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The Cosmological Constant as a Present Effective Dark-Sector Value: A Compact Phenomenological Parametrization for Cosmic Acceleration That Need Not Be Eternal

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Abstract

This work proposes a compact phenomenological parametrization in which the quantity observationally identified with the cosmological constant is interpreted as the present effective value, $\Lambda_{eff}(t_0)$, of a dark component that remains close to the Lambda cold dark matter (ΛCDM) model over the recent cosmological epoch but is not required to remain constant in the remote future. The purpose is not to introduce dynamical dark energy in general, which has already been extensively investigated, but to formulate a minimal subclass in which the present effective value associated with Λ and the asymptotic fate of the expansion are explicitly separated. Within this framework, the current dominance of the dark component does not require an increase in Λ_{eff}: it can arise because the matter density dilutes as a^{-3}, whereas Λ_{eff} varies more slowly. A delayed-decay function is introduced, normalized to the present effective value and allowing $\Lambda_{eff} \to 0$ as $a \to \infty$. In the strong version of the parametrization, identified by $n_f > 2$, the effective component is asymptotically unable, by itself, to sustain cosmic acceleration indefinitely. The proposal is not presented as a microscopic theory of dark energy, but as a background-level effective dark-sector parametrization that may serve as a phenomenological benchmark for particle-cosmology scenarios and can be constrained through measurements of $H(z)$ and cosmological distances, together with observational constraints on $w(a)$.

Keywords: cosmological constant; dark sector; dynamical dark energy; particle cosmology; ΛCDM; transient acceleration; equation of state

1. Introduction

Late-time cosmic acceleration, originally inferred from Type Ia supernova observations, represents one of the central observational results of modern cosmology [1,2]. In the standard Lambda cold dark matter (ΛCDM) model, this acceleration is described by a positive cosmological constant, Λ. Despite the remarkable observational success of this framework, the physical nature of Λ remains unresolved, particularly in connection with the cosmological constant problem and the difficulty of naturally interpreting a small, positive, and apparently constant vacuum energy density [3–6]. Precision cosmological constraints, especially those obtained by Planck, confirm the effectiveness of ΛCDM in describing the observable Universe [7].

The possibility that cosmic acceleration may be produced by a dynamical dark component, rather than by a strictly time-independent fundamental constant, has been extensively explored. Quintessence models and scalar-field descriptions provide classic

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examples [8–10], while broader reviews of dark energy dynamics and its phenomenological reconstruction are given in [11,12]. Decaying, metastable, and transient-acceleration dark energy scenarios have also been considered [13–17]. Therefore, the general idea of dynamical dark energy is not, in itself, new.

The point addressed here is narrower. The aim is not to provide another microscopic model of dark energy, nor to replace scalar-field or metastable dark energy constructions. Rather, the objective is to isolate a minimal background-level distinction that is often hidden in the standard interpretation: the value of the cosmological component inferred at the present epoch need not determine its asymptotic future behavior. In this sense, the parametrization is intended as a compact phenomenological bridge between a present value close to ΛCDM and a future evolution that need not approach a de Sitter state. Accordingly, the quantity observationally identified with Λ is interpreted as the present effective value of a phenomenological cosmological component,

$$\Lambda_{eff}(t_0) \simeq \Lambda_0 \tag{1}$$

Here, t_0 denotes the present cosmic time, and Λ_0 denotes the value associated with the cosmological constant in the standard cosmological description at the present epoch. This interpretation does not require assuming that such a value must coincide with a fundamental constant valid throughout the entire future cosmological evolution. In particular, the hypothesis considered here is that an effective component may exist such that

$$\Lambda_{eff}(t_0) > 0, \ \lim_{t\to\infty} \Lambda_{eff}(t) = 0 \tag{2}$$

The essential physical point is that the current dominance of the dark component does not require an increase in Λ_{eff}. The density of non-relativistic matter dilutes as

$$\rho_m(a) \propto a^{-3} \tag{3}$$

Here, a is the cosmological scale factor, normalized such that $a(t_0) = 1$. Consequently, an effective component that decays more slowly than matter can become dominant even while decreasing in absolute value. The distinction proposed here therefore concerns two concepts that coincide in ΛCDM: the value of the cosmological component observed today and its asymptotic future behavior. In this sense, the parametrization introduced below combines three elements in a minimal form: the effective interpretation of the present value associated with Λ, the possibility of dominance without absolute growth of the dark component, and a delayed decay capable of separating current acceleration from the asymptotic fate of the Universe.

The analysis is deliberately limited to background dynamics. No microscopic derivation of Λ_{eff} is proposed, and no cosmological fit of the parameters is performed. The aim is to introduce a minimal and controlled parametrization, clarify its essential dynamical conditions, and place it in relation to the literature on dynamical dark energy and transient cosmic acceleration. In this context, equation-of-state parametrizations provide the natural phenomenological language for describing deviations from $w = -1$ [18,19], while recent analyses from the Dark Energy Spectroscopic Instrument (DESI) provide an observational motivation for reconsidering dynamical dark energy models [20–22].

