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Bayesian analysis of marked stress release models for time-dependent hazard assessment in the western Gulf of Corinth

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Abstract

We consider point processes defined on the space–time domain which model physical processes characterized qualitatively by the gradual increase over time in some energy until a threshold is reached, after which, an event causing the loss of energy occurs. The risk function will, therefore, increase piecewise with sudden drops in correspondence to each event. This kind of behaviour is described by Reid's theory of elastic rebound in the earthquake generating process where the quantity that is accumulated is the strain energy or stress due to the relative movement of tectonic plates. The complexity and the intrinsic randomness of the phenomenon call for probabilistic models; in particular the stochastic translation of Reid's theory is given by stress release models. In this article we use such models to assess the time-dependent seismic hazard of the seismogenic zone of the Corinthos Gulf. For each event we consider the occurrence time and the magnitude, which is modelled by a probability distribution depending on the stress level present in the region at any instant. Hence we are dealing here with a marked point process. We perform the Bayesian analysis of this model by applying the stochastic simulation methods based on the generation of Markov chains, the so called Markov chain Monte Carlo (MCMC) methods, which allow one to reconcile the model's complexity with the computational burden of the inferential procedure. Stress release and Poisson models are compared on the basis of the Bayes factor.

Keywords: Bayesian inference; Markov chain Monte Carlo methods; Magnitude distribution; Point processes; Self-correcting models

1. Introduction

The stress release (SR) model was introduced by Vere-Jones in 1978; it transposes Reid's elastic rebound theory in the framework of stochastic point processes and expresses the probability of instantaneous occurrence as an increasing function of the stress level accumulated in a region, that is, its hazard function is time dependent in contrast to the constant hazard of the most generally used Poisson process. Although the SR model has been successfully applied to model sequences of strong earthquakes in Japan, New Zealand, Iran and China (Vere-Jones and Yonglu, 1988; Zheng and Vere-Jones, 1991, 1994), its physical basis, like all the models in the literature, does not completely explain the process generating earthquakes which, as is known, is considerably more complicated. In fact, according to the elastic rebound theory we would expect a large earthquake to be

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followed by a period of quiescence whereas in reality a strong earthquake may be succeeded by a period of activation and sometimes by another shock of comparable magnitude. An extension of the original model, the *linked* SR model, describes this clustering behaviour of large earthquakes in terms of stress transfer among interacting subregions of the area investigated (Lu et al., 1999; Bebbington and Harte, 2003; Rotondi and Varini, 2003b). The conjecture that an earthquake can accelerate or delay the following one because of the stress transfer due to short or long-range interactions concurs with recent research on the self-organized criticality of earthquakes.

Every point process model is univocally defined by its conditional intensity function, that is, the probability that an event will occur in an infinitesimal interval. A characteristic of SR models is that their intensity function at any time t depends on the entire history of the process up to that instant, that is, it is a function of the set $\{t_i, m_i\}_{i=1}^{n}$ of the occurrence times and magnitudes of the events recorded before t. Obviously, the longer and more complete the catalogue is used in its estimation, the better the results are. From the spatial viewpoint, the model requires the identification of regions that can be considered independent seismic units on the basis, for instance, of recognized geophysical subdivisions. Hence these models, on the basis of the occurrence of large earthquakes, aim at the long-term prediction of future shocks of comparable magnitude in the same area.

We have applied the original SR model to a single seismogenic zone, the western part of the Gulf of Corinth, in order to examine on a real test site the potential of the model and the issues involved in its application. An innovative element consists in the addition of the probability distribution of the magnitude dependent on the stress level and hence, in the assessment of a bivariate time-dependent hazard function. The zonation and the catalogue we have used are, as far as we know, the best data sets available for Greece. No previous studies applying SR models to forecast earthquakes in Greek seismic sources were found in the literature. Recent research related to earthquake prediction in Greece has mainly concerned precursory phenomena on an intermediate time-scale, magnitude, time and spatial distribution of foreshocks (Papadopoulos et al., 2000), seismic regularities (Papadimitriou and Sykes, 2001) and accelerated moment release (Papazachos and Papazachos, 2000).

