A NEW GENERAL MODEL FOR THE EVOLUTION OF THE SPIN VECTOR OF THE TWO LAGEOS SATELLITES AND LARES AND THE LARASE RESEARCH PROGRAM

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ABSTRACT

We present a new general model to calculate the evolution of the spin of passive, conductive and spherical satellites such as the two LAGEOS and LARES. The knowledge of the spin, both in orientation and rate, is of fundamental importance to model correctly the thermal effects on the surface of these satellites, as in the case of the solar Yarkovsky-Schach effect and of the Earth's Thermal drag. These are very important non-gravitational perturbations that produce long-term effects on the orbit of the cited satellites. Therefore, the improvement of the accuracy of the models developed to handle these perturbations represents a very significant result. Such enhancements, with the possibility of a more reliable orbit determination for the satellites, will be very useful in the field of General Relativity measurements with laser-ranged satellites, as well as in the fields of space geodesy and of geophysics.

1 Introduction

The LAser RAnged Satellites Experiment (LARASE) main goal is to provide accurate measurements for the gravitational interaction in the weak-field and slow-motion limit of General Relativity (GR) by means of a very precise laser tracking of geodetic satellites orbiting around the Earth (the two LAGEOS and LARES). Beside the quality of the tracking observations, also the quality of the dynamical models implemented in the Precise Orbit Determination (POD) software plays a fundamental role to obtain precise and accurate measurements. The models have to account for the perturbations due to both gravitational and non-gravitational forces in such a way to reduce as well as possible the difference between the observed range, from the tracking, and the computed one, from the models.

The main effects on the orbit that we faced so far under the LARASE project are those of thermal origin, those due to neutral drag and, finally, those due to Solid and Ocean tides. The thermal perturbations on the orbit are from the Sun (with the solar Yarkovsky-Schach effect) and from the Earth (with the Earth-Yarkovsky effect, also known as Thermal drag or Rubincam effect) see (Rubincam, 1982) (Rubincam, 1988) (Rubincam, 1990) (Rubincam, et al., 1997) (Afonso, et al., 1989) (Scharroo, et al., 1991) (Farinella & Vokrouhlicky, 1996) (Slabinski, 1997) (Farinella, et al., 1990) (Habib, et al., 1994) (Lucchesi, 2002) (Lucchesi, et al., 2004) (Métris, et al., 1997) (Métris, et al., 1999) (Andrés de la Fuente, 2007).

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These disturbing effects depend from the behavior of the spin of the satellite, hence the knowledge of the spin evolution is of paramount importance in order to model efficiently these perturbations. The spin models developed in the past, and currently available in the literature, were restricted to the so-called rapid spin approximation, that is to say they are valid only when the rotation period of the satellite is much smaller than the orbital period and of the Earth's rotational period. Therefore, within the activities of LARASE, we started to review the previous spin models. We first developed our own spin model in the rapid spin approximation, and then we finally got to a new general model.

2 Dynamical parameters of the satellites

A first step in developing our spin model was to collect, when available, or calculate if possible, the values of the dynamic parameters for the geodetic satellites LAGEOS, LAGEOS II and LARES. In fact, the goodness of a dynamical model, strongly depends on the accurate knowledge of the values of quantities such as the moments of inertia or the resistivity of the materials. For the LAGEOS satellites, part of this information was found in the technical documentation describing the satellites structure and characteristics, other information has been numerically calculated using a realistic 3D model specifically built for each satellite. These models will be also used in future studies and simulations, for instance to assess the thermal exchange between the different components of the satellites. One of the main result of this information collection for LAGEOS satellites was the solution of the problem concerning the material used (see Table 1), the dimensions of the different parts (see Figure 1) and the calculation of the moments of inertia in fly arrangement (see Table 2) (Visco & Lucchesi, 2016).

Satellite	hemispheres	Core	Stud
LAGEOS	Al 6061	Brass	Cu-Be
LAGEOS II	Al 6170	Brass	Cu-Be

Table 1 – Material used to build the different parts of LAGEOS satellites

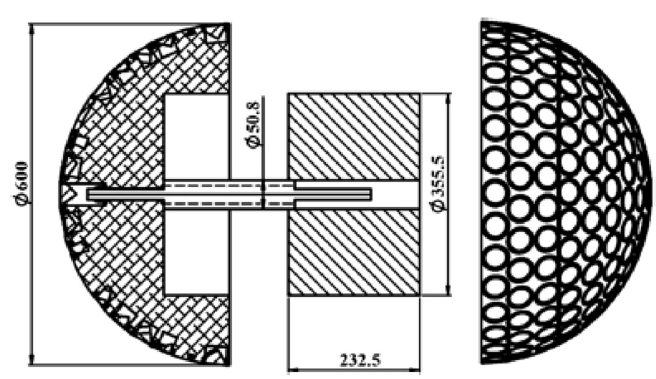

Figure 1- LAGEOS and LAGEOS II section. The dimensions are in mm.