Although the construction does not specify new particles or a fundamental interaction, it can be read as an effective dark-sector layer within the broader particle-cosmology landscape beyond ΛCDM. In this interpretation, the effective quantity $\Lambda_{eff}(a)$ summarizes the background-level consequence of an unresolved dark-sector mechanism rather than a specified microscopic model. Scalar-field dynamics and quintessence provide one possible route to time-dependent dark energy [8–10]. Decaying or metastable dark energy scenarios provide another route, in which the late-time component may evolve without being interpreted as a strictly eternal cosmological constant [13,16]. Interacting dark-sector models and running-vacuum prescriptions offer further effective interpretations of a changing dark component [23–25]. Effective modifications of gravity provide an additional possibility, in which an apparent dark component can emerge from modified background dynamics rather than from a separately specified fluid [26,27]. The present parametrization is not intended to compete with these mechanisms or to claim greater generality. Its specific contribution is to encode, within a single normalized background-level construction, three features that are not kept equally explicit in standard phenomenological descriptions: the present value of the effective component, the onset and sharpness of its departure from near-ΛCDM behavior, and its asymptotic decay law. Chevallier–Polarski–Linder (CPL) forms are primarily used to describe the observable evolution of $w(a)$; by contrast, the delayed-decay form adopted here assigns an explicit asymptotic power-law behavior to $\Lambda_{eff}(a)$ [18,19]. Metastable and transition scenarios may instead connect the future evolution to a lifetime, a decay prescription, or underlying field dynamics [13–17]. The practical advantage of the present form is therefore interpretive transparency rather than an a priori gain in constraining power: the same logarithmic decay-rate function distinguishes slower dilution than matter from the ability of the effective component to sustain acceleration. At the same time, the asymptotic decay rate, transition epoch, and transition sharpness can be partially degenerate when only the finite redshift interval accessible to observations is considered. This limitation is intrinsic to the parametrization and must be retained when interpreting constraints on its parameters.

2. Delayed-Decay Parametrization

The effective cosmological component is described through a quantity $\Lambda_{eff}(a)$, normalized to the value observed at the present epoch. The corresponding mass-equivalent density is defined as

$$\rho_{\Lambda,eff}(a) = \frac{\Lambda_{eff}(a)c^2}{8\pi G} \tag{4}$$

Here, c is the speed of light and G is Newton's gravitational constant. In this expression, $\rho_{\Lambda,eff}$ denotes an equivalent density in mass units; the corresponding energy density would be $\epsilon_{\Lambda,eff}(a) = \rho_{\Lambda,eff}(a)c^2$. In the following, however, the background dynamics is formulated directly in terms of $\Lambda_{eff}(a)$, in order to keep the normalization to the present cosmological value explicit.

The present normalization is fixed by imposing $\Lambda_{eff}(1) = \Lambda_0$, with $a(t_0) = 1$. The general parametrization is then introduced as

$$\Lambda_{eff}(a) = \Lambda_0 \exp\left[-\int_1^a n(x)\, d\, ln\, x\right] \tag{5}$$

where

$$n(a) = -\frac{d\, ln\Lambda_{eff}}{d\, ln\, a} \tag{6}$$

The function $n(a)$ is not introduced to make a fundamental constant vary arbitrarily, but to describe the logarithmic decay rate of an effective term normalized to the present cosmological value. The limit $n(a) = 0$ exactly recovers ΛCDM, whereas $n(a) > 0$ corresponds to an effective component that decreases with the expansion.

To represent a delayed decay, negligible over the recent observable epoch but potentially relevant in the future, the following form is assumed:

$$n(a) = \frac{n_f}{1 + (a_t/a)^s} \tag{7}$$

with $n_f > 0$, $a_t > 1$, and $s > 0$. Here, n_f is the asymptotic value of the logarithmic decay rate, a_t is the transition scale factor in units of the present scale factor, and s controls the sharpness of the transition. For $a/a_t \ll 1$, $n(a)$ remains small; for $a/a_t \gg 1$, $n(a) \to n_f$. In this way, behavior close to ΛCDM can be preserved near the present epoch without requiring it to persist indefinitely.

Integrating Equation (5) with the choice in Equation (7) gives

$$\Lambda_{eff}(a) = \Lambda_0 \left[\frac{1 + a_t^{-s}}{1 + (a/a_t)^s} \right]^{n_f/s} \tag{8}$$

The normalization $\Lambda_{eff}(1) = \Lambda_0$ is automatically satisfied. In the future limit,

$$\Lambda_{eff}(a) \propto a^{-n_f}, a \to \infty \tag{9}$$

and therefore $\Lambda_{eff}(a) \to 0$ for $n_f > 0$. The effective component can thus remain nearly constant at the present epoch while becoming asymptotically decaying in the future.