The paper presents the model in Section 2 and the case concerned in Section 3. In Section 4 we illustrate the results obtained and indicate the weak points of this approach. The estimation problem is tackled in the Bayesian framework; the complexity of the model requires resort to recently developed procedures of stochastic simulation for inference, the Markov chain Monte Carlo methods. For a detailed explanation of these methods in this context we refer to Rotondi and Varini (2003a).

2. Stress release model

Let $\mathcal{H}_t = \{t_i, m_i\}_{i=1}^{n}$ denote the seismic history of an active region, that is, the set of events recorded in the time period (T_0, T_1) of magnitude m_i, $i=1, \ldots, n$, not less than a threshold M_0 and $t_i = T_i - T_0$, T_i being the date of the ith event. We assume that the probability of occurrence depends on an unobserved quantity $X(t)$ which increases linearly between two events and decreases suddenly when the events occur. This quantity may be interpreted as the stress present in the region at any instant and hence it is given by

$$X(t) = X(0) + ct - S(t),$$

sum of the stress $X(0)$ in the region at the initial instant and of the stress accumulated through the constant loading rate $c > 0$, subtracting the stress $S(t)$ released through earthquakes occurred up to t, that is $S(t) = \sum_{i:t_i<t} x_i$ where $x_i = 10^{\beta\cdot(m_i - M_0)}$ is, according to Benioff's formula, an approximation of the stress released by a shock of magnitude m_i. The β constant depends on the characteristics of the region, but is generally set at about 0.75.

To define a point process univocally one must assign what is called the conditional intensity function, that is the instantaneous probability of occurrence. In our case a mark, the magnitude, is also associated with each event. Consequently, to satisfy the assumption relating $X(t)$ and the occurrence probability, we must assign a bivariate conditional intensity function $\lambda(t, m)$ of the following type

$$\lambda(t, m) = \psi(X(t)) f(m|X(t)) \quad (1)$$

where $\psi(X(t))$ is a convex risk function increasing to infinity and $f(m|X(t))$ is the probability density function for the magnitude. In general the choice for this density falls on the exponential; considering the dependence on $X(t)$ we have chosen both right and left truncated exponential distribution on the domain $[M_0, M(t)]$:

$$f(m|X(t)) = \frac{\gamma e^{-\gamma m}}{e^{-\gamma M_0} - e^{-\gamma M(t)}} I_{[M_0, M(t)]}(m) \quad (2)$$

where $M(t)$ is the maximum magnitude that an event may have at time t when the stress level present in the region is $X(t)$. From Benioff's formula one obtains the following expression for $M(t)$:

$$M(t) = M_0 + \frac{\log_{10} X(t)}{\beta}$$

Table 1
Catalogue of the events considered of $M_S \geq 5$ from 1945

Date	Latitude	Longitude	M_S	Date	Latitude	Longitude	M_S
1953/06/13	38.10	22.60	5.1	1977/12/29	38.50	22.30	5.2
1962/01/19	38.35	22.25	5.3	1990/05/17	38.39	22.22	5.0
1964/04/21	38.50	22.25	5.0	1991/10/25	38.28	22.23	5.0
1964/10/16	38.50	22.25	5.2	1992/11/18	38.27	22.33	5.7
1970/04/08	38.30	22.60	5.9	1993/03/18	38.26	22.20	5.4
1970/04/20	38.30	22.60	5.3	1995/06/15	38.37	22.15	6.1
1972/06/15	38.20	22.10	5.1	1995/09/28	38.16	22.00	5.3
1974/12/02	38.40	22.20	5.1	1997/11/05	38.34	22.31	5.4
1975/04/04	38.10	22.10	5.6	2000/04/19	38.10	22.06	5.1
1975/05/19	38.30	22.40	5.2	2000/04/27	38.37	22.10	5.1

Other proposals presented in the literature (Kagan and Schoenberg, 2001; Vere-Jones et al., 2001; Kagan, 2002) concerning probability distributions for the size of the earthquake reduce the tail probability beyond an upper limit through an exponential fall-off. We have preferred a more drastic choice, setting at zero the probability of exceeding $M(t)$ in order to have a unique parameter γ to estimate.

