SIMAI 2000

V Congresso Nazionale della SIMAI
Società Italiana di Matematica Applicata e Industriale

Ischia Porto
Centro Congressi Hotel Continental Terme
05-09 giugno 2000

SOMMARI
ABSTRACTS

SIMAI
Società Italiana di
Matematica Applicata e Industriale
C.P. 385 - 00100 ROMA Centro

Restauro Cieco di Immagini mediante Regolarizzazione Edge-Preserving
Blind Image Restoration through Edge-Preserving Regularization

Anna Tonazzini and Luigi Bedini
Istituto di Elaborazione della Informazione - CNR
Area della Ricerca di Pisa
Via Alfieri, 1, I-56010 Ghezzano (Pisa), Italy
surname@iei.pi.cnr.it

1 Introduction

Blind image restoration, i.e. the simultaneous estimation of the restored image and the parameters of the imaging system (blur coefficients plus noise statistics) is highly underdetermined, so that the adoption of constraints for both the image and the blur becomes necessary. A widely experimented approach consists in forcing positivity and/or size constraints for the image and the blur, iteratively and alternately estimated through inverse filtering, Wiener filtering or simulated annealing [1] [8]. Although useful to reduce the number of admissible solutions, positivity and size constraints are not enough. Since the most part of the information needed for blur identification is located across the discontinuity edges, it can be expected that further improvement can be achieved by exploiting image models which allow for an accurate edge location [10]. For instance, Markov Random Field (MRF) models with explicit, interacting line processes are very flexible in incorporating constraints on the geometrical behaviour of the intensity discontinuities, thus allowing for a more reliable detection of edges [3]. Nevertheless, MRF models are characterized by a set of free parameters (hyperparameters or Gibbs parameters), that must be carefully tuned in order to get effective results. Although the functional form of the Gibbs prior is usually known, the *a priori* information is not enough to determine these values, that are usually selected empirically. A realistic and challenging approach to restoration consists thus in considering both the image field and all the parameters (blur coefficients, noise statistics and model hyperparameters) as variables of the problem, to be jointly estimated from the data. Unfortunately, this approach has an intractable computational complexity, even for small size problems. Thus, some form of reduction must necessarily be adopted. To this purpose, in this paper we adopt a fully Bayesian approach which enables to split the problem into a sequence of less complex MAP estimations of the image field and ML estimations of the degradation and model parameters. Furthermore, we show that the presence of an explicit, binary line process allows for the adoption of suitable approximations that greatly reduce the computational costs. At the same time, we show that the incorporation of edge constraints produces better image reconstructions and blur estimates.

2 The Method

We assume the coupled MRF image model in the form of a prior Gibbs distribution $P(f,l)$ of the intensity process f and the line process l:

$$P(f,l) = \frac{1}{Z(w)} \exp\{-U(f,l)\} \qquad U(f,l) = \sum_r w_r V_r(f,l) \tag{1}$$

where $Z(\mathrm{w})$ is the partition function and the prior energy $U(f,l)$ is given by the scalar product between the hyperparameter vector w and the vector of the clique potential functions $V_r(f,l)$. Taking the common assumption of a noise process whose components are independent, white and Gaussian, with zero mean and variance σ^2, the likelihood function, which models the data formation process, is:

$$P(\mathrm{g}|f) = (2\pi\sigma^2)^{-(mn)/2} \exp\left\{\frac{-\|\mathrm{g} - H\mathrm{f}\|^2}{2\sigma^2}\right\} \tag{2}$$

where $m \times n$ is the image size, f is the vector of the lexicographically ordered notation for f, g is the vector of the data, and H is the matrix associated with the linear operator. When the degradation operator is a blur, H is a block Toeplitz matrix, whose elements derive, according to a known rule, from a usually small size mask d. Let us rewrite H as $H(d)$ and call q the set of the degradation parameters d and σ^2. A fully Bayesian approach to reconstruct the image, estimate the "best" Gibbs parameters, and accomplish the degradation process identification is to assume a uniform prior distribution for w and q, and then simultaneously compute (f,l), w and q, by maximizing the joint distribution $P(f,l,\mathrm{g}|\mathrm{w},\mathrm{q})$:

$$\max_{f,l,\mathrm{w},\mathrm{q}} P(f,l,\mathrm{g}|\mathrm{w},\mathrm{q}) \tag{3}$$