In the present phenomenological interpretation, the component defined by $\Lambda_{eff}(a)$ is treated as a separately conserved effective fluid at the background level. Under this assumption, its background evolution obeys the standard homogeneous continuity equation [11]:

$$\dot{\rho}_{\Lambda,eff} + 3H\left[1 + w_{eff}(a)\right]\rho_{\Lambda,eff} = 0 \tag{10}$$

Here, an overdot denotes differentiation with respect to cosmic time t, $H \equiv \dot{a}/a$ is the Hubble parameter, and $w_{eff}(a)$ is the dimensionless effective equation-of-state parameter. Rewriting Equation (10) using $d/dt = H\, d/d\ln a$ gives $dln\rho_{\Lambda,eff}/dlna = -3\left[1 + w_{eff}(a)\right]$. Since $\rho_{\Lambda,eff}(a) \propto \Lambda_{eff}(a)$ and, from Equation (6), $n(a) = -dln\Lambda_{eff}/dlna$, it follows that

$$w_{eff}(a) = -1 + \frac{n(a)}{3} \tag{11}$$

Equation (11) is therefore not imposed as an independent assumption. It follows from the separately conserved effective-fluid interpretation and the definition of the logarithmic decay rate in Equation (6).

Other interpretations of a time-dependent effective cosmological term could lead to different relations between $\Lambda_{eff}(a)$ and $w(a)$. Interacting dark-sector models provide one possibility, because the effective evolution of the dark energy density can be tied to exchange terms rather than to separate conservation [23,24]. Dynamical- or running-vacuum scenarios provide another possibility, in which the vacuum contribution evolves according to a different phenomenological prescription [25]. Effective modifications of the gravitational field equations may also mimic a time-dependent dark component without requiring the same effective-fluid interpretation adopted here [26,27]. These alternatives are not modeled here; the separately conserved effective-fluid interpretation is adopted only to connect the delayed-decay parametrization with standard background observables and to define a minimal effective dark-sector benchmark.

Closeness to ΛCDM over the recent cosmological epoch requires $n(a) \simeq 0$ for $a \simeq 1$. From Equation (7), in particular,

$$n(1) = \frac{n_f}{1 + a_t^s} \ll 1 \tag{12}$$

Therefore, if n_f is of order unity or larger, qualitative compatibility with behavior close to ΛCDM at the present epoch requires a sufficiently delayed transition, namely $a_t^s \gg n_f$. This condition allows w_{eff} to remain close to -1 over the recent observable domain, while still leaving open the possibility of future variation.

The adopted parameter domain is $n_f > 0$, $a_t > 1$, and $s > 0$. The condition $n_f > 0$ ensures asymptotic decay, whereas $a_t > 1$ places the transition after the present epoch and is therefore essential to the delayed-future interpretation. The parameter s controls the width of the transition: small positive values produce a gradual departure, while larger values produce a sharper one. These conditions define the phenomenological class but do not constitute observational priors. In particular, restrictions imposed on a_t and s can strongly affect the inferred future evolution, because present observations mainly probe the combination $n_f/(1 + a_t^s)$ appearing in the present decay rate. Any constraint on the asymptotic behavior must therefore be interpreted together with the adopted parameter ranges.

3. Relative Dominance Without Absolute Growth and Acceleration That Need Not Be Eternal

A consequence of the parametrization introduced above is the separation between the relative dominance of the effective component and its ability to sustain cosmic acceleration. A component may become dominant relative to matter even while decreasing in absolute value, provided that its decay is slower than the dilution of non-relativistic matter.

The normalized effective term is defined as

$$F(a) = \frac{\Lambda_{eff}(a)}{\Lambda_0} \tag{13}$$

With this definition, the standard background Friedmann equation can be written in normalized form as [6,7,11]:

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{k,0}a^{-2} + \Omega_{\Lambda,0}F(a) \tag{14}$$

Here, H_0 is the present Hubble parameter, while $\Omega_{m,0}$, $\Omega_{r,0}$, $\Omega_{k,0}$, and $\Omega_{\Lambda,0}$ denote the present-day density parameters of matter, radiation, curvature, and the effective cosmological component, respectively.

In the ΛCDM limit, $F(a) = 1$. In the case considered here, instead, $F(1) = 1$, but $F(a)$ may decrease in the future. The ratio between the effective component and matter is

$$R(a) = \frac{\rho_{\Lambda,eff}(a)}{\rho_m(a)} = \frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} a^3 F(a) \tag{15}$$

Thus, $R(a)$ measures the relative importance of the effective component with respect to non-relativistic matter.

Using the definition of $n(a)$, one obtains

$$\frac{dln\ R}{dln\ a} = 3 - n(a) \tag{16}$$

This relation formalizes the central point of the parametrization. The effective component can become more important than matter even while decreasing in absolute value, provided that it decays more slowly than matter, namely provided that $n(a) < 3$. The current dominance of Λ_{eff} therefore does not require absolute growth of the dark component.