For the risk function we have adopted the exponential functional form proposed in most of the articles in the literature:

$$\psi(X(t)) = \exp\{a + bX(t)\} \tag{3}$$

being $a \in \Re$ and $b \geq 0$ parameters to estimate. We can interpret b as the sensitivity of the region to risk; when $b=0$, the form (3) includes the special case $\psi(X(t))=\exp\{a\}$ corresponding to the Poisson process. We remark that $M(t_i)$ must be greater or equal to m_i for each $i=1,\ldots,n$, as the accumulated stress at time t_i must be such as to render the occurrence of an event of magnitude m_i possible. For each $i=1, \ldots, n$, the constraint $M(t_i) \geq m_i$ is equivalent to requiring that $X(t_i) \geq x_i$, that is

$$X(0) \geq x_i - ct_i + S(t_i) \quad \text{for } i = 1, \ldots, n. \tag{4}$$

Hence, having set $\overline{X}_c = \max_{i=1,\ldots,n}\{0; x_i - ct_i + S(t_i))\}$, we obtain the constraint $X(0) \geq \overline{X}_c$ between the parameters $X(0)$ and c.

The fitting of the model to the observations $(t_i, m_i)_{i=1,\ldots,n}$ is given by the likelihood function in the time interval $[t_0, t_{n+1}]$

$$\mathcal{L} = \prod_{i=1}^{n} \lambda(t_i, mg_i) \times \exp\left\{ -\int_{t_0}^{t_{n+1}} \int_{\mathcal{M}} \lambda(t, mg) \mathrm{d}t \, \mathrm{d}mg \right\}$$

where $\mathcal{M}$ is the domain of the magnitude of an event, $t_0=0$, $t_{n+1}=T_1-T_0$. Globally the parameters to estimate are a, b, $X(0)$ and c in the risk function and γ in the density of the magnitude. In the frequentist statistics their estimates can be given, for instance, by the respective values that maximize the likelihood. We work instead within the Bayesian framework in which the unknown value of a, b, $X(0)$, c, and γ is not fixed, but uncertain. Before observing the data, we express our degree of belief on each parameter value by assigning a probability distribution, the prior distribution; we then revise our beliefs in the light of the observations by applying Bayes' theorem to obtain our posterior distribution for each parameter. Summaries of this distribution are given by measures of location, such as

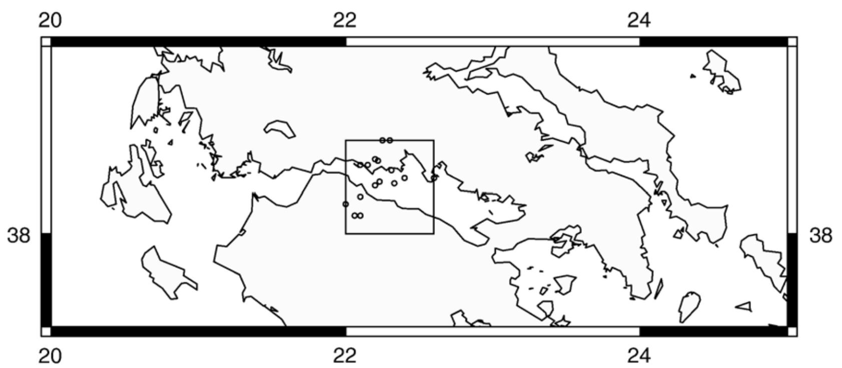

Fig. 1. Seismogenic zone and epicentres of the shocks of magnitude $M_S \geq 5$ in the 1945–2003 time interval.

the mean, which minimizes the expected quadratic loss, and by measures of spread, such as the credible intervals.

3. Seismogenic zone and data set

The area examined is located between latitudes 38.0°–38.5°N and longitudes 22.0°–22.6°E; it covers the western part of the Gulf of Corinth, a region of active normal faults. This zone is very similar to the seismogenic source 43 of the zonation proposed recently by Papaioannou and Papazachos (2000) for Greece and the adjacent area on the basis of seismological data as well as geological and geomorphological information. For a description of the fault systems and of the geotectonic units of the Hellenic arc including the area examined we refer to Mariolakos et al. (2001). Information on the seismicity of Greece exists since the 6th century BC, based on the macroseismic observations made by the ancient Greeks and Romans, followed by those of the Byzantines, but not all the data recorded in those catalogues have the three basic properties (completeness, homogeneity and accuracy) needed for reliable analysis. Within the framework of the ASPELEA project funded by the European Union (1997–2000), we began to analyze the catalogue of the Corinth gulf region assembled by the Institute of Geodynamics of the National Observatory of Athens (Papadopoulos, 1999), where the magnitude assigned to the events, when known, is the surface-wave magnitude. We have updated this data set using the catalogue available on the web page http://www.noa.gr/services/cat.html covering the period from 1964, the year in which the first electromagnetic type satellite stations were set up, to today. The resulting data set is reported in Table 1; it covers 20 events of $M_S \geq 5$ from 1945 on. The map of Fig. 1 shows the epicentres of the shocks.