The joint maximization (3) is a very difficult task. Nevertheless, the separability of the degradation and model parameters allows for the adoption of the following sub-optimal iterative procedure:

$$(f^k, l^k) = arg\max_{f,l} P(f,l|\mathrm{g},\mathrm{w}^k,\mathrm{q}^k) \tag{4}$$

$$\mathrm{q}^{k+1} = arg\max_{\mathrm{q}} P(\mathrm{g}|f^k,\mathrm{q}) \tag{5}$$

$$\mathrm{w}^{k+1} = arg\max_{\mathrm{w}} P(f^k,l^k|\mathrm{w}) \tag{6}$$

that produces a sequence $(f^k, l^k, \mathrm{w}^k, \mathrm{q}^k)$ converging to a local maximum of $P(f,l,\mathrm{g}|\mathrm{w},\mathrm{q})$. It is straightforward to see that the final solution is adaptively obtained by the iterative execution of MAP estimations for (f,l), based on the current parameters, and ML estimations for w and q, based on the current image, in turn [7]. This formulation greatly simplifies the original problem and makes it possible the adoption of relatively cheap and inherently parallel algorithms. With respect to the MAP estimation of the image field for fixed values of the parameters (step (4)) several existing algorithms can be adopted [6] [5] [3], depending on the form of the distribution. Here, owing to the presence of an explicit binary line process, we propose to use the mixed-annealing algorithm [4] in which the continuous variables are updated by deterministic schemes such as gradient descent techniques, while the binary variables are updated in a stochastic way, by means of a low cost binary Gibbs sampler [6]. We are currently studying the possibility to exploit preconditioners for Toeplitz matrices, in order to reduce the number of iterations required by the conjugate gradient in the computation of the intensity process f [2]. Problem (5), i.e. the estimation of the degradation parameters can be reduced to the two following computational steps:

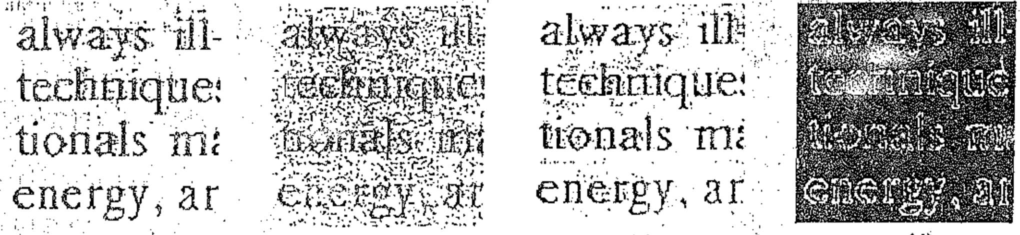
$$d^{k+1} = arg\min_{d} \|\mathrm{g} - H(d)\mathrm{f}^k\|^2 \tag{7}$$

$$(\sigma^2)^{k+1} = \frac{\|\mathrm{g} - H(d^{k+1})\mathrm{f}^k\|^2}{mn} \tag{8}$$

The hyperparameter estimation step (6) entails maximizing the prior distribution of the image field computed in the current image estimate (f^k, l^k). By reformulating this ML estimation problem in terms of the negative log-prior, the minimum can be found as the zero of the gradient, by solving the related system of equations through gradient descent. This leads to the following iterative updating rule:

$$w_r^{t+1} = w_r^t + \eta\left\{E_{\mathrm{w}^t}\left[V_r(f,l)\right] - V_r(f^k,l^k)\right\} \tag{9}$$

starting with $\mathrm{w}^0 = \mathrm{w}^k$. In (9) η is a positive, small parameter, to be suitably chosen to ensure convergence, and E_{w} is the expectation computed with respect to the prior distribution. Unfortunately, the computational charge of the iterative scheme (9) as it stands is unsurmountable, due to the computation of the expectations,



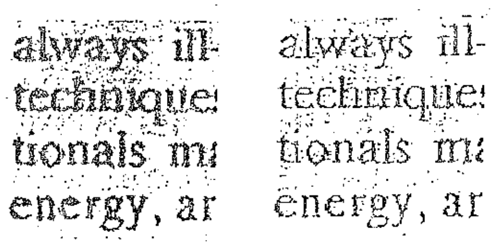
(a) (b) (c) (d)

Figure 1: 128×128 real image, artificially degraded: (a) original image; (b) image degraded by Gaussian noise with $\sigma = 25$; (c) restored image; (d) related edge map.

or equivalently of the partition function, that requires summation over binary variables and integration over continuous variables. Thus, some amount of approximation is unavoidable. Among the others, maximum pseudo-likelihood (MPL), the coding method and the mean field approximations are the most popular [7]. Based on the correlation between intensities and lines, we assume that the line process alone can retain a good deal of information about the hyperparameters that best model the whole image. This allows for the adoption of approximations, such as the saddle point approximation, to feasibly approximate the partition function. We showed that this leads to base the estimation on the conditional prior $P(l|f^k, \mathbf{w})$ rather than on the joint prior $P(f, l|\mathbf{w})$. This means that the expectation in (9) can be substitute with $E_{\mathbf{w}}\left[V_r(f^k, l)\right]$, whose computation requires now summation over binary variables only, since f^k is already available from step (4). If the same assumption to keep f always clamped to f^k is made in the context of a Monte Carlo algorithm, similar performance in terms of computation saving can be obtained when Monte Carlo Markov Chain techniques are applied to approximate, via time averages, the ratio between partition functions [9]. In both cases, the time averages can be computed by means of a low cost binary Gibbs sampler. As a final observation, when the line process is non-interacting the expectations can be computed by means of analytical calculations, with further, significant reduction of the computational cost.