The condition associated with acceleration is distinct. If the effective component is treated as a separately conserved fluid, Equation (11) shows that it has an equation of state sufficiently negative to behave as an accelerating component only if $w_{eff} < -1/3$, namely if

$$n(a) < 2 \tag{17}$$

This condition concerns the ability of the effective component, considered by itself, to have sufficiently negative pressure. It does not automatically coincide with the full condition for acceleration of the total expansion, which also depends on the relative fractions of matter, radiation, curvature, and the effective component. Nevertheless, the distinction between Equations (16) and (17) is essential: $n(a) < 3$ controls the relative growth with respect to matter, whereas $n(a) < 2$ controls the accelerating character of the effective component.

The acceleration of the total cosmological background is conventionally characterized by the dimensionless deceleration parameter [6,11]:

$$q(a) \equiv -\frac{\ddot{a}}{aH^2(a)} = -\frac{a\ddot{a}}{\dot{a}^2} \tag{18}$$

Here, $q(a) < 0$, $q(a) = 0$, and $q(a) > 0$ denote accelerated expansion, a transition between acceleration and deceleration, and decelerated expansion, respectively. For the background expansion in Equation (14), with the effective component described by the equation of state in Equation (11), the deceleration parameter becomes

$$q(a) = \frac{\Omega_{m,0}a^{-3} + 2\Omega_{r,0}a^{-4} + [n(a) - 2]\Omega_{\Lambda,0}F(a)}{2E^2(a)} \tag{19}$$

Here, $E(a)$ is the normalized Hubble function, $F(a) = \Lambda_{\rm eff}(a)/\Lambda_0$, and $n(a)$ is the logarithmic decay rate of the effective component. The three terms in the numerator represent the matter, radiation, and effective-component contributions to the acceleration equation. The curvature term does not appear because it does not contribute directly to $\ddot{a}$. The factor $n(a) - 2$ makes the physical threshold explicit: the effective component contributes negatively to $q(a)$ when $n(a) < 2$, although acceleration of the total background additionally requires this contribution to overcome those of matter and radiation.

For the three illustrative parameter sets used in Figure 1, adopting the same present-day values $\Omega_{\Lambda,0} = 0.70$ and $\Omega_{m,0} = 0.30$, with $\Omega_{r,0} = 0$ and $\Omega_{k,0} = 0$ for this late-time illustration, the first crossing $q(a) = 0$ occurs at $a \simeq 0.598$ for all three sets and marks the onset of the current accelerated phase. The second crossing, corresponding to the return to decelerated expansion, occurs at $a \simeq 7.06$, 11.89, and 20.00 for Sets A, B, and C, respectively. At the present epoch, the corresponding values are $q(1) \simeq -0.55$ in all three cases. These values are illustrative consequences of the selected parameter sets and present-day density parameters, not fitted cosmological constraints.

In the future limit, the chosen parametrization gives $n(a) \to n_f$, and therefore $w_{eff}(a) \to -1 + n_f/3$ as $a \to \infty$. The asymptotic condition for the effective component to retain an accelerating equation of state is therefore $n_f < 2$. The strong version of the parametrization is instead identified by