Actually, five earlier events registered in the first half of the twentieth century were available, the last being that of May 31, 1939. However, there is no information indicating how much of the low activity recorded up to 1964, and in the gap between 1939 and 1953 in particular, must be interpreted as quiescence and how much as missing data. Lacking further evidence, we have set $t_0 = 1945$, the mean point of the gap, as the starting point of the interval under study, and postponed any discussion of the sensitivity of the model to variations of the starting date. Obviously, the fifty-year interval covered by the data set is quite short with respect to the periods of centuries considered in the articles on the same models that have appeared in the literature. Still, observing the list of shocks we can identify two clusters of events, separated by an interval of 12 years: the former going from 1962 up to 1977, or more briefly from 1970 to 1977, and the latter from 1990 to 2000. The stress release is slightly higher in the second period, as is the release rate.

Table 2
Prior distributions for the model parameters

Parameter	Prior	Mean	Variance	Prior parameters	
a	Normal	−5.5	6.25	Mean = −5.5	SD = 2.5
b	Gamma	0.012	0.0001	1.5	0.008
c	Gamma	0.3	0.25	0.36	0.8$\overline{3}$
γ	Gamma	2.3	1.	5.29	0.435
$X(0)$	Unif(0, 100)				

Table 3
Posterior mean and 90% credible interval for the model parameters

Parameter	Posterior mean	90% Credible interval
a	−2.386	(−4.110, −0.909)
b	0.016	($1.625 \cdot 10^{-4}$, 0.0326)
c	1.008	($1.369 \cdot 10^{-7}$, 2.064)
γ	2.898	(1.772, 3.894)
$X(0)$	58.208	(17.186, 99.921)

4. Results

To complete the definition of the model (1) we must now assign the prior distributions. One way of eliciting priors consists in drawing information on the parameters from an earlier, even if less complete, part of the catalogue. This is not possible in our case: the quality of the data before the twentieth century is too limited. We have therefore proceeded as follows: (a) we have set $X(0) < X_U$ with, for instance, $X_U = 100$, allowing in this way that the structures of the zone could generate earthquakes of a

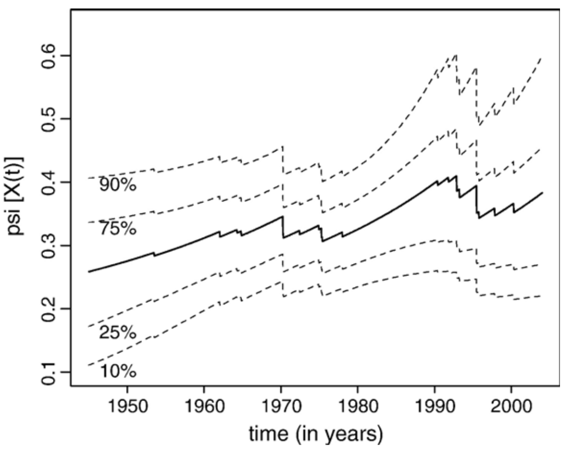

Fig. 2. Ergodic average (solid line) and 90%, 75%, 25%, 10% credible intervals of the $\psi(t)$ risk function.

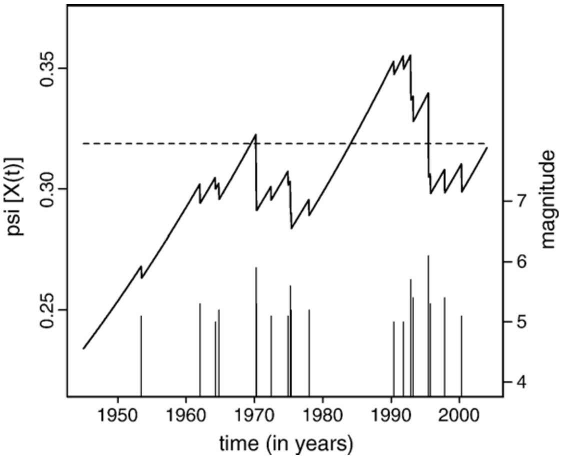

Fig. 3. Plug-in estimate of the $\psi(t)$ risk function. The bars denote the magnitude of the events and the horizontal line the constant risk of the Poisson model.