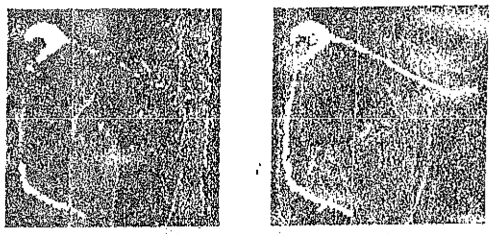
3 Numerical Experiments

The performance of the fully data driven restoration method proposed here was analyzed on piecewise smooth real images, considering, as image model, an isotropic, homogeneous prior energy that accounts for a useful line continuation constraint. For blurred data we started the process from a blur mask in the form of a Dirac function, that represents the most non-informative starting point, and the degraded image itself as initial guess for the restored image. The line process was set to zero everywhere, while the initial hyperparameters were roughly chosen in such a way to give a threshold higher than the noise standard deviation. In all trials convergence of the parameters and stabilization of the reconstructions were reached in less than 30 iterations of the whole procedure. In a first experiment we only added some noise ($\sigma = 25$) to the original image (see Figure 1), and started the process with a uniform blur mask. At convergence, we get a Root Mean Square Error (RMSE) of 0.0032 between the estimated blur mask and the ideal blur mask (that was a Dirac function in this case), and an estimated standard deviation $\sigma = 22.85$. As it can be appreciated also by the inspection of the edge map, the quality of the reconstruction is excellent, thus indicating that the adaptive tracking of the image model parameters was effective in this case. In a second experiment we degraded the same image of Figure 1(a) by a 3×3 non-uniform blur mask, plus addition of modest noise ($\sigma = 5$)(see Figure 2). The obtained results are very similar to those of the only noisy case, with a RMSE for the estimated blur mask of 0.0044. Nevertheless, we found that, for blurred image, the quality of the reconstructions quickly deteriorates as long as the noise level increase. We also experimented our procedure on a 200×200 already degraded microscope image, that represents an isolated euglena photoreceptor (see Figure 3). Although we know that the theoretic Point Spread Function of a microscope can be expressed in the form of an Airy diffraction pattern, in practical situations the true blur mask depends on a number of factors, including light level. Thus, in this specific case we did not know the exact size of the mask and the values of the blur coefficients. We thus run



(a) (b)

Figure 2: 128 × 128 real image, artificially degraded: (a) image degraded by convolution with a 3 × 3 non-uniform mask plus Gaussian noise with $\sigma = 5$; (b) restored image.



(a) (b)

Figure 3: Real degradation: (a) 200 × 200 original image; (b) reconstructed image.

the blind unsupervised image restoration procedure assuming a 7 × 7 size to be sufficient. The satisfactory quality of the reconstructed image validates the choice for the blur size, which is, at the moment, the only *a priori* information requested by the method.

References

[1] G. R. Ayers and J. G. Dainty. Iterative blind deconvolution method and its applications. *Opt. Lett.*, 13:547–549, 1988.

[2] L. Bedini, G. M. D. Corso, and A. Tonazzini. Preconditioned edge-preserving regularization for image restoration. Techn. report, IEI-CNR, 1998.

[3] L. Bedini, I. Gerace, E. Salerno, and A. Tonazzini. Models and algorithms for edge-preserving image reconstruction. *Advances in Imaging and Electron Physics*, 97:86–189, 1996.

[4] L. Bedini and A. Tonazzini. Image restoration preserving discontinuities: the Bayesian approach and neural networks. *Image and Vision Computing*, 10:108–118, 1992.

[5] A. Blake and A. Zisserman. *Visual Reconstruction*. MIT Press, Cambridge, MA, 1987.

[6] S. Geman and D. Geman. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Trans. Pattern Anal. Machine Intell.*, 6:721–740, 1984.

[7] S. Lakshmanan and H. Derin. Simultaneous parameter estimation and segmentation of Gibbs random fields using simulated annealing. *IEEE Trans. Pattern Anal. Machine Intell.*, 11:799–813, 1989.

[8] B. C. McCallum. Blind deconvolution by simulated annealing. *Optics Commun.*, 75:101–105, 1990.

[9] A. Tonazzini, L. Bedini, and S. Minutoli. Joint MAP image restoration and ML parameter estimation using MRF models with explicit lines. In *Proc. IASTED-SIP'97*, New Orleans.

[10] Y. You and M. Kaveh. A regularization approach to joint blur identification and image restoration. *IEEE Trans. Image Proc.*, 5:416–428, 1996.