$$n_f > 2 \tag{20}$$

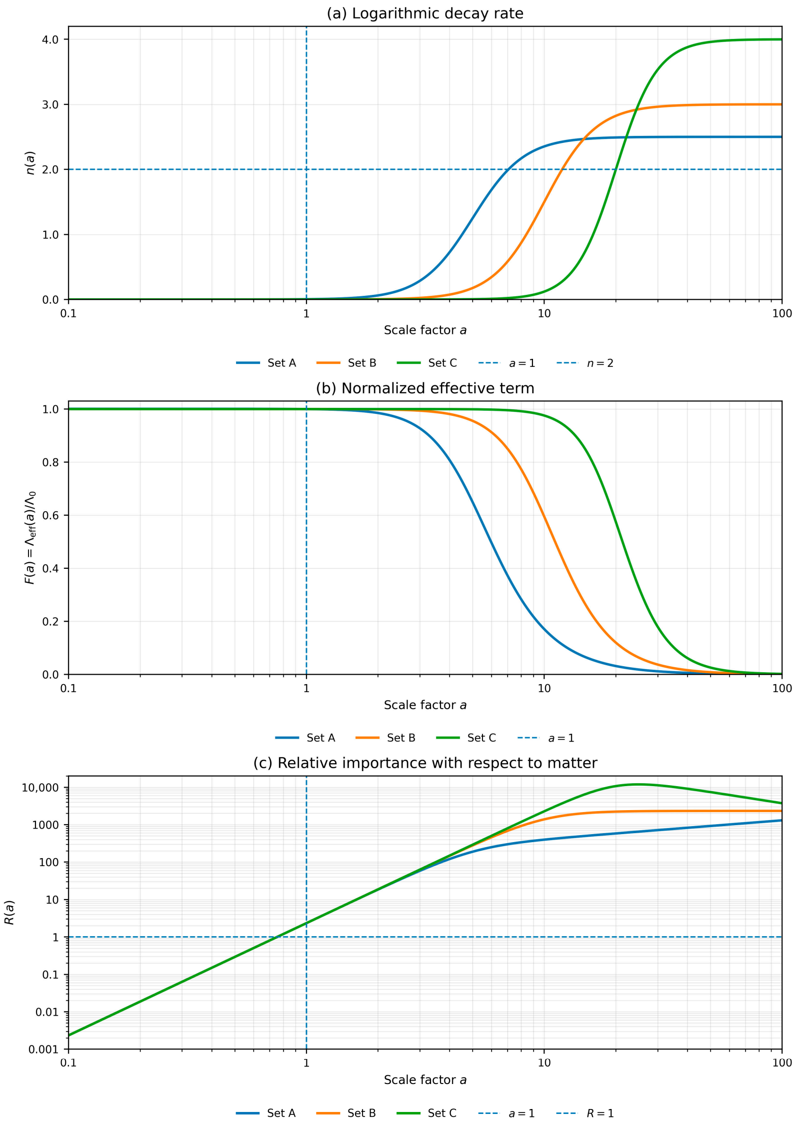

Figure 1. Representative delayed-decay trajectories. The same illustrative parameter sets are used in all panels: Set A, $n_f = 2.5$, $a_t = 5$, $s = 4$; Set B, $n_f = 3$, $a_t = 10$, $s = 4$; and Set C, $n_f = 4$, $a_t = 20$, $s = 5$. (**a**) Logarithmic decay rate $n(a)$, with the horizontal reference line indicating $n = 2$; (**b**) normalized effective term $F(a) = \Lambda_{eff}(a)/\Lambda_0$, showing the future decay of the effective cosmological component; (**c**) relative importance $R(a) = \rho_{\Lambda,eff}(a)/\rho_m(a)$ of the effective component with respect to non-relativistic matter, defined as the unnormalized density ratio in Equation (15), with the horizontal reference line indicating $R = 1$, i.e., equality between matter and the effective component. In panel (**c**), $R(a)$ is evaluated using the illustrative present-day values $\Omega_{\Lambda,0} = 0.70$ and $\Omega_{m,0} = 0.30$, so that $R(1) = \Omega_{\Lambda,0}/\Omega_{m,0} \simeq 2.33$. The vertical dashed line marks the present epoch, $a = 1$. The trajectories are not intended as cosmological fits, but only as qualitative illustrations of a delayed transition from near-ΛCDM behavior around the present epoch to asymptotic decay in the future.

In this case, the effective component is no longer able, by itself, to sustain cosmic acceleration indefinitely. This condition is not required in order to obtain

$$\Lambda_{eff}(a) \to 0 \quad \text{as} \quad a \to \infty \tag{21}$$

which only requires

$$n_f > 0 \tag{22}$$

Rather, the stronger condition in Equation (20) distinguishes a generic asymptotic decay from a scenario in which cosmic acceleration is not necessarily eternal.

The asymptotic behavior can be classified more explicitly according to the value of n_f. For $0 < n_f < 2$, the effective component decays to zero but retains an accelerating equation of state asymptotically. The case $n_f = 2$ is the limiting case, with $w_{eff} \to -1/3$. For $2 < n_f < 3$, the effective component is no longer accelerating by itself, but it still decays more slowly than matter and can therefore continue to grow in relative importance. For $n_f = 3$, the ratio to matter tends asymptotically to a constant. For $n_f > 3$, the effective component decays faster than matter and may eventually lose relative dominance. This last behavior is illustrated by Set C in Figure 1, where $R(a)$ increases during the intermediate epoch but decreases again once the asymptotic decay becomes faster than matter dilution.

The behavior of the delayed-decay parametrization is illustrated in Figure 1 for three representative parameter sets. The same sets are used in all panels and are intended only to show the qualitative roles of n_f, a_t, and s, not to provide cosmological fits.

Figure 1 shows how the three parameters control distinct aspects of the delayed evolution. In panel (a), a_t determines the approximate epoch at which $n(a)$ departs appreciably from zero, s controls the sharpness of that transition, and n_f fixes the asymptotic logarithmic decay rate. All three trajectories remain close to $n(a) = 0$ around $a = 1$, while the horizontal line at $n = 2$ identifies the threshold above which the effective component no longer has an equation of state sufficiently negative to sustain acceleration by itself. Panel (b) displays the corresponding normalized effective term $F(a)$. Its near-constant behavior around the present epoch reflects the delayed transition, whereas its decline at larger a follows the increase of $n(a)$. Panel (c) shows that the ratio $R(a)$ can continue to increase even while $F(a)$ decreases, because matter dilutes as a^{-3}. For Sets A and B, the effective component continues to grow relative to matter over the plotted interval. For Set C, $n(a)$ eventually exceeds 3, so $R(a)$ reaches a maximum and subsequently decreases. The three panels therefore distinguish the absolute decay of the effective component, its relative importance with respect to matter, and its ability to sustain accelerated expansion.

4. Relation to the Literature and Observational Constrainability

The proposed parametrization belongs to the broad class of dynamical dark energy descriptions, but its role is more restricted. Relative to quintessence and scalar-field models, it does not specify a field, a potential, or a microscopic equation of motion [8–10].