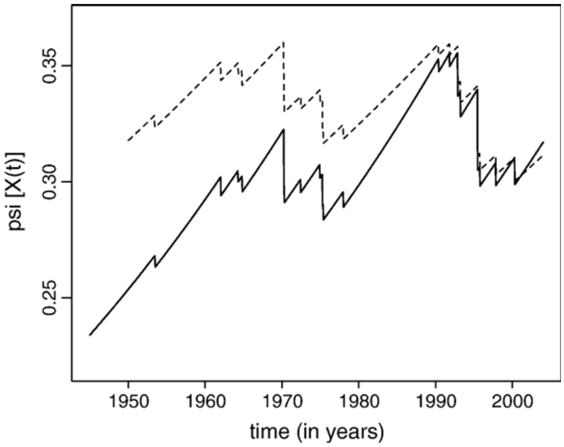

Fig. 5. Estimated risk functions over the intervals (1945–2003) (solid line) and (1950–2003) (dotted line).

magnitude even larger than 7.5; (b) we have collected the estimates of the parameters found in the literature (see, for example, Zheng and Vere-Jones, 1991, 1994), then varying $X(0)$ in $\mathcal{X} = (0, X_U)$; and exploiting the relationships which relate the four parameters a, b, c and $X(0)$, we have obtained the variability ranges for a, b, and c; (c) finally the γ parameter is related to the Gutenberg-Richter formula by the expression $\gamma = b_{GR} \log 10$, which b_{GR} parameter, it is known, generally assumes values between 0.8 and 1.2. On the basis of these considerations we have assigned the prior distributions summarized in Table 2.

With the application of Bayes' theorem the computation of the posterior distributions implies multidimensional integrals that cannot be solved analytically. Consequently we have decided to use recent methods based on the stochastic simulation of Markov chains in order to explore those distributions. We denote by π a distribution to estimate; the Markov chain Monte Carlo (MCMC) methods generate a Markov chain with invariant distribution π, and its sample-path averages are used to estimate the characteristics of π. We refer to Gilks et al. (1995) for details on MCMC theory and algorithms, and to Rotondi and Varini (2003a) for the issue of data-constrained parameters that arises in making inference of our model. In the present case we have generated a chain of 20 million elements for each parameter, deleted the initial 20% of scans, and recorded the output every 500th iteration. On the resulting 32,000 values we have computed the posterior means and the 90% credible intervals given in Table 3. The convergence of the thinned chains has been checked by the

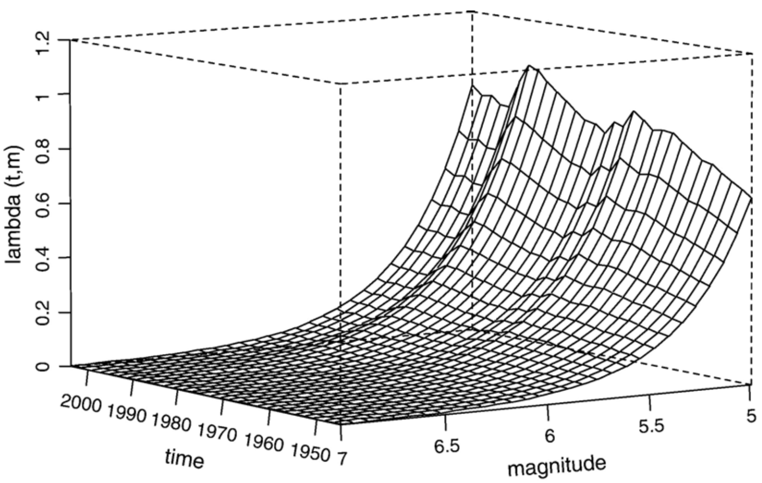

Fig. 4. Plug-in estimate of the $\lambda(t, mg)$ conditional intensity function.

principal diagnostics implemented in the free S-Plus package BOA, version 0.5.0 (Smith, 2000).

