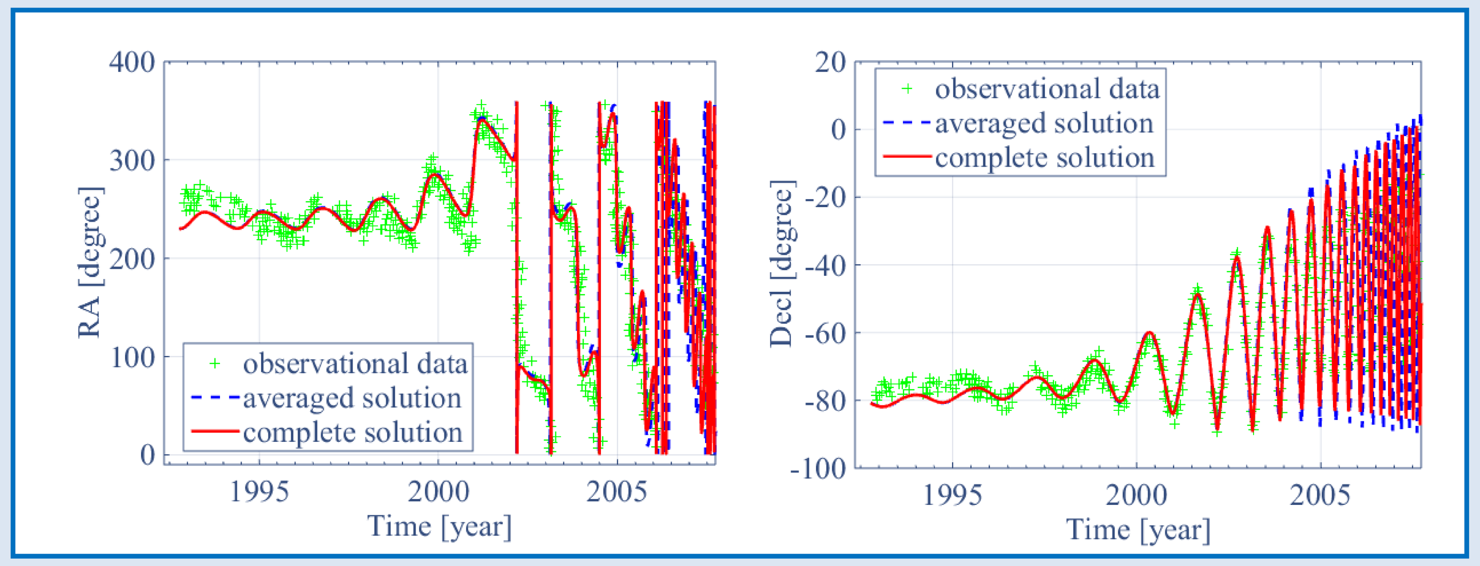
Results with our model

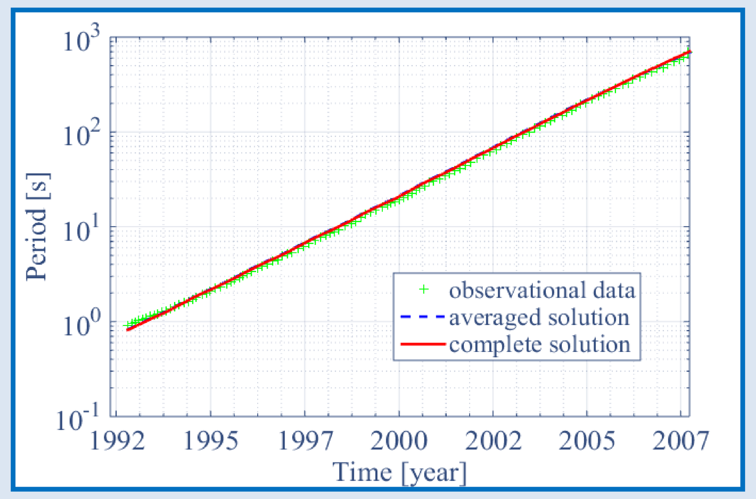
We built a code based on MATLAB routines. We used as independent variables θ, ϕ, ψ, $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$, reducing the equations to a system of six differential equations of first order.

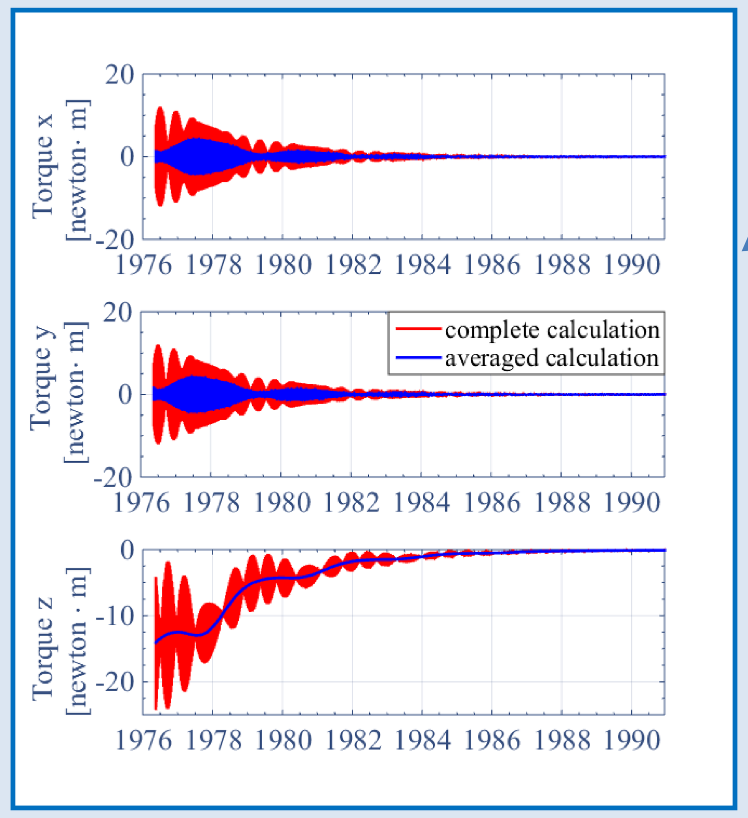
Visual interface of MATLAB code to solve spin equations for different satellites, different equations (averaged or not) and different parameters sets.





The two solutions for the spin of LAGEOS II: in red the complete solution, in blue the solution with averaged torques. The green crosses are experimental data from Kurchasky et al., (2013)





The magnetic torque acting on LAGEOS calculated with our model in body reference frame: in red the complete calculation, in blue the averaged one. The torque along axis z, the initial rotation axis, spins down the satellite