Relative to phenomenological reconstruction approaches, it does not aim to reconstruct a general dark energy history from data [11,12]. Its purpose is instead to describe, at the background level, how a component with the present value usually associated with Λ may remain close to ΛCDM over the recent cosmological epoch while acquiring a non-zero logarithmic decay rate only in the future.

Relative to decaying or metastable dark energy scenarios, the parametrization does not assume a specific decay channel, lifetime, or microscopic instability. The decay is represented through the delayed logarithmic rate $n(a)$, normalized so that $\Lambda_{eff}(1) = \Lambda_0$. This makes the present effective value an explicit input, while the asymptotic behavior is controlled separately by n_f, a_t, and s. Relative to transient-acceleration models, the emphasis is not only on whether acceleration eventually ends, but also on the distinction between relative dominance and accelerating character: $n(a) < 3$ controls growth relative to matter, whereas $n(a) < 2$ controls whether the effective component can sustain acceleration by itself [13–17].

From a particle-cosmology perspective, this separation is useful because different microscopic dark-sector mechanisms may be nearly degenerate around the present epoch, $a = 1$, while implying different asymptotic behaviors.

These distinctions can also be stated directly at the level of functional form. The Chevallier–Polarski–Linder parametrization describes the finite-redshift evolution of the equation of state through $w(a) = w_0 + w_a(1-a)$, but it does not impose a controlled far-future asymptote for the dark energy density [18,19].

Early dark energy parametrizations address the opposite temporal regime by allowing a non-negligible dark energy fraction at high redshift, often through a direct parametrization of the dark energy density fraction $\Omega_{de}(a)$; their characteristic additional parameter therefore describes the early-time fraction rather than a future decay law [28].

Transition and metastable scenarios may instead interpolate between equation-of-state regimes or associate the evolution with a lifetime, decay prescription, or underlying instability [13–17].

The delayed-decay form adopted here directly describes how the effective component evolves with cosmic expansion: a_t determines the characteristic transition epoch, s controls the sharpness of the transition, and n_f fixes the asymptotic power-law decay. Its practical advantage is therefore the direct physical interpretation of the thresholds $n = 2$ and $n = 3$ and of the asymptotic density law, rather than intrinsically improved statistical identifiability. A more transparent parametrization is not necessarily an easier one to constrain when only a finite redshift interval is observed [29].

Equation-of-state parametrizations remain the natural observational context [18,19]. In the present model, deviations from $w = -1$ are controlled directly by the logarithmic decay rate $n(a)$, since

$$w_{eff}(a) + 1 = \frac{n(a)}{3} \tag{23}$$

At the present epoch, this gives

$$w_0 \equiv w_{eff}(1) = -1 + \frac{n_f}{3(1+a_t^s)} \tag{24}$$

Here, w_0 denotes the present-day value of the effective equation-of-state parameter. If $\delta w > 0$ denotes a generic observational tolerance on the present-day deviation from $w = -1$, such that $|w_0 + 1| \lesssim \delta w$, then Equation (24) implies the approximate condition

$$\frac{n_f}{1+a_t^s} \lesssim 3\delta_w \tag{25}$$

Thus, for fixed n_f, compatibility with a value close to $w = -1$ near the present epoch requires the transition to be sufficiently delayed through a_t and s. A local comparison with the Chevallier–Polarski–Linder convention, $w(a) = w_0 + w_a(1-a)$, gives, near $a = 1$,

$$w_a \simeq -\left.\frac{dw_{eff}}{da}\right|_{a=1} = -\frac{s n_f a_t^s}{3(1+a_t^s)^2} \tag{26}$$

Here, w_a is not introduced as an independent fitting parameter, but only as a local low-redshift descriptor of the delayed-decay form. Equation (26) therefore defines only a local correspondence and should not be interpreted as a global equivalence between the present parametrization and the CPL parametrization [30]. For the illustrative parameter sets used in Figure 1, one obtains $w_0 + 1 \simeq 1.3 \times 10^{-3}$, 1.0×10^{-4}, and 4.2×10^{-7} for Sets A, B, and C, respectively. The corresponding local CPL-like slopes are $w_a \simeq -5.3 \times 10^{-3}$, -4.0×10^{-4}, and -2.1×10^{-6}. These values show that the illustrative cases are deliberately very close to ΛCDM at $a = 1$, while still allowing a future departure.

Constraints from $H(z)$ act through the normalized Hubble function in Equation (14). Relative to a ΛCDM background with the same present-day density parameters, the fractional change in $E^2(a)$ is

$$\frac{E^2(a) - E^2_{\Lambda CDM}(a)}{E^2_{\Lambda CDM}(a)} = \frac{\Omega_{\Lambda,0}\,[F(a) - 1]}{\Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{k,0}a^{-2} + \Omega_{\Lambda,0}} \tag{27}$$

Here, $E^2_{\Lambda CDM}(a)$ denotes the normalized ΛCDM expansion function obtained from Equation (14) by setting $F(a) = 1$. To distinguish the composition of the reference background from the model-induced departure, Figure 2a shows the fractional matter and cosmological-term contributions to $E^2_{\Lambda CDM}(z)$, whereas Figure 2b shows the fractional deviation defined in Equation (27) after using $a = (1+z)^{-1}$. The observable interval $0 \le z \le 3$ is considered, with $\Omega_{m,0} = 0.30$, $\Omega_{\Lambda,0} = 0.70$, and $\Omega_{r,0} = \Omega_{k,0} = 0$.