The MCMC method allows us to estimate the risk function pointwise through its ergodic average $\tilde{\psi}(X(t))=1/\eta\sum_{i=1}^{\eta}\psi^{(i)}(X(t))$, where $\psi^{(i)}$ denotes the value taken by the risk function at the ith iteration and η is the total number of the iterations. The average $\tilde{\psi}(\cdot)$ and some sampled quantiles of the variable $\psi(\cdot)$ at any time t are shown in Fig. 2. Another possible result for the risk function is given by its plug-in estimate $\hat{\lambda}$, obtained by substituting the posterior means of the parameters in expression (3). This estimate is represented in Fig. 3 together with the data set. We have estimated the bivariate conditional intensity function (1) in the same way using the posterior mean of the γ parameter in its evaluation as well; the result is shown in Fig. 4.

The choice of the time and magnitude thresholds and the completeness and reliability of the data are related problems. So far we have performed the analysis setting $t_0=1945$ as our starting date; this choice has resulted in the globally increasing trend of the stress level, and consequently in the risk function we can observe in Fig. 3. To try and check the influence of the starting time, the data were refitted from a starting point 5 years later. The two estimated risk functions are compared in Fig. 5. The two curves seem to approach each other after an initial period for the stabilization of the models, but we cannot be sure this behaviour will continue in future. Roughly speaking we would say that for the model over the 1950–2003 interval the two active periods, 1962–1977 and 1990–2000, are comparable. Hence the system has now finished a release period and is entering a recovery period. Instead for the model over the 1945–2003 interval, the current active period is not yet at an end: further events must be expected in the short-term. Discriminating between these two conjectures is only possible through a judgement on the completeness of the catalogue.

The role played by the Poisson model in seismic hazard assessment makes it a natural reference model with which to compare the other models. To do so we have repeated the analysis, setting the b parameter at zero (see Section 2); the MCMC algorithm provides the value -1.1434 as the posterior mean of the a parameter, and hence the estimated constant risk of the Poisson model is $\exp(a)=0.3187$. It is represented in Fig. 3 by the horizontal line. The comparison between the SR and the Poisson model can be given by the ratio of their marginal likelihoods with respect to the prior distributions of the parameters, obtained by the harmonic means of the respective likelihoods $\left\{1/\eta\sum_{i=1}^{\eta}\mathcal{L}_i^{-1}(M)\right\}^{-1}$, η being the number of iterations and $\mathcal{L}_i(\mathcal{M})$ the value taken by the likelihood of the $\mathcal{M}$ model ($\mathcal{M}_{SR}$ for the stress release model, $\mathcal{M}_P$ for the Poisson model) at the ith iteration. This ratio coincides with the Bayes factor if we assume that the prior probability of the two models is the same. In our case we have

$$\frac{\Pr(\mathcal{M}_{\mathrm{SR}}|\mathrm{data})}{\Pr(\mathcal{M}_{\mathrm{P}}|\mathrm{data})}=\frac{2.4548\cdot10^{-18}}{1.8711\cdot10^{-18}}=1.3119$$

which corresponds to a slight evidence in support of the SR model.

5. Conclusions

We have analyzed the set of earthquakes of $M_S\geq5$ in the western zone of the Gulf of Corinth starting from 1945, and evaluated the seismic hazard through a non-stationary point process model. From the methodological viewpoint we underline the dependence of the distribution of the magnitude on the stress level present in the area at any instant. The application of recently developed methods of stochastic simulation for statistical inference has allowed us to provide measures of the uncertainty on the estimates of both the parameters and the risk and intensity functions. We have examined the sensitivity of the model to the choice of the time threshold, and have also compared the stress release with the Poisson model through the ratio of their marginal likelihoods. The value obtained slightly favours the non-stationary model, but the small number of events in the study advices caution in accepting a conclusion based on such a provisional basis. When we consider the components of this study, that is, the statistical tools and geological knowledge involved, we find the former quite powerful and commensurate with the complexity of the problem. What is lacking is the latter, the information and data that can provide a more complete and reliable estimate of the hazard. We hope that the positive, albeit limited, results described here will induce the field experts to take these models into consideration for thorough analyses of hazard assessment.

Acknowledgments

The map was produced using the GMT software (Wessel and Smith, 1998).We are grateful to the referees for their constructive comments.

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