Satellite	I_x (kg m^2)	I_y (kg m^2)	I_z (kg m^2)
LAGEOS	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

Table 2 - Moments of inertia of the LAGEOS satellites calculated using a 3D model. The x axis coincides (nominally) with the symmetry axis of the satellite.

Till now we were not able to recover the full documentation for LARES's dynamic characteristics. Its geometry is much more simple than that of LAGEOS and it is well documented in the technical papers, that contain also a good report on materials used, but to our knowledge it is not reported a measurement of its moments of inertia in the literature. This parameters are critical for the spin modeling and therefore we calculated them with a method similar to that adopted for LAGEOS (Visco & Lucchesi, 2016).

3 LARASE spin model

In the past, different models built to describe the spin behavior of LAGEOS satellite were published in the literature. The authors of the first model were (Bertotti & Iess, 1991), followed by (Habib, et al., 1994), (Farinella, et al., 1996), (Vokrouhlicky, 1996) and. (Andrés de la Fuente, 2007) (Andrés, et al., 2004) with the development of LOSSAM model. These early studies, with the exception of Habib et al., attack and solve the problem of the evolution of the rotation of a satellite in a terrestrial inertial reference system in the so-called *rapid spin approximation*. In these studies were introduced equations for the external torques that have been averaged over characteristic time intervals, as the orbital period and the day. Averaged models, especially LOSSAM, well match the observations, while the model by Habib et al., using a body-fixed reference frame and not-averaged torques, fails to fit the observations properly. Our target was to build a dynamical spin model well-fitting the available experimental data and that is able to predict the future trend of the spin evolution in the general case, and not only in the fast spin approximation.

We have deeply reviewed the previous spin models. The first step of our project involved the construction of a model for the *rapid spin approximation*. This first model has been useful to compare our results with those obtained using the analogous models developed in the past and, at the same time, to validate our *general model* in the regime where both the models can be applied.

The general model was built considering *non averaged torques* and solving the Euler equations in the body frame in order to better describe the misalignment between the symmetry axis and the spin. These equations are:

$$I_x\dot{\omega}_x - \left(I_y - I_z\right)\dot{\omega}_y\dot{\omega}_z = M_x$$
$$I_y\dot{\omega}_y - (I_z - I_x)\dot{\omega}_x\dot{\omega}_z = M_y$$
$$I_z\dot{\omega}_z - \left(I_x - I_y\right)\dot{\omega}_x\dot{\omega}_y = M_z$$

where I_x, I_y and I_z are the principal moments of inertia, ω_x, ω_y, ω_z, and M_x, M_y, M_z are, respectively, the projection of the angular velocity $\boldsymbol{\omega}$ and of the total torque $\boldsymbol{M}$ on the principal inertial axes of the satellite. We adopted as variables the Euler angles with respect to the Earth Centered Inertial (ECI) reference frame, and angular velocities and torques were expressed in term of them. In our model we considered all the main torques introduced in the past, in analogy with the choice of (Andrés de la Fuente, 2007).

The first torque that we considered is the one due to the interaction between the Earth Magnetic field and the magnetic moment induced, by the same field, in the satellite. We adopted a time dependent value for magnetic field along the orbit, and not the average value as done in the past by all the authors. The magnetic field along the orbit has harmonic components at twice the mean motion n of the satellite and its combinations with the Earth's rotational angular velocity $\omega_\oplus$. The expression for the magnetic field is:

$$\boldsymbol{B_E} = \sum_{i=0}^{8} \boldsymbol{B_i} \cos(\omega_i t + \varphi_i)$$

where

$$\omega_0 = 0,\ \ \omega_1 = \omega_2 = \omega_\oplus - 2\,n,\ \ \omega_3 = \omega_4 = \omega_\oplus + 2\,n,$$
$$\omega_5 = \omega_6 = 2\,n,\ \ \omega_7 = \omega_8 = \omega_\oplus$$

and

$$\varphi_i = \pi/2 \text{ for } i = 2,4,6,8 \ \ \varphi_i = 0 \text{ for } i = 1,3,4,7\ .$$

The torque that originates from this magnetic field shows components at $2\omega_i$ (Visco & Lucchesi,).

The second torque considered was that produced by the monopolar gravitational field of the Earth on a not spherically symmetric satellite (Beletskii, 1966):

$$\boldsymbol{M_{grav}} = 3\,\omega_\oplus^2 \left\{\boldsymbol{s} \wedge \left[I_x s_x \boldsymbol{x} + I_y s_y\, \boldsymbol{y} + I_z s_z\, \boldsymbol{z}\right]\right\}$$

where **s** is the unit vector along the Earth-satellite direction having projections s_x, s_y, s_z along the principal inertia axes of the satellite.