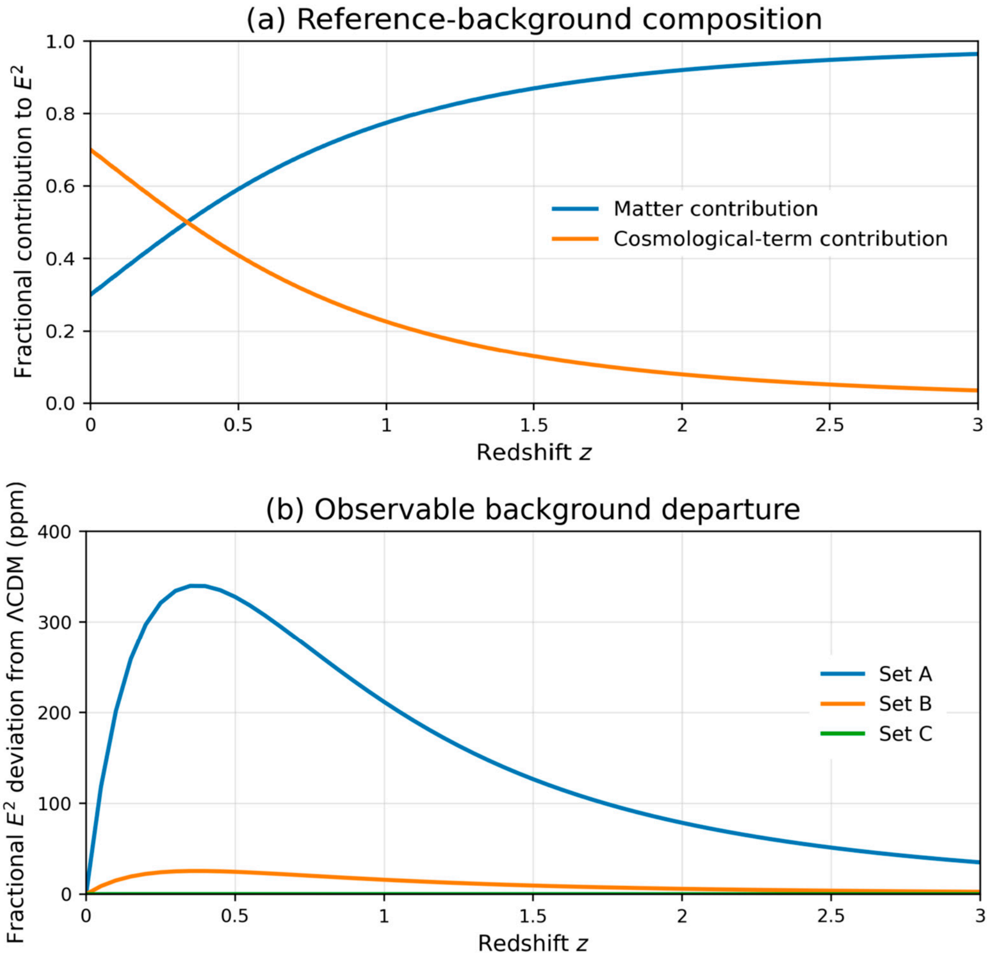

Figure 2. Reference-background composition and observable departure from ΛCDM. (**a**) Fractional contributions of matter and the cosmological term to the reference normalized expansion function $E^2_{\Lambda CDM}(z)$ over $0 \le z \le 3$, assuming $\Omega_{m,0} = 0.30$, $\Omega_{\Lambda,0} = 0.70$, and $\Omega_{r,0} = \Omega_{k,0} = 0$. (**b**) Fractional departure of the delayed-decay background from ΛCDM, calculated from Equation (27) using $a = (1+z)^{-1}$, for Set A ($n_f, a_t,\ s$)=(2.5, 5, 4), Set B (3, 10, 4), and Set C (4, 20, 5). Deviations are expressed in parts per million (ppm), corresponding to $10^6\left[E^2(z) - E^2_{\Lambda CDM}(z)\right]/E^2_{\Lambda CDM}(z)$.

Panel (a) shows that the cosmological term dominates the reference background near $z = 0$, whereas the matter contribution increases progressively with redshift and becomes dominant at earlier epochs. In panel (b), all three trajectories vanish at $z = 0$ because $F(1) = 1$. For $z > 0$, the delayed-decay parametrization gives $F\left[(1+z)^{-1}\right] > 1$, producing a positive fractional departure from the corresponding ΛCDM background. The deviation reaches a small maximum at low redshift and subsequently decreases as the matter term increasingly dominates the denominator of Equation (27). Set A produces the largest observable departure, followed by Sets B and C, consistently with the corresponding hierarchy in $n(1)$ and $w_0 + 1$. Even for Set A, the departure remains below approximately 400 ppm over $0 \leq z \leq 3$.