Finally, we considered two torques due to the sun light pressure over the satellite surface. The first of these torques is due to the misalignment between the geometrical center of application of the solar radiation pressure, and the center of mass of the satellite (Vokrouhlicky, 1996):

$$\boldsymbol{M_{off}} = \pi\, R^{\boldsymbol{2}}\, \frac{\Phi_\odot}{c}\, C_R(\boldsymbol{h} \wedge \boldsymbol{s}_\odot)$$

This torque, whose effects are modulated by the satellite's eclipses from the Earth, depends from the satellite radius R, from the solar constant $\Phi_\odot$, from the radiation coefficient C_R (related with the reflectivity of the surface of the satellite), and finally from the cross product between the vector $\boldsymbol{h}$ from the center-of-mass to the geometrical center of the satellite and the satellite-Sun unity vector $\boldsymbol{s}_\odot$.

The last torque considered, which is present in the case of the two LAGEOS satellites, depends from the relative difference $\Delta\rho$ between the reflectivity of the two hemispheres of the satellite along the principal axis $\boldsymbol{z}$ (Vokrouhlicky, 1996):

$$\boldsymbol{M_{ar}} = \frac{2}{3}\, R^{\boldsymbol{3}}\, \frac{\Phi_\odot}{c}\, \Delta\rho\ \ C_R\, (\boldsymbol{z} \wedge \boldsymbol{s}_\odot) \left|\boldsymbol{z} \wedge \boldsymbol{s}_\odot\right|$$

4 – Results and conclusions

We have built a code based on MATLAB routines to solve numerically the Euler motion equations with the torques introduced in section 3 and we compared the time evolution of the spin obtained with the available measurements. By way of example, in Figures 2 and 3 the results obtained for LAGEOS both for the general model and for the rapid spin average model are shown. As we can see, there is a good agreement between the model predictions with the available measurements. A reliable model for the spin is a very important tool and represents the first step to model the effects due to thermal forces on the dynamics of the two LAGEOS and LARES. Work is currently underway to reach a better understanding of the temperature distribution on these satellites.

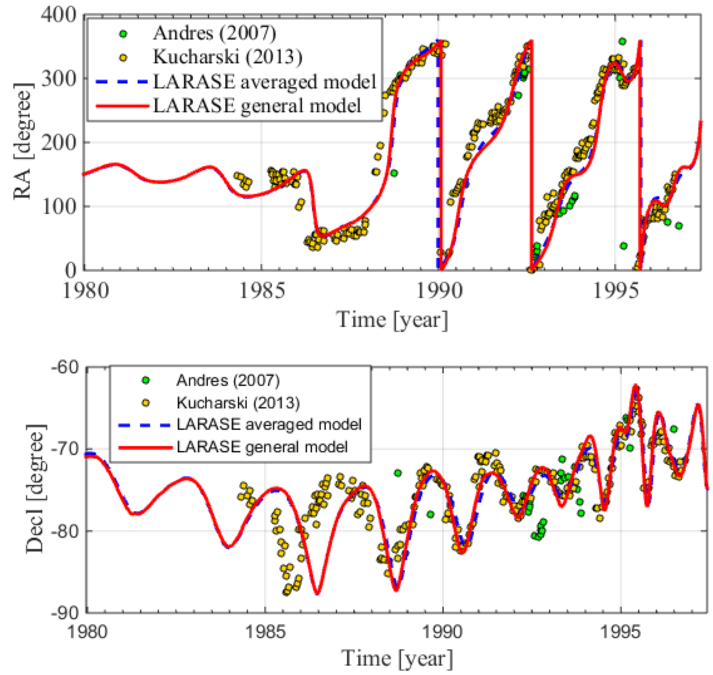

Figure 2 - Time evolution of the right ascension and declination of the spin vector of LAGEOS. Two set of measurements were considered (Andrés de la Fuente, 2007) (Kucharski, et al., 2013).

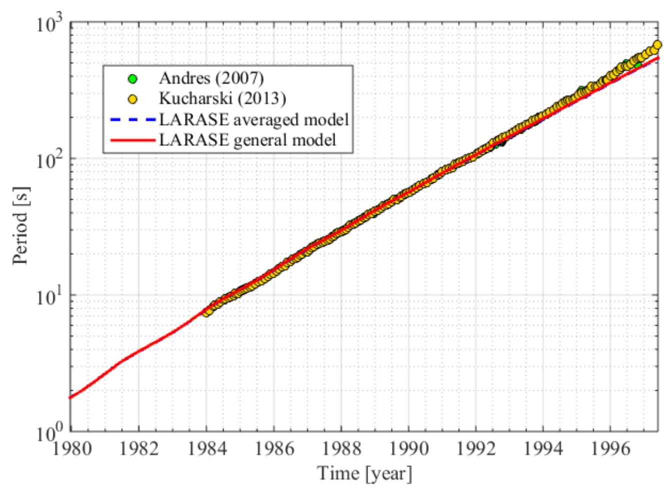

Figure 3 Time evolution of the rotational period of the spin vector of LAGEOS. Two set of measurements were considered (Andres de la Fuente, 2007) (Kucharski, et al., 2013).

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