The same background expansion function also enters cosmological-distance observables through an integral over the expansion history. For example, the comoving distance is given by [31]:

$$D_C(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}, \qquad a = \frac{1}{1+z} \tag{28}$$

Here, $D_C(z)$ is the comoving distance, z is the redshift, z' is an integration variable, and $E(z')$ denotes the normalized Hubble function evaluated at $a' = 1/(1+z')$.

Background observations constrain the delayed-decay parametrization only over the finite redshift interval sampled by measurements of the expansion history and cosmological distances [29,31]. Near the present epoch, the leading departure from ΛCDM is governed by the present decay rate $n(1)$ defined in Equation (12), whereas the local variation depends on the correlated combination $sn_f a_t^s/(1+a_t^s)^2$ appearing in the CPL-like slope in Equation (26). Consequently, substantially different combinations of n_f, a_t, and s can produce nearly indistinguishable values of w_0, w_a, $H(z)$, and cosmological distances over the observable redshift interval. The three parameters are therefore not expected to be independently identifiable from background data alone. In particular, restrictions imposed on a_t or s can propagate directly into the inferred value of n_f, thereby affecting conclusions about the asymptotic future.

Because $a_t > 1$, the characteristic transition lies beyond the present epoch. Existing observations can test the low-redshift tail of the parametrization and constrain quantities such as $n(1)$, w_0, w_a, and $F(a) - 1$, but they cannot directly measure the asymptotic limit $n(a) \to n_f$. The future decay is therefore indirectly constrainable and prior-dependent rather than directly observed. Parameter constraints should accordingly be reported together with the adopted parameter ranges and the joint covariance among n_f, a_t, and s.

The same distinction between finite-redshift constraints and broader interpretations beyond ΛCDM is central in discussions of alternative cosmological scenarios, where observational viability does not by itself identify the microscopic origin of the dark sector [32]. More generally, the physical origin of cosmic acceleration remains open, and phenomenological parametrizations are useful only if they keep separate what is observationally constrained from what is theoretically inferred [33].

Recent DESI results motivate the exploration of dynamical dark energy models and parametrizations in which $w(z)$ may deviate from -1 [20–22]. However, these data do not demonstrate a decay of Λ_{eff}, nor do they establish that cosmic acceleration is transient. In the present work, they are therefore treated as observational motivation for dynamical parametrizations, not as confirmation of the proposed scenario.

5. Conclusions

This work has proposed a compact phenomenological parametrization in which the cosmological contribution inferred at the present epoch is interpreted as the present effective value, $\Lambda_{eff}(1) = \Lambda_0$, of a dark-sector component that remains close to ΛCDM over the recent cosmological epoch but is not required to remain constant in the remote future.

The main conceptual result is that the current dominance of the dark component does not require an increase in Λ_{eff}. Since the matter density dilutes as a^{-3}, an effective component that decays more slowly can become dominant even while decreasing in absolute value. This observation allows the present cosmic acceleration to be interpreted as a phase of dominance of an effective component, without automatically identifying the value observed today with an eternal fundamental constant.

The delayed-decay form introduced through $n(a)$ contains ΛCDM as the limiting case $n(a) = 0$, preserves qualitative compatibility with $w \simeq -1$ near $a = 1$, and allows $\Lambda_{eff}(a) \to 0$ as $a \to \infty$. In the strong version, defined by $n_f > 2$, the effective component does not asymptotically possess an equation of state sufficiently negative to sustain cosmic acceleration indefinitely.

For the adopted illustrative parameter sets and present-day density parameters, the return to decelerated expansion occurs at $a \simeq 7.06$, 11.89, and 20.00, whereas the fractional departure from the corresponding ΛCDM background remains below approximately 400 ppm over $0 \leq z \leq 3$. These results illustrate how expansion histories that are nearly indistinguishable around the present epoch can nevertheless imply markedly different asymptotic futures.

The parametrization remains deliberately phenomenological. No microscopic derivation of $\Lambda_{eff}(a)$ is proposed, and no claim is made that current data demonstrate a decay of the cosmological component. The specific contribution lies in the controlled separation between the present effective value and the asymptotic future behavior. This separation is implemented without specifying a scalar-field potential, a microscopic decay channel, or a fitted dark energy history, and is therefore intended as a minimal phenomenological layer rather than as a replacement for existing dynamical dark energy models.

Accordingly, the parametrization may also serve as a compact background-level benchmark for effective dark-sector or particle-cosmology scenarios that remain close to ΛCDM today but need not approach an asymptotic de Sitter state. In this sense, the work proposes a phenomenological distinction between Λ as an effective parameter inferred at the present epoch and Λ as an eternal fundamental constant, allowing a present expansion compatible with ΛCDM without necessarily implying a future de Sitter state.

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